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GEOMAGNETISM

BY

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VOLUME II

ANALYSIS OF THE DATA, AND PHYSICAL THEORIES

> Magnus magnes ipse est globus terrestris.
> W. GILBERT, *De Magnete*, 1600.

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PART 2

THE ANALYSIS AND SYNTHESIS OF GEOMAGNETIC DATA

IN the study of geophysical phenomena, an intermediate stage is usually required between the making of the actual observations, and the discussion of the physical causes of the observed phenomena; the data obtained must be both analysed, by statistical or other methods, and also synthesized, so that particular features of the phenomena, as observed at many points on the earth, may be conveniently represented by diagrams or mathematical formulae.

In laboratory physics, experiments can be carefully planned and executed with a view to the settlement of definite isolated questions; in such work the intermediate analysis and synthesis of the data are seldom needed. It is a necessary stage, however, in geophysics, and particularly in geomagnetism, because Nature continually makes a multitude of 'experiments' simultaneously upon the earth, as, for example, by the sun and moon. The results of these many influences must be observed over long periods at many stations distributed over the earth, and then the records obtained must be patiently studied, in order to disentangle the various individual processes. Order must be found among the constant vagaries of geomagnetic phenomena, by applying suitable statistical methods to adequate amounts of observed data; and even the variability must be made to elucidate the physical causes in operation.

Statistical studies in geomagnetism are closely associated with questions of periodicity, but many of the usual statistical methods, and the theory of Fourier's series as found in mathematical text-books, cannot easily be directly applied to geomagnetic data. In fact many serious misunderstandings that have arisen are due to the fact that methods suitable in biometrical studies have been inappropriately applied to geomagnetism. This applies particularly to statistical theorems concerning tests for the reality of periodicities in geophysical time series. Such tests are often based on assumptions as to randomness, which ignore the important characteristic of 'conservation' which is a notable feature in many kinds of geophysical data.

For this reason Chapter XVI, on harmonic analysis, discusses not only the ordinary processes which can be found in most text-books on practical analysis, but also indicates the statistical basis for the

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For this reason Chapter XVI, on harmonic analysis, discusses not only the ordinary processes which can be found in most text-books on practical analysis, but also indicates the statistical basis for the treatment of periodicities in geomagnetism. Such conceptions as conservation, the harmonic dial, the random walk, persistence and quasi-persistence, are described and illustrated by means of geomagnetic and solar data, in relation to sunspot, annual and other periods, and the 27-day recurrence-tendency.

Chapter XIX, dealing with the variability of the solar and lunar daily variations, continues this discussion by means of 'clouds of points' in harmonic dials.

Another mathematical aspect is introduced by the spherical form of the earth: Chapter XVII describes spherical harmonic functions, in their dual capacity as functions suitable for the mathematical expression of arbitrary distributions given on the surface of a sphere, and as appropriate solutions of Laplace's equation, allowing the separation of the earth's magnetic field into parts caused within and outside the earth's surface. Chapters XVIII and XX apply these general theorems to the main geomagnetic field, and to the solar and lunar daily variations.

XVI

PERIODICITIES AND HARMONIC ANALYSIS IN GEOPHYSICS

16.1. Periodic and non-periodic functions. A function $f(t)$ of the time t is said to be periodic, with the period T, if for all integral values k (or at least for many successive values)

$$f(t) = f(t+kT). \tag{1}$$

The time-changes of many geophysical phenomena can be represented as a combination of a non-periodic part and of many periodic components with unequal periods. Among the principal periodic times are the solar and lunar days, the solar rotation period, a month, a year, and the 11-year sunspot cycle, and sub-multiples of these times.

Suppose that a periodic component of the form (1) is present in an unbroken series of observations. It may be determined by dividing the series into sections of length T, superposing these sections with the same initial and end-points, forming their sum, and dividing by the number of sections. The usual calculation of the mean daily variation of the earth's vertical magnetic intensity at any station, from hourly observations throughout N successive days, may be cited as an example. In this case the period T is a mean solar day. The hourly values (or means for hourly intervals) are written in horizontal rows, each containing twenty-four values, and therefore representing one day. The different values in the same column all refer to the same solar hour on different days. Let $v(t)$ denote the actual vertical intensity (a function of time); let $w(t)$ denote the daily variation, which for $k = 1, 2,..., N$ satisfies the condition (1); and let $g(t)$ denote the remainder of $v(t)$, viz.

$$v(t) = w(t)+g(t) \quad (t = 0 \text{ to } t = NT). \tag{2}$$

The average value in the column for the hour τ $(0 \leqslant \tau \leqslant T)$ is

$$\{v(\tau)+v(\tau+T)+v(\tau+2T)+...+v(\tau+(N-1)T)\}/N \equiv \bar{v}(\tau). \tag{3}$$

Let $\bar{w}(t)$ and $\bar{g}(t)$ be similarly defined; then by (2),

$$\bar{v}(\tau) = \bar{w}(\tau)+\bar{g}(\tau). \tag{4}$$

The greater the number of days (N), the more exactly will $\bar{v}(\tau)$ represent $w(\tau)$; for by (1),

$$\bar{w}(\tau) = (1/N)\{w(\tau)+w(\tau+T)+...+w(\tau+NT-T)\}$$
$$= (1/N)Nw(\tau) = w(\tau). \tag{5}$$

On the contrary it is found by experience that in the great majority of cases $\sum_{k=0}^{N-1} g(t+kT)$ increases with N only in the ratio $\surd N$, so that $\bar{g}(t)$, equal to $1/N$ times this sum, decreases approximately as $1/\surd N$. This $\surd N$ law for non-periodic functions $g(t)$, which in geodesy and general statistics is called the '*law of propagation of errors*', forms the starting-point for most investigations of periodicity in geophysics.

16.2. The non-cyclic variation. Suppose that, in determining the above daily variation—whatever the quantity denoted by $v(t)$—every row of 24 hourly values is extended by adding the first value for the next day. Then in columns 1 and 25 the same numbers will be found, except for $v(0)$ in column 1 of row 1, and $v(NT)$ in column 25 of row N. The mean values of the columns 1 and 25 therefore differ by

$$d = (1/N)\{v(NT)-v(0)\}; \tag{6}$$

if $d \neq 0$, the function $v(t)$ is said to have a *non-cyclic variation*; by (1) it is clearly due wholly to $g(t)$.

A typical example of a non-cyclic variation is the slow systematic secular variation of the earth's magnetic elements. It contributes to $g(t)$ a part which in the course of a single year may be regarded as a linear function of the time; it will consequently appear also in $g(\tau)$, without change. It is therefore usual to correct the daily variation $v(\tau)$ determined from the 25 columns, by subtracting a linear function, so as to make the initial and final values equal. This procedure is followed also with other geophysical data, e.g. in finding the monthly mean daily variation of air temperature; in this case d is due mainly to the annual variation of temperature. The same procedure is adopted even when nothing is known as to the nature of $g(t)$ or the cause of d.

16.3. The choice of days. So far the calculation has been supposed to refer to an unbroken series of observations. We next consider the more general case in which, after dividing the material into sections of length T, some only of the sections are picked out, according to certain criteria, and used as the basis of investigation of the periodic component of duration T. The condition (1) need then apply only to the numbers k of the selected sections. As before, the calculation is made with an extra final column, e.g. in the case of a daily period, with 25 hourly values. But the average difference between the first and last mean value is no longer given by (6), because many of the values in the final column do not reappear in the first column of the next row; they are the initial values of days not included in the selection. The average non-cyclic variation d often depends systematically on the nature of the selected days. For example, in the daily variation of the earth's horizontal magnetic intensity, d is in general positive on quiet days, and negative on disturbed days; d for the north component at Potsdam, for the mean of the years 1918–22, was $+3{\cdot}9\ \gamma$ for the international quiet days (5 per month), and $-8{\cdot}6\gamma$ for the disturbed days (also 5 per month), whereas the mean secular variation, given by d from all days, is only $-0{\cdot}05\ \gamma$. These differences are characteristic; the disturbed days fall mainly in the time of strong decrease of horizontal intensity during the main phase of major or minor magnetic storms, while the quiet days occur during the slow recovery to the normal value. Hence in every daily variation the non-cyclic variation should be stated, whether it is eliminated or not.

16.4. The effect of curvature. The linear elimination of the non-cyclic variation usually precedes the harmonic analysis (§ 12) of the daily variation. It is also of value when the variation is not to be thus analysed, because it largely removes any influence depending on the choice of origin ($\tau = 0$) of the interval T. For example, in calculating the mean daily variation of the north component of magnetic intensity on quiet days at Batavia, we may wish to use the international quiet days, which begin and end at Greenwich midnight; it appears that the mean value for 23^h G.M.T. exceeds that for 0^h G.M.T. by about 5γ. If the twenty-four mean values from 0^h to 23^h G.M.T. were used to obtain the daily variation according to *local* time, simply by transposition of the last seven before the remaining seventeen (since 0^h G.M.T. approximately corresponds to 7^h of Batavia local time), the transposed sequence of numbers would show a sharp decrease of 5γ between 7^h and 8^h of local time, which obviously does not correspond to anything real. By using 25 hourly values and removing d linearly we avoid this influence of the limits of the day on the daily variation.

But the criterion of selection of days often proves to be associated with a systematic character of $g(t)$, which cannot be treated as a merely linear change. An example is the variation of air pressure on bright and cloudy days at non-tropical stations; this caused much confusion until it was elucidated by Bartels [36]. Bright days occur predominantly during the passage of anticyclones, associated with high pressures; hence the pressure is near its maximum, and the barograph curves are convex ($d^2g/dt^2 < 0$). This convexity is naturally not subject to the $\sqrt{N}$ law, but will also appear undiminished in the mean curve from many bright days; it cannot be removed by a linear correction. But it does not belong to the daily periodic variation, as appears when the initial time ($\tau = 0$) is altered, say from midnight to some other hour. The convex variation always appears in much the same form relative to the chosen limits of the day, whereas the daily periodic variation must depend on solar time: this property of the convex variation may be used to determine it separately, for the purpose of subsequent elimination; for example, it suffices to form daily variations for four or six different uniformly spaced origins $\tau = 0$, and then to superpose these.

The possibility of such systematic curvature is often overlooked. For example L. A. Bauer [40, 41] and, following him, H. H. Clayton [42], concluded that there is a pronounced annual variation in the relative sunspot numbers, the maximum being near aphelion, in July, and the minimum near perihelion, in January; they called this an 'earth-effect' on sunspots. The evidence for this was that, in the mean of the years 1913–22, the mean sunspot number for the northern-summer half-year (April to September) was 46, as against 42 for the winter half-year. But this was only an effect of curvature in the variation over the whole period 1913–22; this period coincided with the rise to, and fall from, the sunspot-maximum of 1917 (p. 369): the apparent mean yearly variation derived from such a set of years must be convex, and will give a higher mean value for the months April to September, which are in the middle of the selected yearly intervals, than for the other six months.

The misleading effect of curvature appears still more clearly in the following calculation, which gives the half-yearly mean sunspot numbers derived from the 100 years 1826–1925, and also, separately, from two sets of selected years, namely, 5 around each of the sunspot-maxima, and 5 around the sunspot-minima. The year is supposed to begin in January.

Average sunspot numbers, 1826–1925

	Northern summer	*Northern winter*	*Difference*
45 maximum years . .	77	73	+4
45 minimum years. . .	17	19	−2
All years (100) . . .	46	46	0

Other examples of this spurious curvature effect have been given by Chapman [38, 39].

16.5. Elimination of the non-periodic part. In calculating periodic components it is desirable to eliminate the mean non-periodic part $g(t)$ as far as possible. Several methods have been devised for the case when the amplitude of the periodic component to be determined is small compared with the other fluctuations ($w(t) \ll g(t)$), as, for example, for the lunar daily variation of pressure in non-tropical latitudes.

The simplest plan is to use only the quiet days, on which $v(t)$, and therefore also $g(t)$, is small. The upper limit of the range of $v(t)$ on the selected days must be chosen so that, on the one hand, the most disturbed days are excluded, since otherwise they may spoil the mean curve $\bar{v}(\tau)$, while on the other hand there remain a sufficient number of days to give scope to the $\sqrt{N}$ law. The possibility of a curvature effect and other systematic effects is to be considered (see, for instance, 8.18).

If the available material is small, as, for example, from polar stations, this procedure can be refined by dealing separately with the days of different degrees of disturbance, that is, with days of different range; the periodic components calculated from each set of days must then be combined with different weights [37*a*]. Chapman [26] has proposed a quite different method for the elimination of a great part of $g(t)$ in certain cases.

In calculations of periodic components it is usually necessary to rearrange a large collection of numbers. It is therefore often convenient (unless Hollerith methods are used, cf. 8.1) to deal, not with the hourly or bi-hourly values themselves, but with their successive *differences*, which in general are much smaller [25]. The reversion to the values themselves is made at the end of the calculation by successive addition of the differences or otherwise (see § 19).

16.6. Sine waves: Fourier series. One of the simplest periodic physical phenomena is the motion of a pendulum. When the amplitude is small the displacement is proportional to sin nt. All simple mechanical and electric wave-motions may likewise be associated with the sine function. The *period* T is $2\pi/n$; the *frequency* n is the number of complete swings in 2π units of time.

Mathematically the problems that lead to sine waves are characterized by the differential equation

$$\frac{d^2y}{dt^2} = -n^2y; \tag{7}$$

if y denotes the displacement from equilibrium (e.g. for a pendulum), then (7) states that the acceleration d^2y/dt^2 is directed towards the origin and is proportional to the displacement from the origin.

In geophysics, as in mechanics, there are *physical* reasons for the occurrence of trigonometric functions in connexion with waves and tides. Mathematically their importance even in the case of complicated periodic phenomena depends on Fourier's theorem (1807). This states that every function $\psi(t)$, which is defined in the interval $t = 0$ to $t = 2\pi$, and satisfies certain conditions, (e.g. that $\psi(t)$ must have a continuous first derivative except, at most, at a finite number of points in the interval) can be expressed as an infinite series of trigonometric functions:

$$\psi(t) = a_0+(a_1\cos t+b_1\sin t)+(a_2\cos 2t+b_2\sin 2t)+\dots \tag{8}$$

with the coefficients

$$\left.\begin{aligned} a_0 &= \frac{1}{2\pi}\int\limits_0^{2\pi} \psi(t)\,dt, \\ a_n &= \frac{1}{\pi}\int\limits_0^{2\pi} \psi(t)\cos nt\,dt, \qquad b_n = \frac{1}{\pi}\int\limits_0^{2\pi} \psi(t)\sin nt\,dt \quad (n = 1,2,\dots). \end{aligned}\right\} \tag{9}$$

Thus a_0 is the mean value of $\psi(t)$ in the interval 0 to 2π. In § 8 a modified form of this theorem, appropriate to geophysics, is derived, and in § 10 the convergence of the series is discussed. In geophysical cases the conditions of Fourier's theorem are practically always fulfilled.

The terms of frequency $n \geqslant 1$ can also be written in the form

$$a_n\cos nt+b_n\sin nt = c_n\sin(nt+\epsilon_n), \tag{10}$$

where
$$a_n = c_n\sin\epsilon_n, \qquad b_n = c_n\cos\epsilon_n, \tag{11 a}$$

$$c_n = +\sqrt{(a_n^2+b_n^2)}, \qquad \tan\epsilon_n = a_n/b_n. \tag{11 b}$$

The amplitude c_n is taken positive; the signs of a_n and b_n determine the quadrant in which the phase-angle ϵ_n lies.

The equation (10) shows that $\psi(t)$ can be expressed as a combination of sine waves of frequencies $n = 1, 2, 3,\dots$. Because of its acoustic applications the process of obtaining this development of a function in the form of a sine series is named *harmonic analysis*, and the factors a_n and b_n are called *harmonic coefficients*. The representation is unique—every function $\psi(t)$ corresponds to one and only one set of amplitudes c_n and phases ϵ_n.

If $\psi(t)$ is periodic in an interval of length T, it is convenient to introduce the new variable $\tau = 2\pi t/T$, which increases from 0 to 2π as t increases from 0 to T.

For a function which is symmetrical with respect to the instant $t = \pi$, i.e.

$$\psi(\pi+t) = \psi(\pi-t),$$

all the coefficients b_n disappear; for an antisymmetrical function, such that

$$\psi(\pi+t) = -\psi(\pi-t),$$

all the terms a_n disappear. This follows from (9) by reason of the corresponding properties of $\sin nt$ and $\cos nt$.

16.7. Orthogonality. A set of real functions

$$\phi_0(t),\quad \phi_1(t),\quad \phi_2(t),\quad \ldots, \tag{12}$$

which are defined in a finite or infinite interval $t_1 < t < t_2$, and which, as also their squares and products, are integrable in this interval, form an *orthogonal* system, if

$$\int\limits_{t_1}^{t_2} \phi_n(t)\phi_m(t)\,dt = 0 \quad \text{for} \quad n \neq m. \tag{13}$$

A function $\phi_n(t)$ is said to be *normalized* if

$$\int\limits_{t_1}^{t_2} \phi_n^2(t)\,dt = 1. \tag{14}$$

If this integral is i_n ($\neq 1$) instead of unity, the function $\phi_n(t)/\sqrt{i_n}$ is the normalized function corresponding to $\phi_n(t)$; i_n is of course necessarily positive.

In the interval $0 < t < 2\pi$ the functions

$$1,\quad \cos t,\quad \sin t,\quad \cos 2t,\quad \sin 2t,\quad \ldots,\quad \cos nt,\quad \sin nt,\quad \ldots \tag{15}$$

form an orthogonal system, since

$$\int\limits_0^{2\pi} \sin nt\,dt = 0, \qquad \int\limits_0^{2\pi} \cos nt\,dt = 0 \quad (n \text{ being any integer})$$

$$2\sin nt \sin mt = \cos(n-m)t - \cos(n+m)t \tag{16}$$

(with similar equations for $2\cos nt\cos mt$, $2\sin nt\cos mt$), so that

$$\left.\begin{aligned} &\int\limits_0^{2\pi} \sin mt \sin nt\,dt = 0, \qquad \int\limits_0^{2\pi} \sin mt \cos nt\,dt = 0,\\ &\qquad\qquad\qquad\qquad \int\limits_0^{2\pi} \cos mt \cos nt\,dt = 0 \quad (n \neq m)\\ &\int\limits_0^{2\pi} 1\,dt = 2\pi, \quad \int\limits_0^{2\pi} \sin^2 nt\,dt = \int\limits_0^{2\pi} \cos^2 nt\,dt = \pi \quad (n \geqslant 1), \end{aligned}\right\} \tag{17}$$

the equations in the last line being derived from (16) by taking $m = n$.

Thus the functions (15) satisfy the orthogonality conditions (13).

The set of normalized functions corresponding to (15) is

$$\frac{1}{\sqrt{(2\pi)}},\quad \frac{1}{\sqrt{\pi}}\cos t,\quad \frac{1}{\sqrt{\pi}}\sin t,\quad \ldots,\quad \frac{1}{\sqrt{\pi}}\cos nt,\quad \frac{1}{\sqrt{\pi}}\sin nt,\quad \ldots, \tag{18}$$

but it is usual to work with the non-normalized set (15).

The development of a function ψ in a series of orthogonal functions ϕ_n is in some ways similar to the geometrical problem of resolving an arbitrary vector $\boldsymbol{R}$ in k-dimensional space into components $R_m\,\boldsymbol{i}_m$ along k orthogonal vectors $\boldsymbol{i}_m$ of unit length, in which case $R_m = \boldsymbol{R}\cdot\boldsymbol{i}_m$, $\boldsymbol{i}_m\cdot\boldsymbol{i}_n = 0$ $(n \neq m)$, and $\boldsymbol{i}_m\cdot\boldsymbol{i}_m = 1$, where the dot is the sign for the scalar product; these equations have some analogy with (9), (13), and (14); it is for this reason that the functions ϕ_n are called *orthogonal*, and the single terms (10) of the Fourier series are called the *harmonic components* of the function $\psi(t)$.

16.8. Approximation to a function by a finite trigonometrical series. Fourier's theorem that every function $v(t)$ can be represented *exactly*, within any chosen interval, by an *infinite* series, is of less importance in geophysics than the practical problem of *approximating* to $v(t)$ by a *finite* series, namely, by finding a nearly equal function $\psi_k(t)$ of the form

$$\psi_k(t) = a_0+a_1\cos t+b_1\sin t+\ldots+a_k\cos kt+b_k\sin kt. \tag{19}$$

In this problem it is required to find $(2k+1)$ harmonic coefficients a_0, $a_1,\ldots, a_k$ and $b_1,\ldots,b_k$ such that the difference $v(t)-\psi_k(t)$ over the whole interval shall be as small as possible. As a measure of the closeness of the approximation, the square of this difference, averaged over the interval, is chosen,

$$\mu^2 = \frac{1}{2\pi}\int\limits_0^{2\pi} [v(t)-\psi_k(t)]^2\,dt. \tag{20}$$

On squaring the integrand, and substituting for $\psi_k(t)$ from (19), the part $\frac{1}{2\pi}\int\limits_0^{2\pi} \{\psi_k(t)\}^2\,dt$ of this integral can be evaluated by means of (17): thus

$$\frac{1}{2\pi}\int\limits_0^{2\pi} \{\psi_k(t)\}^2\,dt = \tfrac{1}{2}\{2a_0^2+a_1^2+b_1^2+\ldots+a_k^2+b_k^2\}. \tag{21}$$

Incidentally it follows from this equation that if a series of trigonometric functions (such as ψ_k) vanishes identically for all values of t—in which case the integral of its square must be zero—all the harmonic coefficients a and b must be zero. This result serves to establish the uniqueness of the Fourier expansion of a function.

From (20) and (21) it follows that

$$\mu^2 = \frac{1}{2\pi}\int\limits_0^{2\pi} \{v(t)\}^2\,dt - \frac{1}{\pi}\left\{a_0\int\limits_0^{2\pi} v(t)\,dt + a_1\int\limits_0^{2\pi} v(t)\cos t\,dt + \ldots + \right.$$
$$\left. + b_k\int\limits_0^{2\pi} v(t)\sin kt\,dt\right\} + \tfrac{1}{2}\{2a_0^2 + a_1^2 + b_1^2 + \ldots + a_k^2 + b_k^2\}. \qquad (21\text{ a})$$

The conditions that μ^2 shall be a minimum with respect to variations of the $(2k+1)$ coefficients a and b are that

$$\frac{\partial\mu^2}{\partial a_0} = 0, \qquad \frac{\partial\mu^2}{\partial a_1} = 0, \qquad \frac{\partial\mu^2}{\partial b_1} = 0, \quad \ldots, \quad \frac{\partial\mu^2}{\partial b_k} = 0. \qquad (22)$$

On performing the differentiation we obtain the formulae (9).

This result is remarkable in that the coefficients a_n and b_n $(n \leqslant k)$ for each value of n are independent of k, that is, of the number of periodic terms included in ψ_k. It is even possible to leave out any assigned intermediate terms in $\psi_k(t)$ without affecting the values of a_n and b_n (as derived by this minimum criterion) for the terms retained. By adding further terms to $\psi_k(t)$, the approximation is in general improved, but without altering the coefficients already found.

16.9. Approximation to a function by a finite series of any orthogonal functions. This property of the series (19) is characteristic of all developments of arbitrary functions in series of orthogonal functions, because in deriving (21 a) from (20) use was made only of the orthogonality of the trigonometrical functions. The approximation to $v(t)$ by a finite series of *normalized* orthogonal functions,

$$\psi_l(t) = a_0\phi_0(t) + a_1\phi_1(t) + \ldots + a_l\phi_l(t), \qquad (23)$$

subject to the condition of minimum μ^2, where now

$$\mu^2 = \frac{1}{t_2-t_1}\int\limits_{t_1}^{t_2} \{v(t)-\psi_l(t)\}^2\,dt, \qquad (24)$$

therefore leads to the coefficients

$$a_0 = \int\limits_{t_1}^{t_2} v(t)\phi_0(t)\,dt, \quad \ldots, \quad a_l = \int\limits_{t_1}^{t_2} v(t)\phi_l(t)\,dt; \qquad (25)$$

for example, the set of normalized functions (18) leads to the Fourier formulae (8), (9). If the functions ϕ are not normalized, the equations (25) are only altered by the insertion of numerical factors, as in (9).

16.10. Bessel's inequality. On substituting from (9) for the coefficients in (21 a), we find that

$$\mu^2 = \frac{1}{2\pi}\int_0^{2\pi} \{v(t)\}^2\,dt - a_0^2 - \tfrac{1}{2}(a_1^2+b_1^2+\ldots+a_k^2+b_k^2), \tag{26}$$

or, by (11 b),

$$\mu^2 = \frac{1}{2\pi}\int_0^{2\pi} \{v(t)\}^2\,dt - a_0^2 - \tfrac{1}{2}(c_1^2+c_2^2+\ldots+c_k^2). \tag{27}$$

Since $\mu^2 \geqslant 0$, by reason of its definition (20), it follows that

$$a_0^2+\tfrac{1}{2}(c_1^2+c_2^2+\ldots+c_k^2) \leqslant \frac{1}{2\pi}\int_0^{2\pi} \{v(t)\}^2\,dt; \tag{28}$$

The infinite series $$c_1^2+c_2^2+\ldots+c_k^2+\ldots \tag{29}$$ must therefore converge, since it consists wholly of positive terms and has a finite upper limit. The inequality (28) is known as *Bessel's inequality*, and there is an analogous inequality for every development in a series of orthogonal functions.

It follows from (27) that the representation of $v(t)$ by $\psi_k(t)$ is the better, the greater the value of k (but cf. § 16).

From (9) it is obvious that the harmonic coefficients of a sum of functions f_1 and f_2 are simply the sums of the corresponding coefficients for the separate functions. The same is true also for every linear combination of functions

$$\alpha_1 f_1+\alpha_2 f_2+\ldots+\alpha_l f_l \tag{30}$$

with arbitrary constants $\alpha_1,\ldots, \alpha_l$.

Dirichlet in 1827 stated conditions which ensure that the infinite trigonometric series (8) shall converge and shall represent $v(t)$; these conditions, and also the more recent conception of 'almost periodic functions', are described in text-books on analysis but need not be discussed here.

16.11. Orthogonal polynomials. Legendre functions. A typical example of a *non-orthogonal* set of functions is the series of powers

$$1,\ x,\ x^2,\ldots \tag{31}$$

relative to the interval $-1 < x < +1$. The non-orthogonality is obvious, and it has the consequence that the condition of minimum μ^2, applied to the approximation

$$\psi_l(x) = a_0+a_1 x+a_2 x^2+\ldots+a_l x^l, \tag{32}$$

leads to different sets of coefficients for every different value of l.

But an orthogonal set of functions, for the same interval, can be constructed by combinations of the powers, that is, in the form of polynomials. The following polynomials, known as the Legendre functions, are such a set; they are here ordered according to their degree in x.

$$\left.\begin{array}{ll} P_0(x) = 1, & P_4(x) = \tfrac{1}{8}(35x^4-30x^2+3), \\ P_1(x) = x, & P_5(x) = \tfrac{1}{8}(63x^5-70x^3+15x), \\ P_2(x) = \tfrac{1}{2}(3x^2-1), & \quad . \quad . \quad . \quad . \quad . \\ P_3(x) = \tfrac{1}{2}(5x^3-3x), & \end{array}\right\} \tag{33}$$

The general formula is

$$P_n(x) = \frac{(2n)!}{2^n(n!)^2}\left\{x^n - \frac{n(n-1)}{2(2n-1)}x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2.4(2n-1)(2n-3)}x^{n-4} - \ldots\right\}, \tag{34}$$

where $\qquad n! \equiv 1\times 2\times 3\ldots\times n, \qquad 0! = 1.$

By integrating the series in (34) n times, we obtain a certain multiple of the binomial expansion of $(x^2-1)^n$; this corresponds to the following alternative expression for P_n:

$$P_n(x) = \frac{1}{n!\,2^n}\frac{d^n}{dx^n}(x^2-1)^n. \tag{35}$$

By means of this formula, and repeated partial integrations, it is easy to prove that

$$\int\limits_{-1}^{1} P_n(x)P_m(x)\,dx = \begin{cases} 0 & \text{if } n \neq m, \\ \dfrac{2}{2n+1} & \text{if } n = m; \end{cases} \tag{36}$$

when $n \neq m$ these represent the orthogonality conditions (13). It is clear that the functions P_n do not satisfy (14), so that they are not normalized.

It follows from § 9 that we may approximate to a function defined in the interval -1 to 1, by a series of Legendre functions, i.e.

$$f(x) \doteqdot b_0 P_0(x)+b_1 P_1(x)+\ldots+b_l P_l(x), \tag{37}$$

wherein $\doteqdot$ denotes 'approximately equals', and the coefficients are independent of l. Other sets of orthogonal polynomials in the same interval (the *associated Legendre functions* or *spherical functions*) are described in 17.3.

It may be shown that, given a series of linearly independent functions of any kind, a series of normalized orthogonal functions, each of which is a linear combination of functions of the given series, can be constructed, relative to any chosen interval. The formation of the Legendre set of orthogonal functions from the powers of x is a particular example of this theorem.

16.12. Harmonic analysis of a set of equidistant values of a function. It often happens that the function $v(t)$ to be approximated to, as in § 8, over a given range of t, here taken to be from 0 to 2π, is not completely specified over this range, but is given only by a number of values at equal intervals within it, e.g. by hourly or monthly values. Let the range of t, from 0 to 2π, be divided into r equal parts, at the epochs $t_0, t_1,..., t_r$, so that

$$t_\sigma = 2\pi\sigma/r \quad (\sigma = 0, 1,..., r). \tag{38}$$

Let the corresponding values of the function be $y_0, y_1,..., y_r$; we suppose for the present that any non-cyclic variation of $v(t)$ from $t = 0$ to $t = 2\pi$ has already been removed, so that

$$y_0 = y_r.$$

We consider how to determine the coefficients of $\psi_k(t)$ in (19) so that μ^2, now defined by

$$\mu^2 = \frac{1}{r}\sum_{\sigma=1}^{r} \{y_\sigma - \psi_k(t_\sigma)\}^2 \quad . \tag{39}$$

instead of by (20), shall be a minimum. In the present case we must take

$$2k+1 \leqslant r, \tag{40}$$

because otherwise the number of unknowns, namely

$$a_0, a_1,..., a_k, b_1,..., b_k,$$

to be determined by this minimum condition, would exceed the number of given values of the function. The equations (22) which express the minimum condition now involve sums (instead of integrals, as in § 8); these are best evaluated as follows. Let

$$\cos nt_\sigma + i \sin nt_\sigma = \exp(int_\sigma) = \exp(in2\pi\sigma/r) = q^\sigma, \tag{41}$$

where $i = \sqrt{(-1)}$, and

$$q = e^{2\pi in/r}. \tag{42}$$

Then

$$\sum_{\sigma=1}^{r} \cos nt_\sigma + i \sum_{\sigma=1}^{r} \sin nt_\sigma = \sum_{\sigma=1}^{r} q^\sigma = q\frac{q^r-1}{q-1}. \tag{43}$$

Clearly $q^r = e^{2i\pi n} = 1$, whereas $q \neq 1$ for $2n+1 \leqslant 2k+1 \leqslant r$. Consequently the right-hand side of (43) is zero, and therefore also both the series on the left are zero. Hence, with the aid of the three product formulae, of which (16) is an example, we obtain the following series formulae, analogous to the integral formulae (17); n and m here denote any positive integers from 1 to k.

$$\left.\begin{aligned} \sum_{\sigma=1}^{r}\cos nt_\sigma &= \sum_{\sigma=1}^{r}\sin nt_\sigma = 0,\\ \sum_{\sigma=1}^{r}\cos^2 nt_\sigma &= \sum_{\sigma=1}^{r}\sin^2 nt_\sigma = \tfrac{1}{2}\sum_{\sigma=1}^{r}(1\pm\cos 2nt_\sigma) = \tfrac{1}{2}r,\\ \sum_{\sigma=1}^{r}\cos nt_\sigma\cos mt_\sigma &= \sum_{\sigma=1}^{r}\sin nt_\sigma\sin mt_\sigma\\ &= \sum_{\sigma=1}^{r}\sin nt_\sigma\cos mt_\sigma = 0 \quad (n\neq m).\end{aligned}\right\}\tag{44}$$

By means of these formulae it is easy to show that the minimum conditions (22) give the following expressions for the coefficients a and b;

$$a_0 = (1/r)\sum_{\sigma=1}^{r} y_\sigma = \text{the mean of the } y_\sigma\text{'s},\tag{45 a}$$

$$a_n = (2/r)\sum_{\sigma=1}^{r} y_\sigma\cos nt_\sigma,\qquad b_n = (2/r)\sum_{\sigma=1}^{r} y_\sigma\sin nt_\sigma.\tag{45 b}$$

Here also, as in § 8, the coefficients are independent of k. The simplest form of numerical integration shows that the sums (45 a, b) are approximations to the integrals (9).

From (39), (44), and (45), writing $c_n^2 = a_n^2+b_n^2$ as in (11 b), we find that

$$\mu^2 = (1/r)\sum y_\sigma^2-a_0^2-\tfrac{1}{2}(c_1^2+c_2^2+\ldots+c_k^2).\tag{46}$$

Here the first term is simply the mean value of y_σ^2. We introduce a number m (not to be mistaken for the integers m in (44)), defined by

$$m^2 \equiv (1/r)\sum y_\sigma^2-a_0^2;\tag{47}$$

m^2 is the mean value of the squares of the deviations of y_σ from the mean y_σ, or a_0, since, by (45 a),

$$(1/r)\sum(y_\sigma-a_0)^2 = (1/r)\sum y_\sigma^2-(2/r)a_0\sum y_\sigma+a_0^2 = (1/r)\sum y_\sigma^2-a_0^2 = m^2.\tag{47 a}$$

The number m is called the *standard deviation* of the set of values y_σ (§ 22); Fisher [11.1] calls m^2 their *variance*.

The standard deviation η_k of the set of values $\psi_k(t_\sigma)$ given by (19), for $\sigma = 1$ to $\sigma = r$, is found, by direct application of (44), to be given by

$$\eta_k^2 = \tfrac{1}{2}(c_1^2+c_2^2+\ldots+c_k^2).\tag{47 b}$$

Hence, by (46), (47),

$$\mu^2 = m^2-\eta_k^2;\tag{48}$$

we note also that μ is the standard deviation of the set of *residuals* $y_\sigma-\psi_k(t_\sigma)$, whose mean value is zero.

It is convenient for practical calculation if r is a multiple of 4, or

$$r = 4p$$

where p is an integer; this is the case when a daily variation is expressed by 24 hourly or 12 bi-hourly values, or an annual variation by 12 monthly means. The same numerical values of the sine and cosine in (45b) then recur in each quadrant, and this symmetry of the trigonometrical functions can be used (§ 19) to group various terms in the sums of products (45b); many calculating-schemes using this fact have been published.

In this case the $4p$ values of the function can be *exactly* represented by the formula

$$\psi(t) = a_0+(a_1\cos t+b_1\sin t)+\dots$$
$$+\{a_{2p-1}\cos(2p-1)t+b_{2p-1}\sin(2p-1)t\}+a_{2p}\cos 2pt, \qquad (49)$$

which has $4p$ adjustable constants. These are given by (45); in this case, however, the last constant, a_{2p}, is given by

$$a_{2p} = (-y_1+y_2-y_3+-\dots+y_r)/r. \qquad (45\text{c})$$

It is easy to infer from (46) that in this case $y_\sigma = \psi_k(t_\sigma)$, since we find, on using (44) and (45), that $\mu = 0$.

If r is an odd number, the exact representation of the set of values y_σ is given by (19), if we take $k = \frac{1}{2}(r-1)$.

Bartels [22] has given a direct proof, not using the differential calculus, of the minimum theorems of §§ 8, 12. The formulae (45) and (46) are valid also if, from the outset, some terms in (19) are omitted.

16.13. The connexion between harmonic analysis and correlation. The correlation coefficient R for a series of pairs of associated values $u_1, v_1; u_2, v_2; \dots; u_r, v_r$, such that the mean u and the mean v are each zero (i.e. $\sum u_\sigma = \sum v_\sigma = 0$), is defined (see also (11.10), p. 367, where, however, the notation is different), by

$$\mathrm{R} \equiv \frac{\sum u_\sigma v_\sigma}{\sqrt{\{\sum u_\sigma^2 \sum v_\sigma^2\}}}. \qquad (50)$$

We will now suppose that the values y_σ of the function discussed in § 12 are given as deviations from their mean (so that $\sum y_\sigma = 0$ or $a_0 = 0$), and we will identify them with the numbers u_σ; we will also take the numbers $\cos nt_\sigma$ (or $\sin nt_\sigma$), for any value of n, as the v_σ's. Also let

$$(1/r)\sum y_\sigma^2 = m^2, \qquad (51)$$

agreeing with (47) since now $a_0 = 0$. Then by (44) and (45) the correlation coefficient is

$$\frac{\sum y_\sigma \cos nt_\sigma}{\sqrt{(rm^2\times\frac{1}{2}r)}} = \frac{a_n}{m\sqrt{2}}. \qquad (52)$$

Similarly the correlation coefficient between y_σ and $\sin nt_\sigma$ is $b_n/m\surd 2$. The factor $1/m\surd 2$ is independent of the frequency n, so that the harmonic coefficients a_n and b_n, apart from this factor, are the correlation coefficients between the y_σ's and the values of $\cos nt$ or $\sin nt$ at the points t_σ.

A further connexion with the typical product-sums associated with the correlation coefficient is indicated in the following equation (54). If $v(t)$ is integrable and bounded, then it is known that, in (20), $\mu^2 \to 0$ as $k \to \infty$; from (26) it therefore follows that

$$\frac{1}{2\pi}\int\limits_0^{2\pi} \{v(t)\}^2\,dt = a_0^2+\tfrac{1}{2}\sum_1^\infty (a_n^2+b_n^2). \quad (\textit{Parseval's theorem.}) \tag{53}$$

If $f(t)$ and $g(t)$ are two such functions, whose harmonic or Fourier coefficients are a_n', b_n', and a_n'', b_n'', then the mean value of the product fg is given by

$$\frac{1}{2\pi}\int\limits_0^{2\pi} f(t)g(t)\,dt = a_0'a_0''+\tfrac{1}{2}\sum_1^\infty (a_n'a_n''+b_n'b_n''); \tag{54}$$

this is because the function $f+g$ has the Fourier coefficients $a_n'+a_n''$ and $b_n'+b_n''$, by § 10, and similarly for $f-g$; and equations analogous to (53), for $v=f+g$ and $v=f-g$, on subtraction from one another, give (54).

16.14. The relation between the Fourier series for a continuous function and the series obtained from equidistant values. Suppose that an approximate Fourier series for a continuous function $f(t)$ is obtained, as in § 12, from r equidistant values $y_\sigma = f(t_\sigma)$. Let $a_0, a_1,..., b_1, b_2,...$ be the coefficients of this series, and let the *infinite* series

$$f(t) = a_0^*+a_1^*\cos t+b_1^*\sin t+..., \tag{55}$$

be the *complete* representation of $f(t)$ in the interval 0 to 2π. Then the coefficients a_n and b_n of the finite series ($n \leqslant \frac{1}{2}(r-1)$), which are given by (45), are related as follows to the coefficients of the infinite series;

$$\left.\begin{aligned} a_n &= a_n^*+a_{r-n}^*+a_{r+n}^*+a_{2r-n}^*+a_{2r+n}^*+\cdots, \\ b_n &= b_n^*-b_{r-n}^*+b_{r+n}^*-b_{2r-n}^*+b_{2r+n}^*-\cdots. \end{aligned}\right\} \tag{56}$$

This is readily seen to follow from the fact that, if m is an integer, and $t_\sigma = 2\pi\sigma/r$, $\sigma = 1, 2,..., r$,

$$\begin{aligned} \cos(mr+n)t_\sigma &= \cos mrt_\sigma \cos nt_\sigma - \sin mrt_\sigma \sin nt_\sigma \\ &= \cos nt_\sigma = \cos(mr-n)t_\sigma, \\ \sin(mr+n)t_\sigma &= \sin mrt_\sigma \cos nt_\sigma + \cos mrt_\sigma \sin nt_\sigma \\ &= \sin nt_\sigma = -\sin(mr-n)t_\sigma, \end{aligned}$$

so that the cosine terms in (55) which have the frequencies $mr \pm n$ make the same contributions to the values y_σ as if their frequencies were all changed to n, and the corresponding sine terms contribute as if their frequencies were $\pm n$.

For example, if $r = 3$,

$$a_1 = a_1^* + a_2^* + a_4^* + a_5^* + \ldots;$$

hence if a 3-yearly cosine wave is derived from yearly values ($r = 3$, $n = 1$), any 1·5-yearly wave ($n = 2$) that may also be present will add its coefficient to that of the 3-yearly wave: this case arises in the investigation of the 3-yearly wave of air pressure at tropical stations. Fig. 1 illustrates the reason for this.

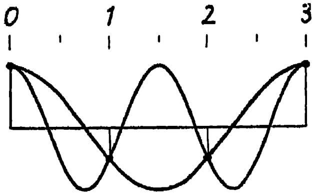


FIG. 1. Illustrating why cosine-waves of frequencies 1 and 2 cannot be separated, if only 3 equidistant ordinates are given

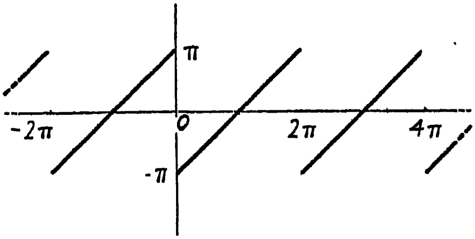


FIG. 2. The discontinuous periodic function which, between $t = 0$ and $t = 2\pi$, represents the non-cyclic variation $f(t) = t-\pi$

16.15. Corrections for non-cyclic variations. It is sometimes convenient to eliminate the non-cyclic variation *after* the harmonic analysis. To determine the correction, a linear function $f(t) = t-\pi$, from $t = 0$ to 2π, and its periodic continuation (Fig. 2) must be developed in a trigonometric series. The formulae (9), after partial integration, give

$$t-\pi = -2(\sin t + \tfrac{1}{2}\sin 2t + \tfrac{1}{3}\sin 3t + \ldots); \tag{57}$$

This function represents a discontinuity 2π at the end of each period. If the non-cyclic variation is d, (57) must be multiplied by $d/2\pi$.

Hence, if harmonic coefficients are calculated by applying (9) to a function $\psi(t)$ which includes, besides a part which is periodic in the interval 0 to 2π, a part which increases steadily by the amount d in this interval, the coefficients b_n may be corrected for the non-cyclic variation d by adding Δb_n, given by

$$\Delta b_n = d/\pi n; \tag{58}$$

there is no correction to a_n.

Where the harmonic analysis refers to a set of equidistant values y_σ, instead of to all values of the function, the correction is slightly different. The analysis in this case is made by using (45), and it is usual to replace y_r by $(y_0+y_r)/2$. The calculation of the harmonic coefficients for the linear function $(t-\pi)$ is made by applying (45) to the corresponding equidistant values of this function; it leads to sums of the form

$$\sum \sigma \cos nt_\sigma, \qquad \sum \sigma \sin nt_\sigma.$$

These can be obtained [22] by differentiating (43).

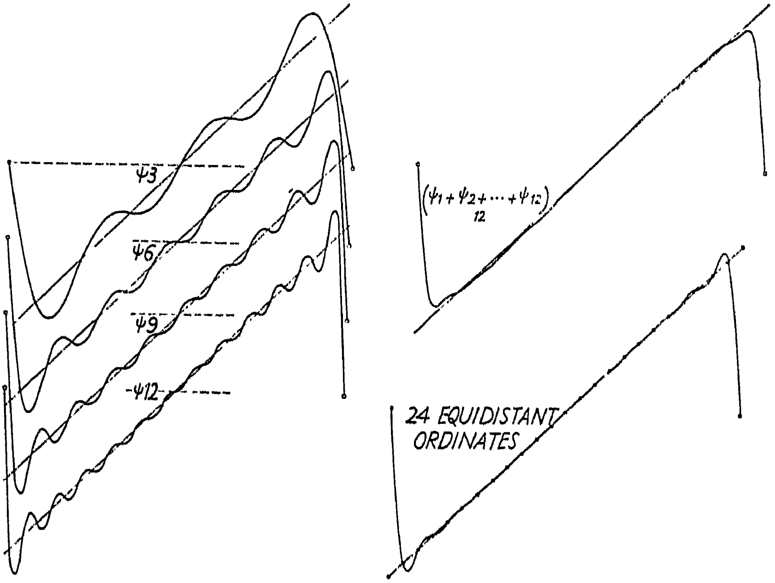


FIG. 3. The left-hand curves, which represent the sums of the first 3, the first 6, the first 9, and the first 12 terms (respectively) in the Fourier series for the discontinuous periodic function $f(t) = t-\pi$, illustrate the Gibbs phenomenon. The upper right-hand curve represents the Cesàro sum of the first 12 terms. The lower right-hand curve represents the sum of the Fourier series (of 24 terms) which agrees exactly with the 24 equidistant ordinates of $f(t)$

It is found that, as before, the coefficients a_n need no correction, while to the coefficients b_n we must add [22 or 25a]

$$\Delta b_n = \frac{d}{r}\cot\frac{n\pi}{r}; \tag{59}$$

for $r = \infty$ this agrees with (58).

16.16. Gibbs's phenomenon and the convergence of the Fourier series. The left part of Fig. 3 illustrates the approximation of the 'partial' sums of the first 3, 6, 9, and 12 terms of (57) to the function $t-\pi$. In accordance with (27), the representation steadily improves as the number of terms increases. But near the points of discontinuity $x = 0$ and $x = 2\pi$, the partial sums show appreciable oscillations. In the partial sum the last wave before the discontinuity always rises by about $0{\cdot}09d$ above the function, however many terms be taken (d here denotes the discontinuity, in this case 2π); the next nearest wave falls below the function by $0{\cdot}05d$, the third rises above by $0{\cdot}033d$, and so on. This wave pattern is confined within a narrower interval round the points of discontinuity, the greater the number of terms, but the amplitude of the waves remains finite. At the discontinuity itself, the partial sums represent the mean of the values to which y tends on the right and the left—in the present case this is zero.

This behaviour of the partial sums is known as the *Gibbs phenomenon*; it occurs similarly in expansions in series of other functions, e.g. of Legendre functions. It is not contradictory to the minimal condition (20), which is fulfilled by each of the partial sums; because (20) only requires the best possible approximation over the whole interval, and at individual points the difference between the function and the partial sum can be arbitrarily great. But it can be shown that such large differences must be confined to the points of discontinuity; for in every interval in which (including the end-points) the derivative $f'(t)$ of $f(t)$ is continuous, the Fourier series converges *uniformly*. That is to say, the absolute value of the difference $f(t)-\psi_k(t)$ in the *whole* of such an interval can be made less than any assigned arbitrarily small number ϵ, if the number of terms, k, exceeds a definite limit K, which naturally depends on ϵ.

It is obvious that sharp bends or angles in $f(t)$ can only be approximated to by terms of high frequency. Thus the harmonic analysis of the daily variation of air temperature *near the ground* in general requires a long series of terms, because of the sudden rise near dawn.

The averages of the partial sums—the so-called Cesàro sums—do not show the Gibbs phenomenon (Fig. 3, upper right). Fig. 3, lower right, shows also the exact representation for $r = 24$ ordinates, with the coefficients (59).

16.17. The influence of smoothing. The values to be analysed often do not refer to a single instant, but are averages over a time-interval, to whose centre they are ascribed—for example, hourly or monthly means. As the operation of averaging is a linear one (in the sense of equation (30)), its influence can be examined with reference to each single term of the trigonometrical series. Let the range 0 to

2π be divided into p sections of length $2\pi/p$. The mean value of $\sin nt$ in the interval $t-(2\pi/2p)$ to $t+(2\pi/2p)$, of length $2\pi/p$, centred at t, is

$$\overline{\sin nt} = \int\limits_{t-\pi/p}^{t+\pi/p} \frac{\sin nt\,dt}{2\pi/p} = \frac{p}{2\pi}\frac{1}{n}\{\cos n(t-\pi/p)-\cos n(t+\pi/p)\}$$

or, because of (16),

$$\overline{\sin nt} = \frac{p}{\pi n}\sin\frac{n\pi}{p}\sin nt. \tag{60}$$

For $\sin(nt+\epsilon_n)$ a similar result holds. The averaging over intervals of length $2\pi/p$ therefore leaves the form and phase of each single wave, for $n < p$, unaltered, but reduces its amplitude in the ratio

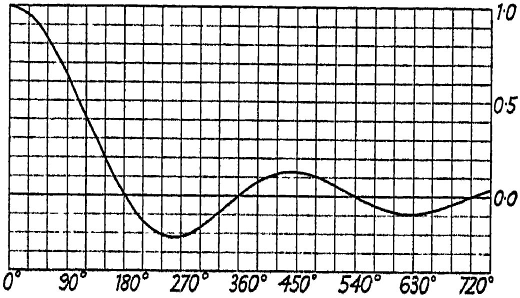


FIG. 4. The function $(\sin\omega)/\omega$, where ω is the circular measure of an angle (shown here in terms of degrees)

$$(\sin\omega)/\omega, \quad \text{where} \quad \omega = n\pi/p. \tag{61}$$

Fig. 4 is a graph of $(\sin\omega)/\omega$.

Conversely, if a series of mean values is analysed, the effect of the averaging or *smoothing* can be corrected for by multiplying the amplitudes by $\omega/\sin\omega$. For hourly means, and the periods 24, 12, 8, and 6 hours, these factors are 1·003, 1·012, 1·026, and 1·047, respectively.

The amplitudes of higher frequency are more reduced by the smoothing or averaging than those of lower frequency. For $n = p$ the factor (61) is zero, and for $n > p$ it becomes negative (Fig. 4). This possibility of the inversion of a wave by smoothing can be illustrated by the case of the curve $\sin x$, when averaged over intervals 3π: (61) then gives ($n = 1, p = \tfrac{2}{3}$) $\omega = 3\pi/2$, and the reduction factor $-\dfrac{2}{3\pi} = -0{\cdot}213$.

16.18. The harmonic dial. In geophysics it is often desirable in connexion with different series of data, to compare their periodic components of the same frequency n, by using a graphical representation which indicates both the amplitude and the phase. The vector character of a sine wave enables this to be done. In a plane coordinate system, in which a_n is measured upwards, and b_n to the right (Fig. 5) each periodic component is associated with a point P having the coordinates a_n, b_n. The vector from the origin to this point has the length c_n, by (11), and its inclination to the b_n axis is ϵ_n; a_n, b_n, and c_n, ϵ_n are therefore the cartesian and polar coordinates of P.

We note also that, if $i = \sqrt{(-1)}$, and $\exp \alpha = e^{\alpha}$,

$$\begin{aligned} c_n \cos(nt+\epsilon_n)+ic_n \sin(nt+\epsilon_n) &= c_n \exp i(nt+\epsilon_n) \\ &= c_n \exp(i\epsilon_n)\exp(int). \qquad (62) \end{aligned}$$

Hence the sine wave $c_n \sin(nt+\epsilon_n)$ can be regarded as the imaginary part of a complex periodic function with the time-factor e^{int}. This illustrates the connexion between our vector representation and the customary representation of complex numbers—in this case, of the amplitude $c_n \exp(i\epsilon_n)$; this analogy has, of course, found manifold applications in electrotechnics, e.g. to the characteristics of alternating currents.

The maximum of the sine wave occurs when $(nt+\epsilon_n) = \frac{1}{2}\pi$, therefore at

$$t_{\max} = \frac{1}{n}(\tfrac{1}{2}\pi-\epsilon_n). \qquad (63)$$

Suppose that, besides the scale for angle, ϵ_n, measured from the b_n axis in the anti-clockwise direction (Fig. 5), a scale of time is also inserted on the diagram, measured from the a_n axis in the clockwise sense. By choosing the time scale suitably, the vector which indicates c_n and ϵ_n will also indicate the time of the maximum of the sine wave. For the 24-hourly (diurnal) component of the daily variation, in which from midnight to midnight t increases from 0 to 2π, the time-scale is 15° per hour, and goes from 0 to 24 hours; for a semi-diurnal wave ($n = 2$) the scale is 30° per hour, and goes from 0 to 12 hours like the dial of an ordinary clock. Hence the name '*harmonic dial*' introduced by Chapman [27 *a*] for this mode of representation for any value of n.

The additive property of the Fourier coefficients a_n, b_n (§ 10) for components of equal frequency n extends also to the vectors, where it appears as *vector addition*. Again, a change of time-origin, as, for example, from local time to world time, can be made by a simple rotation of the coordinate system; if

$$t^* = t+t_0^*, \qquad (64)$$

then $\sin(nt+\epsilon_n) = \sin(nt^*+\epsilon_n^*)$, where

$$\epsilon_n^* = \epsilon_n-nt_0^*; \qquad (65)$$

thus, keeping the vector fixed, the axes of a_n^* and b_n^*, and with them the time scale for t^*, must be turned anti-clockwise through the angle nt_0^* relative to the a_n, b_n axes (Fig. 6).

The 'blank' for a harmonic dial of a certain frequency contains the origin O, a linear scale for the amplitudes c_n (or a number of circles around O corresponding to certain values of c_n) and a circular scale at the edge, graduated in angle and also in time; this time-scale, incidentally, indicates the length of the period T. Changes either in the units in which c_n is expressed, or in the time-origin, are easily made by renumbering the respective scales. Each point P entered, as a dot, on this blank represents, by the vector OP, a sine-wave of the period T. Since the blanks for harmonic dials of the period T for the intervals $t = 0$ to T, T to $2T$, $2T$ to $3T$, etc., are identical except for the numbering of the circular scales, which differ by multiples of T, they can all be combined into that for the interval $t = 0$ to T, because the dots derived from various intervals can be indicated by marking the dots P with explanatory letters or symbols. For instance, 24-hourly waves for different days can all be shown in the same dial.

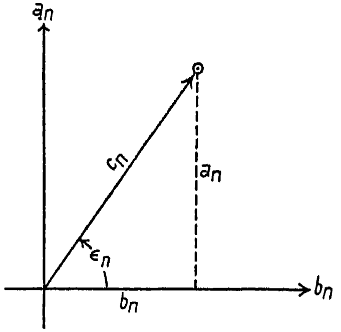


FIG. 5. Illustrating the properties of a harmonic dial for the sine wave $a_n \cos nt + b_n \sin nt = c_n \sin(nt + \epsilon_n)$

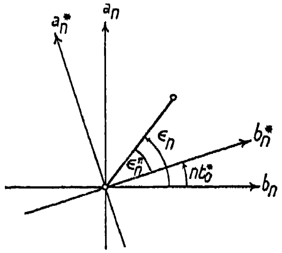


FIG. 6. Illustrating how the harmonic dial is changed by an alteration of the time origin ($t^* = t + t_0^*$)

Fig. 7 contains three harmonic dials, whose significance is indicated below them. The end-points of the vectors for the 24-hourly waves derived from the international quiet days (5 per month), the ordinary days, and the international disturbed days (5 per month) are indicated, and joined together. Especially in the vertical intensity it is strikingly shown how, with increasing degree of disturbance, the 24-hourly wave not only increases in amplitude, but also changes in phase. The diagrams illustrate, in a clear and concise manner, various characteristic features of the daily magnetic variation, already discussed in Chapter VII, as, for example, the differences between the seasons of the year.

16.19. Numerical harmonic analysis. The actual computing labour involved in evaluating the fundamental formulae (45) may be minimized by suitable arrangement of the work. Elaborate tables [20] have been worked out by Pollak which give the multiples of various sines and cosines occurring in (45) for different numbers ($r = 3, 4, \dots, 40$) of coordinates y_σ. It must be realized, however, that the use of many digits in the calculations would in many cases be unjustified, because of the only moderate degree of precision with which the values y_σ are known. Therefore if a great number of harmonic analyses have to be made for similar material, the following simplifications are convenient:

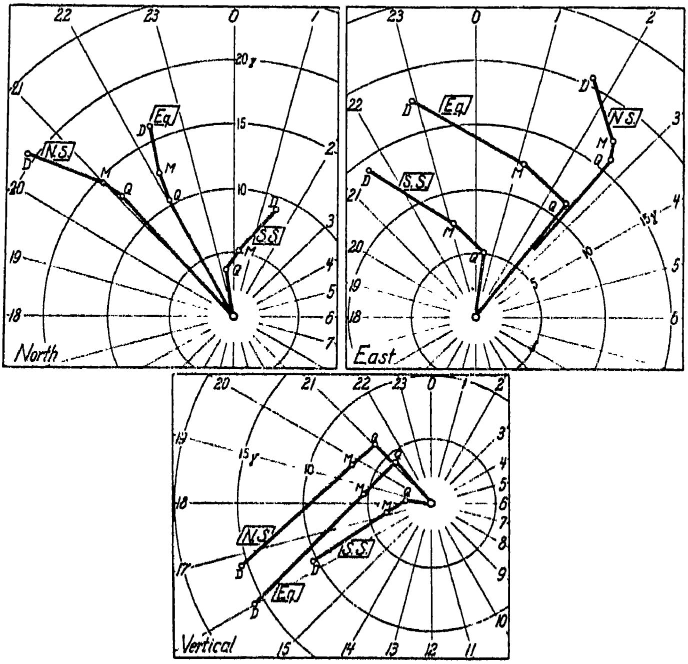


FIG. 7. Harmonic dials for the diurnal (24-hourly) sine-waves in the daily variations of the north, east, and vertical components of magnetic force at Potsdam-Seddin. Means 1915 to 1926, for the international quiet days (Q), disturbed days (D) and the ordinary days (moderately disturbed, M), for the three seasons: northern solstice (NS, May to August), southern solstice (SS, November to February), and equinox (Eq). The amplitude scale is shown by the circles, drawn at 5γ intervals; the direction from the central origin to each dial-point (Q, M, or D) indicates the time of maximum departure. (Local time is obtained by adding 2 hours to the values read from the time scales)

(*a*) The angles nt for $r = 24$ (hourly values) or $r = 12$ (monthly or bi-hourly values) are all multiples of 15° or 30°. Hence the numerical factors by which the values y_σ must be multiplied are limited to 0, 0·259, 0·500, 0·707, 0·866, and 0·966. Multiplication by means of Crelle's or Cotsworth's tables [45] or a slide-rule is usually sufficient. The division by r may partially or wholly be embodied in the multiplication factor. Since $12 = 2\times3\times4$, when $r = 12$ the first four harmonics are most easily calculated by using the following scheme: Divide the set y_0 to y_{11} into two half-sets to be added and subtracted:

$$\begin{matrix} y_0 & y_1 & y_2 & y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 & y_9 & y_{10} & y_{11} \end{matrix}$$

For $n = 1$ and $n = 3$, we first form the differences

$$d_0 = y_0-y_6,\quad d_1 = y_1-y_7,\quad \ldots,\quad d_5 = y_5-y_{11}.$$

Then (45 b) becomes:

$$\begin{aligned} a_1 &= \{d_0+0{\cdot}866(d_1-d_5)+0{\cdot}5(d_2-d_4)\}/6, \\ b_1 &= \{0{\cdot}5(d_1+d_5)+0{\cdot}866(d_2+d_4)+d_3\}/6; \\ a_3 &= (d_0-d_2+d_4)/6, \\ b_3 &= (d_1-d_3+d_5)/6. \end{aligned}$$

For $n = 2$ and 4, we use the sums

$$s_0 = y_0+y_6,\quad s_1 = y_1+y_7,\quad \ldots,\quad s_5 = y_5+y_{11}.$$

Then
$$\begin{aligned} a_2 &= \{(s_0-s_3)+0{\cdot}5(s_1-s_2-s_4+s_5)\}/6, \\ b_2 &= 0{\cdot}866(s_1+s_2-s_4-s_5)/6; \\ a_4 &= \{(s_0+s_3)-0{\cdot}5(s_1+s_2+s_4+s_5)\}/6, \\ b_4 &= 0{\cdot}866(s_1-s_2+s_4-s_5)/6. \end{aligned}$$

Simple factors occur also for $r = 8$, namely 1, 0·707 and 0. Where many sets y_σ are to be analysed, it is advisable to write all the calculations for one set (row) of r ordinates on the same line; the analysis can then be checked by adding the columns for, say, every ten rows; see also p. 605.

(*b*) For other values of r, it is sometimes advantageous to transform the series into one with a more suitable value of r. Such a case arises in treating the 27-day recurrence tendency. To find the first, second, and third harmonic from a set of 27 daily values, it suffices to form a new set of 8 values, each being the sum of 3·5 original ordinates,

$$\begin{aligned} v_1 &= y_1+y_2+y_3+0{\cdot}5y_4, & v_2 &= 0{\cdot}5y_4+y_5+y_6+y_7, \\ v_3 &= 0{\cdot}5y_7+y_8+y_9+y_{10}, & v_4 &= y_{11}+y_{12}+y_{13}+0{\cdot}5y_{14}, \\ v_5 &= 0{\cdot}5y_{14}+y_{15}+y_{16}+y_{17}, & v_6 &= 0{\cdot}5y_{17}+y_{18}+y_{19}+y_{20}, \\ v_7 &= y_{21}+y_{22}+y_{23}+0{\cdot}5y_{24}, & v_8 &= 0{\cdot}5y_{24}+y_{25}+y_{26}+y_{27}. \end{aligned}$$

If the time to which y_1 refers is called the day 1·0, then the v-values are not exactly equidistant, because they refer to the days 2·3, 5·7, 8·7, 12·3, 15·7, 18·7, 22·3, and 25·7, whereas the days equidistantly spaced (beginning with 2·1) would occur a few tenths of a day (on the average 0·15 day) earlier or later. It is possible to make a correction for the irregularity, and the equivalent case of the harmonic analysis of monthly means (owing to the unequal length of the months) has attracted the attention of some geophysicists [13]; however, in geophysics it is hardly worth while to apply such corrections, or (expressed differently) where such small corrections may seriously alter the conclusions, these do not invite much confidence.

(*c*) The use of successive *differences* (or progressive changes) of the ordinates, namely $z_1 = y_1 - y_0$, etc., has already been mentioned (§ 5) as sometimes simplifying the numerical work. For sufficiently small time-intervals $\Delta t = 2\pi/r$,

$$z_1 \doteqdot (dy/dt)_1 \Delta t;$$

differentiation of (10) shows that, corresponding to any term

$$c_n \sin(nt+\epsilon_n)$$

in y, there is a term $c'_n \sin(nt+\epsilon'_n)$ in z, with $c'_n = nc_n \Delta t$, $\epsilon'_n = \epsilon_n + 90°$. For differences with a *finite* time-interval $2\pi/r$, the corresponding *accurate* relations [25] between the harmonic results c'_n and ϵ'_n for the set z_σ, and c_n, ϵ_n for the set y_σ, are as follows. For the amplitudes,

$$c_n = c'_n/2\sin(n\pi/r);$$

for instance, for $r = 12$, $c_1 = 1{\cdot}93c'_1$, $c_2 = c'_2$, $c_3 = 0{\cdot}707c'_3$. As to the phases, the relation depends on the time t_0 assigned to that difference z which is multiplied, in the harmonic analysis, by $\cos 0°$ or $\sin 0°$; if the time for the full period is 2π, t_0 is half-way between the times for the two values y_σ from which that difference was computed (for instance in the case of hourly means from midnight to midnight, the difference in question may be the change from 0·5 to 1·5 o'clock; t_0 then corresponds to 1 o'clock, or 15°). Then [25 *a*]

$$\epsilon_n = \epsilon'_n - (nt_0 + 90°).$$

If a set of points on a harmonic dial gives the results c'_n, ϵ'_n for a number of sets of differences z_σ, the corresponding results for the sets of original values y_σ can be inferred by means of a modified scale for the amplitudes, and by taking new axes Oa (for a_n) and Ob (for b_n), which are rotated anti-clockwise, relative to the Oa', Ob' axes, through the angle $(90°+nt_0)$.

(*d*) Many formal schemes for harmonic analysis have been devised, but for practical use in geophysics those mentioned are found the best. The beginner is apt to calculate too accurately; as a curiosity, a scientific paper may be mentioned in which the phases of the diurnal temperature waves, from ordinary thermographs, were given to seconds. In ϵ_n for diurnal waves, tenths of a degree (corresponding to less than 1 minute in time) may be retained in results from long series, but have no meaning for single days.

The phase-changes which must be made for the various harmonic components of a solar daily variation, in order to change their time basis from mean to apparent solar time, have already been given in 2.15.

16.20. Mechanical and optical harmonic analysers. In practice it is sufficient in most cases to make numerical calculations according to the formulae of §§ 12 and 19. For the evaluation of the integrals (9), integrating instruments have been devised, such as Mader's harmonic analyser [25], or the optical devices due to Douglass [10] and Stumpff [9]. The use of Hollerith machines, working with cards with holes punched in them, has been developed by Pollak [21], for investigations of periodic terms. Chapman has also independently developed and extensively used Hollerith machines in the investigation of lunar daily variations in geophysics, in which large masses of data have to be treated. Tide-heights in harbours are analysed and predicted by special tidal calculating machines (cf. Rauschelbach [46]).

16.21. Graphical harmonic analysis: physical analogies. The following geometrical interpretation of the formulae (45) helps towards the understanding of the subsequent discussion of errors.

As an example, we use international magnetic character-figures C in a row of 27 days, preparatory to discussion in § 33. The 27 magnetic character-figures C for the solar rotation starting April 14, 1926, have been plotted in the top part of Fig. 8. This set of 27 ordinates can be regarded as a sum of 27 *primitive sets*, in each of which all the ordinates are zero except one (the first three of these sets are plotted in Fig. 8 *A*). That is to say, the set of ordinates

$$y_1, y_2, y_3, \ldots, y_r$$

is equivalent to the sum of the *primitive sets*

$$\begin{array}{cccccccc} y_1 & 0 & 0 & . & . & . & 0 \\ 0 & y_2 & 0 & . & . & . & 0 \\ 0 & 0 & y_3 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & y_r. \end{array}$$

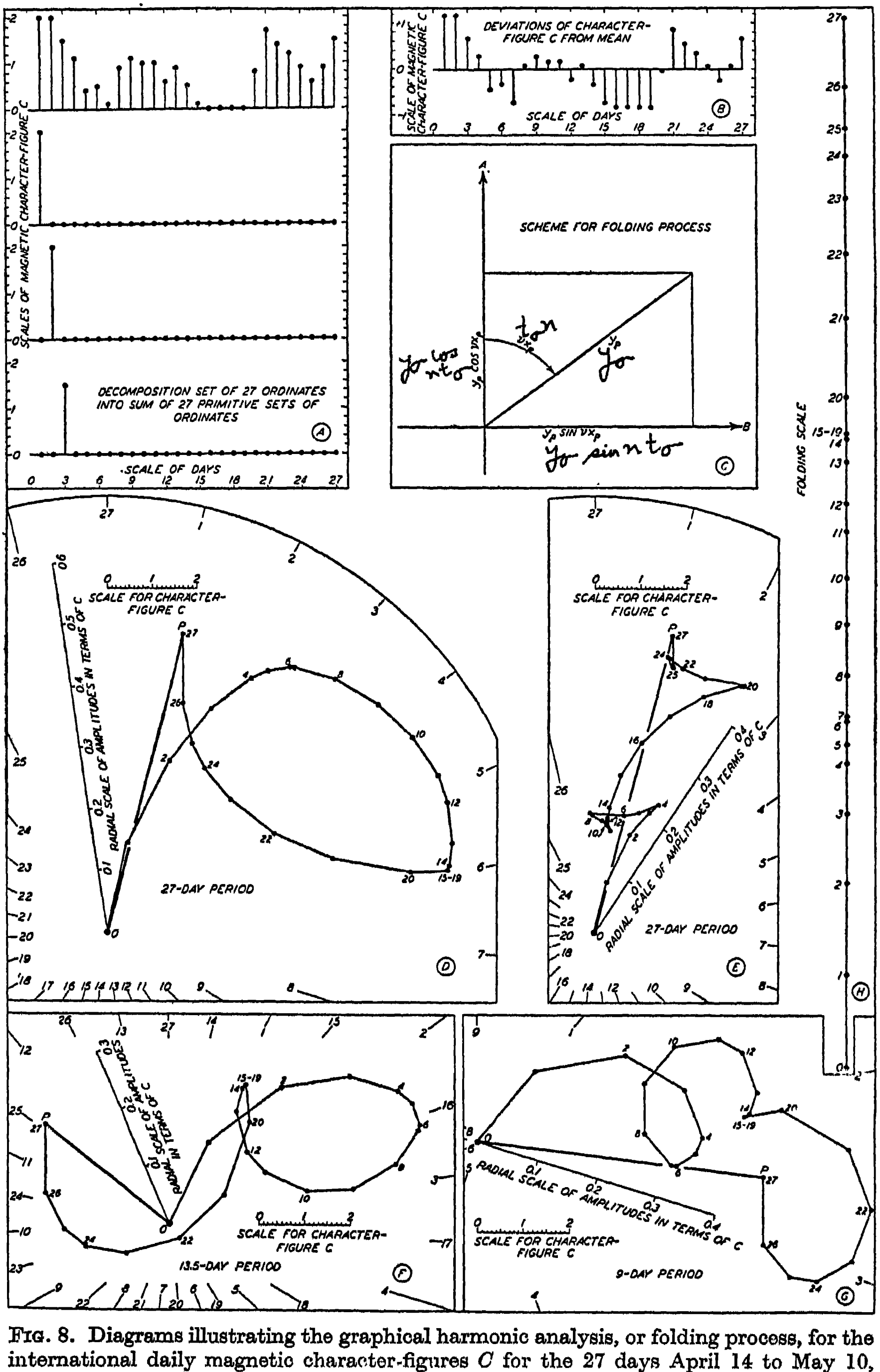

SCALES OF MAGNETIC CHARACTER-FIGURE C
DECOMPOSITION SET OF 27 ORDINATES INTO SUM OF 27 PRIMITIVE SETS OF ORDINATES
SCALE OF DAYS
0 3 6 9 12 15 18 21 24 27
Ⓐ
DEVIATIONS OF CHARACTER-FIGURE C FROM MEAN
SCALE OF MAGNETIC CHARACTER-FIGURE C
SCALE OF DAYS
Ⓑ
SCHEME FOR FOLDING PROCESS
A
B
$y_p \cos \nu x_p$
$y_p \sin \nu x_p$
y_p
νx_p
$t_\sigma n$
$y_\sigma \cos n t_\sigma$
y_σ
$y_\sigma \sin n t_\sigma$
Ⓒ
SCALE FOR CHARACTER-FIGURE C
RADIAL SCALE OF AMPLITUDES IN TERMS OF C
27-DAY PERIOD
P
Ⓓ
Ⓔ
13.5-DAY PERIOD
Ⓕ
9-DAY PERIOD
Ⓖ
FOLDING SCALE
Ⓗ

Fig. 8. Diagrams illustrating the graphical harmonic analysis, or folding process, for the international daily magnetic character-figures C for the 27 days April 14 to May 10, 1926, for the periods of 27, 13·5, and 9 days. (In Fig. C, for x_p, y_p, ν read t_σ, y_σ, n)

According to (45), the harmonic coefficients, multiplied by $\frac{1}{2}r$, are given for each primitive set, for the frequency n, by $y_\sigma \cos nt_\sigma$, $y_\sigma \sin nt_\sigma$ (where $\sigma = 1, 2,..., r$): the representation in the harmonic dial (Fig. 8 *C*) is a vector of length y_σ forming the angle nt_σ with the direction OA, because the projections of this vector are equal to the coefficients multiplied by $\frac{1}{2}r$. The sum of these r primitive vectors has, according to (45), the projections $\frac{1}{2}ra_n$, $\frac{1}{2}rb_n$, and therefore represents the sine wave for the original set. In our example (the set of 27 ordinates in the top part of Fig. 8 *A*) $r = 27$, $t_1 = 2\pi/r = 13{\cdot}3°$: for a sine wave of 27-day period, $n = 1$, and the angles nt_σ for the successive primitive vectors are 13·3°, 26·7°, 40·0°,..., 360·0°; the process of summing the vectors consists in joining together a set of lines equal in length to the ordinates y_σ, at inclinations increasing successively by 13·3° (Fig. 8 *D*). The vector between O and the end-point, P, should be divided by $\frac{1}{2}r$ ($= 27/2$) to obtain the amplitude c_1: instead, we can measure it on a scale enlarged (27/2) times (the radial scale for OP is indicated in Fig. 8 *D*). Thus we see from Fig. 8 *D*, on comparing it with Fig. 5, that the orthogonal coordinates of P, in units of *C*, are $a_1 = +0{\cdot}49$, $b_1 = +0{\cdot}12$, and its polar coordinates are $c_1 = 0{\cdot}50$, $\alpha_1 = 76°$: the sine wave, therefore, is

$$+0{\cdot}49 \cos t + 0{\cdot}12 \sin t = 0{\cdot}50 \sin(t+76°).$$

Its maximum occurs at about the time $t = 14°$, or

$$t = 14 \times (27/360) = 1{\cdot}05$$

days, or, since the time 1 day designates Greenwich noon of April 14, 1926, about 1 o'clock in the afternoon of that day.

If all the ordinates $y_1, y_2,..., y_r$ were equal, the construction in Fig. 8 *D* would lead to a regular polygon ending at the origin, that is, to vanishing coefficients, as would be expected. From the principle of superposition it follows, therefore, that a positive or negative constant can be added to all the ordinates y_σ without changing the harmonic coefficients. For instance, the arithmetic mean a_0 can be subtracted, which amounts to measuring the ordinates as positive or negative *deviations* from the level a_0 (Fig. 8 *B*); the corresponding construction of Fig. 8 *E*, plotting negative ordinates in the reverse direction, leads, of course, to the same point P as in Fig. 8 *D*.

Figs. 8 *F* and 8 *G* are analogous to Fig. 8 *D*, and show the construction of the harmonic coefficients for the frequencies $n = 2$ and $n = 3$, or for periods of 13·5 and 9 days. The scales for the time of maximum are indicated on Figs. 8 *F* and 8 *G*, while the scales for OP are the same in all the diagrams 8*D*, 8*E*, 8*F*, and 8*G*. The sine waves of frequencies 2 and 3, in units of *C*, are

$+0{\cdot}16\cos 2t-0{\cdot}20\sin 2t = 0{\cdot}26\sin(2t+141^\circ)$, with maxima on the days 11·6 and 25·1; and

$-0{\cdot}06\cos 3t+0{\cdot}46\sin 3t = 0{\cdot}46\sin(3t+353^\circ)$, with maxima on the days 2·4, 11·4, and 20·4.

The construction in Figs. 8*D*, 8*F*, and 8*G* can be interpreted as follows. Imagine a *folding scale* having links with the lengths of the ordinates y_σ. If stretched out, as illustrated in Fig. 8*H*, its entire length is equal to the sum of the ordinates, that is, in the general case, ra_0. Suppose now that each link is turned so as to make the angle $360^\circ/27 = 13{\cdot}3^\circ$ with its neighbour; we then obtain Fig. 8*D*; similarly if the angle is $2\times 360^\circ/27$ or $3\times 360^\circ/27$, we obtain Figs. 8*F* and 8*G*. In general, this turning of the folding links through the angles $n\times 2\pi/r$ furnishes $(\frac{1}{2}r)a_n$ and $(\frac{1}{2}r)b_n$, and the distance of the end-point from the origin is $(\frac{1}{2}r)c_n$. The conception of this *harmonic folding process* will be helpful later. Graphical harmonic *synthesis* has been discussed elsewhere [25*a*].

Usually only the result of the folding process, that is, the point P, is retained in the harmonic dial. Nevertheless, and although for most actual computations numerical or mechanical harmonic analysis is preferable to graphical analysis, it is sometimes useful to recall the folding process as producing the vector OP, because it reveals the contribution of each single ordinate to the final vector. This contribution is particularly clear in the folding of the deviations from the arithmetic mean (the folding scale itself then having positive and negative links); in Fig. 8*E*, the large positive ordinates 1 to 3 and the large negative ordinates 15 to 19 make the largest strides towards P. For illustration, Fig. 9 shows, for an exact cosine wave of 9 days' period, the folding process for the frequencies 1 to 4 (periods 27, 13·5, 9, and 6·75 days) in a set of 27 days.

The folding process is also a good illustration of the remarks in § 19 (*a*).

In the wave theory of optics the superposition of sine waves of equal frequency but of different phases and amplitudes is the fundamental problem of diffraction phenomena. We may mention the Cornu spirals as an example of the representation of single waves by vectors, and their superposition by graphic addition. In electrotechnics alternating currents are discussed with the aid of diagrams similar to the harmonic dial.

16.22. The theory of errors. For scalar magnitudes, each specified by a single number, there exists a set of mathematical theorems often called the 'Theory of Errors', though they are applicable also in the statistical discussion of frequency distributions of magnitudes that are free from errors. The same considerations may be extended to vectors;

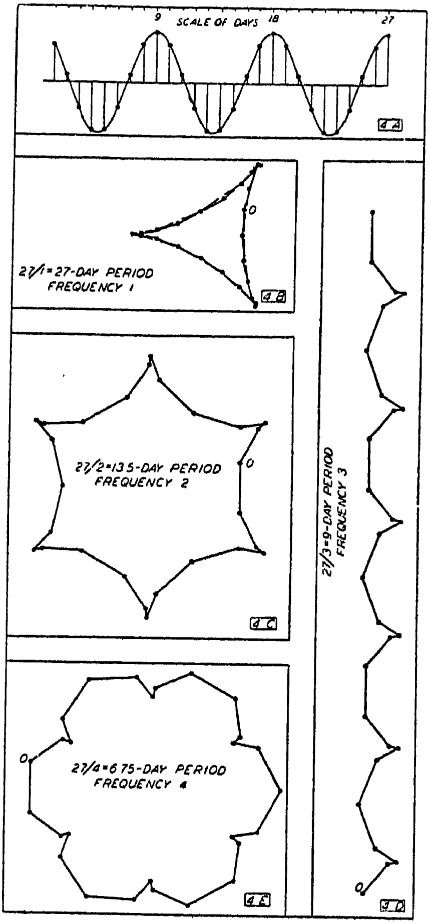


Fig. 9. The graphical harmonic analysis, or folding process, illustrated for the periods of 27 days, 13·5 days, 9 days, and 6·75 days, in a series of 27 ordinates representing a pure 9-day wave

as the basis of the generalization we choose, from among the various alternative foundations of the theory of errors, the so-called hypothesis of *elementary errors*. This assumes that every single error of observation (or, more generally, every deviation of an individual value of a series of similar observations from their mean value) is due to the superposition of very many small errors, which are of random sign, positive or negative. A series of assumptions—which are often, but not always, justifiable—then leads to the known *normal law* of error-frequency, which can be formulated as follows.

Let the observed errors be $\xi_1, \xi_2,..., \xi_N$; since they are deviations of individual observations from the mean, $\sum \xi_n = 0$. The probability that an error lies between ξ and $\xi+d\xi$ is given by

$$\phi(\xi)\,d\xi = \frac{d\xi}{m\sqrt{(2\pi)}}\exp(-\xi^2/2m^2), \tag{66}$$

where $$m^2 = (\xi_1^2+\xi_2^2+...+\xi_N^2)/N. \tag{67}$$

The distribution law (66) is therefore determined by a single parameter m, which is called the *standard deviation*, or by m^2, called the *variance* (§ 12).

Two other quantities are sometimes used to characterize the magnitude of the errors ξ_n. If the distribution of the errors follows the law (66), these quantities stand in a fixed ratio to m. The first is the average of the *numerical* values of the errors, namely

$$(|\xi_1|+|\xi_2|+...+|\xi_n|)/N;$$

this equals $0{\cdot}798m$. The second is the so-called *probable* error ρ; this is defined by the property that, in a large number of errors distributed according to the normal law, there are just as many errors numerically greater than ρ as smaller than ρ. This definition suggests the method by which we may find an approximate value of ρ from an actual finite set of data, namely, by arranging the data according to their size, finding the two values below and above which one-quarter of the total number of data lie, and halving their difference. For the normal law, it can easily be shown that

$$\rho = 0{\cdot}6745m.$$

The normal law, as well as other frequency-distributions of scalar quantities, is discussed in text-books on statistics and the theory of probability, to which the reader is referred [11.1–8]. The much discussed distinction between 'true' and 'apparent' errors—depending largely on the difference between N and $(N-1)$—is mostly irrelevant in geophysics if the number N of observations is large enough. Much more important is the property of conservation, discussed in § 27.

In the case of a distribution of errors not merely along a line but in a *plane*, the hypothesis of elementary errors leads to the question of the '*random walk*' (Pearson's problem [28]): Someone proceeds l_1 metres in a straight line from a point O, then turns through an arbitrary angle and goes l_2 metres farther, and so on successively. The relative frequency of the lengths of the straight lines l_n is supposed to be given. What is the probability that the end P of the zigzag path, after N such steps, is at a distance from the origin O, which lies between r and $r+dr$? (Fig. 10 (*a*)). Here the 'probability' means, as usual, the relative frequency, i.e. the ratio of the number of the selected cases to the whole number of cases, in the average over a very great number of trials of N steps each.

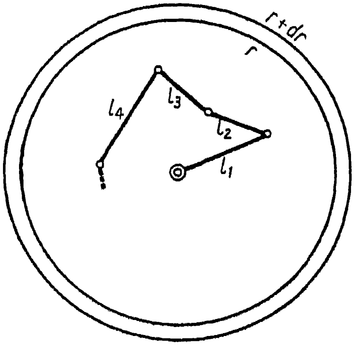


FIG. 10 *a*. Illustrating the random walk

16.23. The asymptotic solution for the random walk. The problem of the random walk has been solved by Pólya [28*a*] (following Lord Rayleigh [1] and Kluyver) for a finite number of displacements $l_1, l_2, ...$; he has also generalized it to spaces of any number of dimensions. We here consider only the asymptotic solution for very many, very small, displacements. This is given by the following probability theorem due to Markoff, which Laue [29] has newly derived, and further generalized, for the discussion of the superposition of light-vibrations (for a more exact mathematical treatment, see, for instance, Tornier [11.5] or Khintchine [11.8]).

Consider N independent variables $x_1, x_2,..., x_N$, where N is a large number; these variables are supposed to be able to take any value within a range X to X' (the same for all), and the probability that any variable x_n shall lie within any small interval of this range, x to $x+dx$, is supposed to be the same for all the variables; it will be denoted by $\rho(x)\,dx$, such that

$$\int\limits_{X}^{X'} \rho(x)\,dx = 1. \tag{67 a}$$

Let $f_1(x_1), f_2(x_2),..., f_N(x_N)$ and $g_1(x_1), g_2(x_2),..., g_N(x_N)$, be two series of functions of these variables. The probable value of any such function is

$$\int\limits_{X}^{X'} f_n(x_n)\rho(x_n)\,dx \quad \text{or} \quad \int\limits_{X}^{X'} g_n(x_n)\rho(x_n)\,dx;$$

this is called the *expectance* of the function. We will suppose that for the functions f and g the expectance is zero, so that

$$\int\limits_{X}^{X'} f_n(x_n)\rho(x_n)\,dx_n = 0, \qquad \int\limits_{X}^{X'} g_n(x_n)\rho(x_n)\,dx_n = 0 \tag{68}$$

for all values of n.

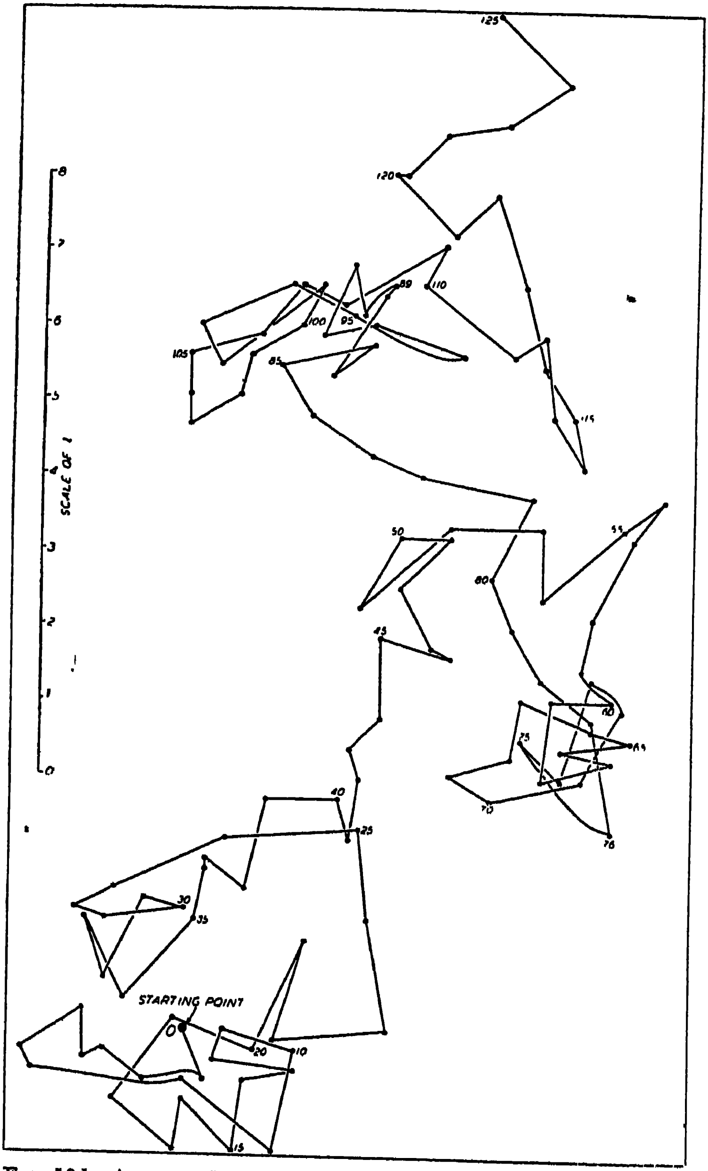


FIG. 10 *b*. An example of a random walk including 125 unequal steps (constructed with the aid of Tippet's random numbers)

Let

$$\left.\begin{aligned} J_{ff} &\equiv \sum_n \int\limits_X^{X'} f_n^2(x_n)\rho(x_n)\,dx_n, \\ J_{fg} &\equiv \sum_n \int\limits_X^{X'} f_n(x_n)g_n(x_n)\rho(x_n)\,dx_n, \\ J_{gg} &\equiv \sum_n \int\limits_X^{X'} g_n^2(x_n)\rho(x_n)\,dx_n, \end{aligned}\right\} \tag{69}$$

and let

$$J \equiv J_{ff}J_{gg}-J_{fg}^2. \tag{70}$$

Then J_{ff}, J_{fg}, and J_{gg} are the expectances for the sums of the functions f_n^2, $f_n g_n$, and g_n^2.

Let $w(u,t)\,dudt$ denote the probability that the two following conditions are both satisfied:

$$t < \sum_n f_n(x_n) < t+dt, \qquad u < \sum_n g_n(x_n) < u+du.$$

The theorem (which will not be proved here) states that

$$w(u,t) = \frac{1}{2\pi\sqrt{J}}\exp\{-(J_{gg}t^2-2J_{fg}tu+J_{ff}u^2)/2J\}. \tag{71}$$

In connexion with the problem of the random walk in a plane, f_n and g_n may be interpreted as the rectangular components of the nth displacement l_n, and x_n as the inclination of the displacement to the f-axis, so that $f_n = l_n\cos x_n$, $g_n = l_n\sin x_n$; the length l_n is supposed to be some function of x_n, which itself may take any value between 0 ($= X$) and 2π ($= X'$). If all directions x_n are equally probable, $\rho(x) = 1/2\pi$. The zero expectances of f_n and g_n (condition (68)) imply that positive and negative components of l_n, along either axis, are equally probable. The function $w(u,t)$ gives the probability (per unit area) that the end point P of the random walk of N steps shall lie in the neighbourhood of the point (t,u) in the plane; according to (71), the locus of points of equal probability is the curve

$$J_{gg}t^2-2J_{fg}tu+J_{ff}u^2 = \text{constant},$$

which is an ellipse because J_{ff} and J_{gg} are necessarily positive.

In the case of sets of functions f_n, g_n whose expectances (F_n, G_n) are not zero, the theorem is applicable to their deviations (f_n', g_n', say) from these expectances, that is, to the sets of functions f_n-F_n, g_n-G_n.

We may note that the correlation coefficient between the f_n's and g_n's, by (50), is equal to

$$\mathrm{R} = J_{fg}/\sqrt{\{J_{ff}J_{gg}\}}. \tag{72}$$

Because of this relation, the surface whose height above the (f, g) plane is $w(u,t)$, given by (71), is called the *correlation surface.*

The equations take their simplest form if the sets of functions f_n and g_n are *normalized,* each being multiplied by such factors as will make $J_{ff} = J_{gg} = 1$, $J_{fg} = \mathrm{R}$. In this case, if the functions f_n and g_n are independent of each other, so that $\mathrm{R} = 0$, the ellipses become circles.

16.24. The random walk with unit displacements. In the simplest case of the random walk all the displacements are of the same length unity ($l_n = 1$). Hence, in the formulae of § 23,

$$\left.\begin{aligned} X = 0,\ X' = 2\pi;\quad f_n(x_n) = \cos x_n,\ g_n(x_n) = \sin x_n;\\ \rho(x_n) = \frac{1}{2\pi};\quad J_{ff} = J_{gg} = \tfrac{1}{2}N,\ J_{fg} = 0,\ J = \tfrac{1}{4}N^2;\\ w(u,t)\,dudt = \frac{1}{N\pi}\exp\{-(t^2+u^2)/N\}\,dudt. \end{aligned}\right\} \tag{73}$$

In polar coordinates, writing $t^2+u^2 = r^2$, and taking the surface element to be a thin ring $2\pi r\,dr$, the probability $w(r)\,dr$, that the end-point is in the ring with inner and outer radii r and $r+dr$, is given by

$$w(r) = (2r/N)\exp(-r^2/N) \quad (\textit{Rayleigh's formula}\ [1]). \tag{74}$$

By simple integration the *expectance* (that is, the value to be expected in the average of a large number of cases) for the average distance of the end-point from the origin, for a large number of such random walks, is clearly

$$E_1 = \int\limits_0^\infty rw(r)\,dr = \tfrac{1}{2}\sqrt{(\pi N)} = 0{\cdot}886\sqrt{N}; \tag{75}$$

this is the expectance of the distance OP. The mean value of the *square* of the distance is

$$E_2^2 = \int\limits_0^\infty r^2w(r)\,dr = N. \tag{76}$$

The probability $W(R)$ that the distance of the end-point from the origin exceeds a fixed limit

$$R = \kappa\sqrt{N} = \kappa E_2$$

can be given explicitly, by

$$W(R) = \int\limits_R^\infty w(r)\,dr = \exp(-R^2/N) = \exp(-\kappa^2). \tag{77}$$

(In the case of linear errors, the analogous expression cannot be integrated explicitly.)

The formula (76) for the expectance E_2 is more fundamental than the distribution law (74), and can be derived directly, and in the more general form corresponding to unequal displacements l_n, as follows. Suppose that the displacements are taken at random from a large number of values $c_{n,m}$ ($n = 1,..., N$; $m = 1,..., M$); N may be any number (2, 3,...), while M is supposed to be large. The average square of the displacements is given by

$$c^2 = \sum_n \sum_m c_{n,m}^2/NM.$$

The mth of M random walks, with N displacements $c_{1,m}, c_{2,m},..., c_{N,m}$, at angles $x_{1,m}, x_{2,m},..., x_{N,m}$ respectively, leads to a point P such that OP² is equal to

$$(c_{1,m}\cos x_{1,m}+...+c_{N,m}\cos x_{N,m})^2+(c_{1,m}\sin x_{1,m}+...+c_{N,m}\sin x_{N,m})^2$$
$$= c_{1,m}^2+...+c_{N,m}^2+\text{'mixed' terms,}$$

the latter being of the form

$$c_{1,m}c_{2,m}(\cos x_{1,m}\cos x_{2,m}+\sin x_{1,m}\sin x_{2,m}) = c_{1,m}c_{2,m}\cos(x_{1,m}-x_{2,m});$$

because of the randomness of the angles $x_{n,m}$ the sum of the mixed terms is of the order $M^{\frac{1}{2}}$ only (by the law of propagation of errors, § 1), while the sum of the squares is MNc^2. Therefore for large values of M the average squared distance, E_2, is given by

$$E_2^2 = Nc^2, \quad \text{and} \quad E_2 = c\sqrt{N},$$

which is a generalization of (76) (that equation refers to the special case $c_{n,m} = 1 = c$), and valid for $N = 2, 3,...$. The distribution formula (74) is, however, only the *asymptotic* formula for *large* values of N; Pearson [28] derived the exact distribution formulae for $N = 2, 3,...$. Fortunately, for practical purposes, (74), or its more general form for displacements of mean square length c, namely

$$w(r) = (2r/c^2N)\exp(-r^2/c^2N), \tag{78}$$

is a good approximation to these exact laws, even for such comparatively small numbers as $N = 10$, provided that r is sufficiently small compared with cN, the maximum value of OP; this value can be reached only by N displacements c which are all in the same direction; naturally this is an extremely rare case.

16.25. The harmonic components of a random set of numbers. The probability theorem of § 24 will now be applied to find the probable distribution of the harmonic coefficients a_s and b_s, or of the amplitude c_s and phase ϵ_s of the sth harmonic $c_s \sin(st+\epsilon_s)$, in the expression of the type (19) by which, for the values $t_n = 2\pi n/N$, we may represent a set of N numbers x_n ($n = 1$ to $n = N$), when these numbers are chosen at random from the whole range of positive and negative numbers; in this case, therefore, the limits X and X' of § 24 are $\pm\infty$.

For this purpose we choose the functions f_n and g_n as follows:

$$f_n(x_n) = (2/N)(\cos st_n)x_n, \quad g_n(x_n) = (2/N)(\sin st_n)x_n. \tag{79}$$

Then (in the present notation) the equations (45) indicate that

$$a_s = \sum_{n=1}^{N} f_n(x_n), \quad b_s = \sum_{n=1}^{N} g_n(x_n), \tag{80}$$

and a_s, b_s are the coordinates of a random walk of N steps (f_n, g_n).

Since the numbers x_n are chosen at random from the whole positive and negative range,

$$\int_{-\infty}^{\infty} x_n \, \rho(x_n)\, dx_n = 0. \tag{81 a}$$

Let

$$\mu^2 \equiv \int_{-\infty}^{\infty} x_n^2 \rho(x_n)\, dx_n, \tag{81 b}$$

so that μ^2 is the variance, or square of the standard deviation, of the set of numbers x_n to be analysed. Also let

$$m^2 = \mu^2/N, \tag{82}$$

so that m^2, by the law of propagation of errors (§ 1), is the variance for the *average* of the N numbers.

The numbers J_{ff}, J_{fg}, J_{gg}, and J of (69, 70) can be evaluated in terms of m^2 with the aid of (44); we obtain $J_{fg} = 0$, $J_{ff} = J_{gg} = 2m^2 = \sqrt{J}$, and from (71) it follows that the probability $w(r)\,dr$ that the amplitude c_s lies between r and $(r+dr)$ is

$$w(r) = (r/2m^2)\exp(-r^2/4m^2). \tag{83}$$

This result contains only m, but not the frequency s, and is therefore *independent of the period.*

Since (83) may be derived from (74) by merely writing $4m^2$ instead of N, it follows from (74, 75) that the expectances E_1 for the amplitude and E_2 for the square root of the average squared amplitude are

$$E_1 = m\sqrt{\pi}, \tag{84}$$

$$E_2 = 2m. \tag{85}$$

The probability that the amplitude R is exceeded is (cf. (78))

$$W(R) = \exp(-R^2/4m^2). \tag{86}$$

Expressing R as a multiple of E_1 or E_2, in the form

$$R = \kappa_1 E_1, \quad R = \kappa_2 E_2,$$

(86) can be written as follows:

$$W(\kappa_1 E_1) = \exp(-\pi\kappa_1^2/4), \quad W(\kappa_2 E_2) = \exp(-\kappa_2^2). \qquad (86\text{ a})$$

The following table contains several values of κ_2, which correspond to simple values of $W(\kappa_2 E_2)$; $W(0{\cdot}32E_2) = 0{\cdot}9$ implies, for example, that on harmonic analysis of a large number of sets of N random numbers, such that each set has the same standard deviation μ, it will be found that $\frac{9}{10}$ of all the amplitudes c_s exceed $0{\cdot}32E_2$ or $0{\cdot}64\mu/\sqrt{N}$.

TABLE 1

$\kappa_2 =$	0	0·32	0·47	0·60	0·72	0·83	0·96	1·10	1·27	1·52	2·15	2·63	3·04	3·72	4·29	4·80	5·26
$W(\kappa_2 E_2) =$	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·2	0·1	10^{-2}	10^{-3}	10^{-4}	10^{-6}	10^{-8}	10^{-10}	10^{-12}

The circles with the radii $\kappa_2 E_2$, which correspond to $W = 1{\cdot}0, 0{\cdot}9, \ldots, 0{\cdot}1$, divide the plane of every harmonic dial (for whatever frequency) into ten circular rings, within each of which one-tenth of all the end-points are situated. The *probable-error circle*, which divides the plane into two regions containing equal numbers of end-points ($W = 0{\cdot}5$), has the radius $0{\cdot}833E_2$. The relation between the average amplitude E_1 and the root of the mean squared amplitude, E_2, is, by (84) and (85),

$$E_1 = \tfrac{1}{2}E_2\sqrt{\pi} = 0{\cdot}886E_2. \qquad (87)$$

Both the parameters E_1 and E_2 are commonly used, but E_2 is preferable.

16.26. Effects of observational errors on harmonic coefficients.

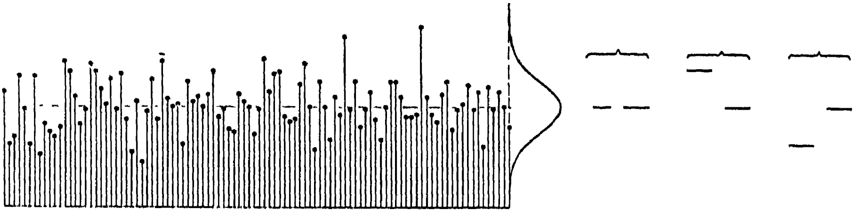
The preceding formulae enable us to determine the uncertainty in the harmonic coefficients for a component of any period, calculated from a set of N equidistant values of some observed quantity, when each of these values includes not only the sum of the contributions made by the truly periodic variations to which the data are subject, but also includes a random part x_n (of known standard deviation). The random part may be made up of errors of observation, and of the aperiodic variation $g(t)$ (in the notation of § 1).

In geophysical data the part of x_n which corresponds to mere errors of observation (or errors whose effects are similar to theirs) can often be neglected. But as already shown in § 4, in geophysical applications $g(t)$ often includes great waves, whose effect on successive ordinates (e.g. hourly values) changes only slowly. Hence this part does *not* satisfy the criterion of randomness, which is presupposed in the deduction of (83). A direct application of the formulae (82)–(86) would consequently lead to inadequate estimates of the actual errors.

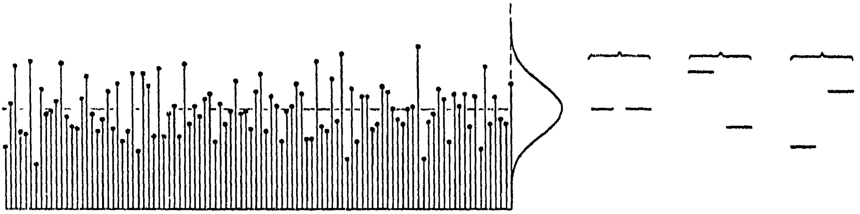
Nevertheless the formulae (83) to (87) can usually be formally adopted; but in these cases the parameter m (or, what is equivalent, E_1 or E_2) cannot be deduced from the 'scatter' μ of the individual ordinates, but must be determined, subsequently, in other ways. This can be done when the material from which the harmonic components are to be calculated is subdivided into several equal parts, and each part separately analysed. For each harmonic component the coefficients a_s, b_s, derived from the separate parts of the material, are plotted as points on the harmonic dial. The deviations of these points from their centroid or mean position are used in estimating the parameter m, just as the uncertainty of the measurements of a scalar magnitude must be found from the deviations of repeated measurements from their mean value.

In order to show clearly the considerations involved in determining m, we start with a discussion of what may be called the '*morphology*' of geophysical time-series [23].

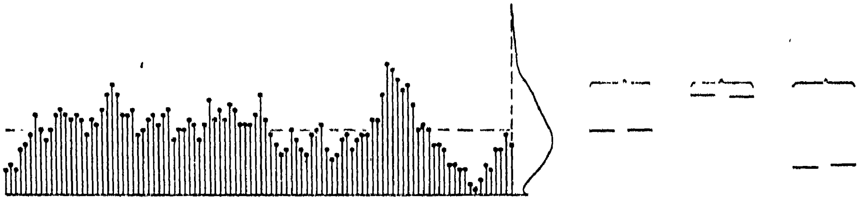
16.27. Random series, and series with conservation. Consider a number of values (ordinates) y_σ of a geophysical quantity given at equidistant time-intervals Δt. Regarded as a 'population', in the sense of mathematical statistics, it can be submitted to the ordinary procedures for computing averages, frequency-distributions, standard deviations, etc. These statistical conceptions, and the laws concerning them, have mostly been developed with reference to random series (such as dice-throws), or to biometrical data. Now there is a fundamental difference between such sets of data (for instance, lists of heights of individuals arranged in the alphabetical order of their names) and the time-series of geophysical observations [23]. Successive data in the former case are mostly independent or random, whereas geophysical data mostly show positive *conservation*, by which it is meant that high values are likely to be followed by relatively high values, and low values by relatively low values, because of the usual gradual variation of the data; this is so, for instance, for the hourly values of the atmospheric pressure, for the daily sunspot figures, and so on. In Fig. 11 *a* three examples are given: (1) for a set of random ordinates, constructed with the help of Tippett's [24] Random Sampling Numbers, (2) for a set of *differences* of such random ordinates (see below), and (3) for a set of daily relative sunspot numbers. The curves in the centre give the frequency distribution of the ordinates, and the three pairs of horizontal lines on the right indicate the degree of conservation, as follows. In each of the three pairs the first line gives the average of certain selected ordinates, and the second line gives the average of the ordinates immediately following the selected ordinates. The selected ordinates are, in the first pair, those near the general average, and in the second and third pairs, the high and the low ordinates respectively. In the case of the random numbers, the ordinates succeeding the selected ordinates always give the general average, however the selected ordinates are chosen; this corresponds to no conservation, either positive or negative. Negative conservation, of course, means that high ordinates are, on the average, followed by low ordinates, and low ordinates by high ordinates; this kind of conservation is shown by the differences between successive random numbers (Fig. 11 *a*), because whenever one of the latter stands out from the ones before and after, the preceding and succeeding differences are of opposite sign. The sunspot numbers, on the other hand, illustrate the more usual case of positive conservation.



RANDOM ORDINATES—ZERO CONSERVATION



DIFFERENCES OF SUCCESSIVE RANDOM ORDINATES—NEGATIVE CONSERVATION



RELATIVE SUNSPOT-NUMBERS JUNE-SEPTEMBER 1929—POSITIVE CONSERVATION

FIG. 11 *a*. Examples of series of ordinates having no conservation, negative conservation, and positive conservation

The standard deviation of the ordinates y_σ will be denoted by m, or preferably (here) by $m(1)$; m here is not the same as the m of § 26. The symbol $m(h)$ will be used to denote the standard deviation of the series of numbers $y_\sigma(h)$, defined as the averages of sets of h successive ordinates, such that

$$y_\sigma(h) = (y_\sigma + y_{\sigma+1} + \dots + y_{\sigma+h-1})/h.$$

If the numbers y_σ have a random distribution—that is, if they have the same statistical properties as if they were drawn from a large supply in the manner of lottery tickets—the fundamental law of propagation of errors (§ 1) asserts that

$$m(h) = m/\sqrt{h}. \tag{88}$$

(Example: $m = 40$; $h = 400$; $m(400) = 40/\sqrt{400} = 2$.) This law is the basis for the confidence felt in averages derived from large samples of that kind.

From any such random series y_σ we may construct a new series y'_σ by repeating each ordinate w times. Then the average of h of the new ordinates, if $h = wh'$, and h' is an integer, is simply the average of h' random ordinates y_σ. Hence the standard deviation $m'(h)$ for these averages of the series y'_σ is given by

$$m'(h) = m(h') = m/\sqrt{h'} = m/\sqrt{(h/w)} = (\sqrt{w})(m/\sqrt{h}), \tag{89}$$

and

$$w = \left(\frac{m'(h)}{m/\sqrt{h}}\right)^2 = h\{m'(h)\}^2/m^2. \tag{90}$$

(Applied to the preceding example, and taking $w = 4$, this implies that $m'(400) = 40/\sqrt{(400/4)} = 4$.)

These simple equations lead to a natural convenient measure of the degree of conservation in any series. We compute the standard deviations $m(h)$, for the original ordinates ($h = 1$), and also for the averages of every h successive ordinates ($h = 2, 3,\dots$). Writing

$$\epsilon(h) = \left(\frac{m(h)}{m/\sqrt{h}}\right)^2 = hm^2(h)/m^2, \tag{91}$$

we call $\epsilon(h)$ the *equivalent number of repetitions*, because of the obvious analogy to w in the formula (90); and we regard $h/\epsilon(h)$ as the *effective number of random ordinates* in the sets of h successive ordinates y_σ. In the case of random ordinates, $\epsilon(h) = 1$; in series of ordinates which show positive conservation (the usual case) $\epsilon(h) > 1$.

In the example of negative conservation (which is the rarer case) given above, namely the series formed by taking the differences between successive random ordinates (Fig. 11 *a*), if μ is the standard deviation of the original random series, the standard deviation m of the differences $y_{\sigma+1} - y_\sigma$ is obviously given by $m^2 = 2\mu^2$. In this case the sums of h successive ordinates $y_{\sigma+1} - y_\sigma$ are simply the differences between the first and the hth succeeding random number, so that the *sums* also have the standard deviation m, and the corresponding *averages* have the standard deviation $m(h) = m/h$. Therefore, by (91), $\epsilon(h) = 1/h < 1$ in this case.

It may be that $\epsilon(h)$ tends towards a limit $\epsilon(\infty)$ as $h \to \infty$. If so, and if h_0 is the smallest number for which $\epsilon(h_0)$ is approximately equal to $\epsilon(\infty)$, the time-interval $h_0 \Delta t$ (of course not strictly defined) may be called an *appropriate statistical time-unit*, since the ordinates separated by such time-intervals will be substantially independent.

It may be shown [23] that, in general,

$$\mathrm{R}(h) = \{\epsilon(2h)/\epsilon(h)\} - 1, \tag{92}$$

where $\mathrm{R}(h)$ denotes the correlation-coefficient between the two series of successive averages, $y_\sigma(h)$ and $y_{\sigma+h}(h)$.

16.28. Conservation in magnetic character-figures, sunspots, and mortality data. The degree of conservation shown by the following material, for the years of high sunspot activity 1917–19, 1925–9, has been studied: (i) the daily relative sunspot numbers s for the whole disk, (ii) the daily sunspot numbers s_c for the central circle of the disk, of half the diameter, (iii) the international magnetic character-figures C. The standard deviations for the daily values of these three quantities, in the order given, are $m(1) = 36{\cdot}6$, $23{\cdot}1$, $0{\cdot}484$. The values of $\epsilon(h)$ are as follows, for values of h from 1 to 16 days.

TABLE 2

$h =$		*1*	*2*	*4*	*8*	*16 days*
Sunspots, whole disk, s	$\epsilon(h) =$	1·00	1·99	3·49	5·91	8·19
Sunspots, central disk, s_c	$\epsilon(h) =$	1·00	1·76	2·83	3·71	4·25
Magn. char.-fig. C	$\epsilon(h) =$	1·00	1·56	2·09	2·33	2·47
R, or $\{\epsilon(2h)/\epsilon(h)\} - 1$, for s		0·99	0·75	0·70	0·38	0·64

For s the conservation has been studied for longer intervals, using the full series of observations since 1749: the results are as follows.

TABLE 3. *Conservation in the daily sunspot numbers, s, whole disk.*

h (days) . .	32	64	128	256	512	1,024	2,048	4,096
$\epsilon(h)$ . . .	14·0	26·8	51·7	99	190	325	323	201
$h/\epsilon(h)$ . . .	2·3	2·4	2·5	2·6	2·7	3·1	6·3	20·4
R, or $\{\epsilon(2h)/\epsilon(h)\} - 1$	0·92	0·93	0·92	0·92	0·71	−0·01	−0·38	. .

It is remarkable that in 1,024 days, or nearly 3 years, the equivalent number of independent daily values thus estimated (namely $h/\epsilon(h)$, $h = 1024$) is not more than 1024/325 or 3·1. In other words, the standard deviation of a set of means of the daily sunspot numbers s, over intervals of 1,024 days (or about 3 years), whether these intervals overlap or not, far exceeds $1/\sqrt{(1024)}$ times the standard deviation $m(1)$ of the set of daily sunspot numbers themselves, being, in fact, $\sqrt{325}$ times $m(1)/\sqrt{1024}$, or $m(1)/\sqrt{3 \cdot 1}$; this is a striking indication of the lack of random character in such a set. The high conservation is, of course, explained by the life-history of single spot-groups and by the solar cycle; the influence of the 11-year cycle in *reducing* $\epsilon(h)$ only becomes predominant from $h = 2048$ onwards. In the correlation-coefficients $\mathrm{R}(h)$ or $\{\epsilon(2h)/\epsilon(h)\}-1$ for successive averages of h values, the sun's rotation period is responsible for the small coefficient 0·38 for the 8-day means (corresponding to the comparative independence of the numbers of spots on the visible and invisible hemispheres; the 10- or 12-day means, not examined, might yield even smaller correlations); the 11-year cycle reverses the correlation for means over periods of 3 or 6 years.

The tables show clearly why, for the statistical treatment of problems involving such data as s and C, the series used must be very long.

In this connexion it may be mentioned that correlations have been claimed [48] between s or C and certain causes of mortality. Some figures will, however, show the small degree of conservation found in the daily figures, for Berlin, for suicides, and for deaths by accident or from cancer. The values of $\epsilon(10)$ in these three cases are only 1·18, 1·09, and 1·00 respectively, as compared with values exceeding 2·3 for s or C. The correlation, if any, between phenomena with such different degrees of conservation can only be small, and this inference has been confirmed by more detailed calculations [47]. But deaths from pneumonia and other pulmonary complaints show high positive conservation—$\epsilon(10) = 3 \cdot 12$—due to the contagiousness of some of these diseases.

16.29. Statistical test for periodicities; the persistence curve. The process described in § 1 for the computation of periodic functions of period $T = r\Delta t$ consists essentially in arranging the data in rows (*individual sets*) of r ordinates, and forming *average sets* which consist of the averages of the r columns of h individual sets. The method of § 27 can be generalized to apply to this case, as follows. Let ζ_r^2 denote the average square of the deviations of the single ordinates of an individual set from the average for the set, or rather let ζ_r^2 denote the average of such average squares, for a large number of individual sets, so that ζ_n is the average standard deviation for the individual sets. In like manner, let $\zeta_r(h)$ denote the average standard deviation for a large number of the average sets.

Incidentally we may note that if the total number of ordinates N is large compared with r, and if $\zeta \equiv m(1)$ is the standard deviation from the mean for the *whole* series, and $m(r)$ the standard deviation for averages of r successive ordinates, we may show, as in (47 a), that

$$\zeta_r^2 = \zeta^2 - m^2(r); \tag{93}$$

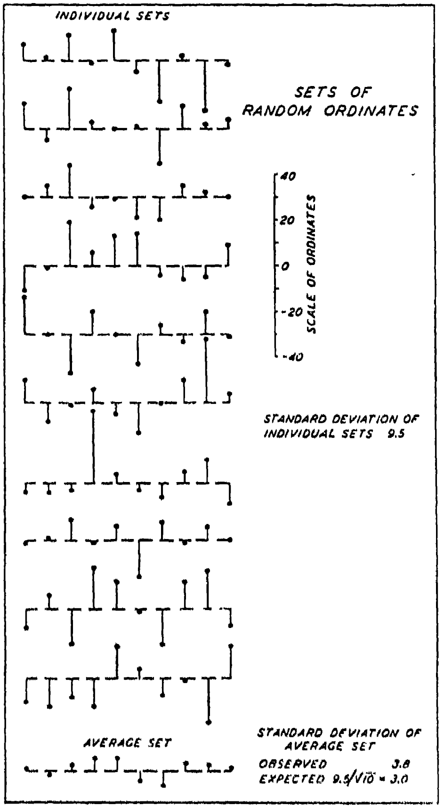


Fig. 11 *b*. Individual sets of 10 (= r) random ordinates, and the average set for 10 (= h) individual sets

hence if there is no conservation, that is, if $m^2(r) = m^2/r$, according to (88), then

$$\zeta_r^2 = (r-1)\zeta^2/r. \tag{94}$$

For random ordinates (Fig. 11 *b*)

$$\zeta_r(h) = \zeta_r/\sqrt{h}. \tag{95}$$

By analogy with (91), let

$$\sigma_r(h) = \left(\frac{\zeta_r(h)}{\zeta_r/\sqrt{h}}\right)^2 = h\zeta_r^2(h)/\zeta_r^2; \tag{96}$$

we call $\sigma_r(h)$ the *equivalent number of repeated sets* in h successive sets, and $h/\sigma_r(h)$ the *effective number of independent sets*; a repeated set here means a succession of sets in which the *variations* from each ordinate to the next are the same for every set, so that the sets differ only in their averages. For h repeated sets, therefore, the average set likewise differs from the individual sets only as regards its average value, so that in this case (96) gives $\sigma_r(h) = h$; for statistically independent sets, on the other hand, $\sigma_r(h) = 1$. In actual sets derived from geophysical observations, $\sigma_r(h)$ lies between these limits, because certain features are repeated in successive sets, while others occur only once.

If as $h \to \infty$ the equivalent number of sequences $\sigma_r(h)$ approaches a limiting value

$$S_r = \sigma_r(\infty), \tag{96 a}$$

we have the case of *asymptotic 'quasi-persistence'*, described as a tendency for the recurrence of certain similarities in successive rows, which, however, vanishes gradually, so that there is no relation between rows sufficiently far apart. In its pure form this case is represented by the oscillations of a pendulum which is set in motion and, after irregular intervals during which it is allowed to swing freely, is given irregular impulses that change the phase and amplitude of the oscillations; the movement thus consists of trains of regular oscillations of a definite period, interrupted from time to time by sudden changes of phase and amplitude. A typical geophysical case, discussed in § 33, is that of the 27-day recurrence-tendency in magnetic activity.

From (96), in cases where $\sigma_r(h) \to S_r$ as $h \to \infty$,

$$\zeta_r(h) = \zeta_r/\sqrt{(h/S_r)}, \qquad \zeta_r^2(h) = \zeta_r^2 S_r/h, \tag{95 a}$$

for sufficiently large values of h.

The case of a random non-periodic part $g(t)$ (§ 1) is governed by (95). A truly periodic (*persistent*) function $w(t)$ of period $r\Delta t$ provides identical average rows for all values of h, so that $\zeta_r(h) = \zeta_r(1)$, and $\sigma_r(h) = h$. In the case of a function $w(t)+g(t)$, where $g(t)$ is a truly random function, the average rows for sufficiently large values of h will show $w(t)$ in its pure form, since the effect of $g(t)$ is reduced in the ratio $1/\sqrt{h}$. If the ordinates are written $y = w+g$, as in § 1, and if, in individual sets of r ordinates each, the standard deviations of y, w, and g are ζ_r, ω_r, and γ_r, respectively, then, if there is no correlation between w and g, i.e. if $\sum wg = 0$, obviously

$$\zeta_r^2 = \omega_r^2+\gamma_r^2. \tag{97}$$

In average sets, the periodic part w remains unchanged, while the averaged non-periodic part g decreases, so that $\gamma_r(h) = \gamma_r(1)/\sqrt{h}$.

Hence
$$\zeta_r^2(h) = \omega_r^2 + \gamma_r^2/h,$$
and
$$\sigma_r(h) = h\zeta_r^2(h)/\zeta_r^2 = h(\omega_r/\zeta_r)^2 + \gamma_r^2/\zeta_r^2,$$
or, by (97),
$$\sigma_r(h) = 1 + (\omega_r/\zeta_r)^2(h-1). \tag{96 b}$$

Finally we consider the threefold combination $y = w+g+a$, where a, not correlated with w and g, has the property of asymptotic quasi-persistence, with a standard deviation α_r in individual sets of r ordinates,

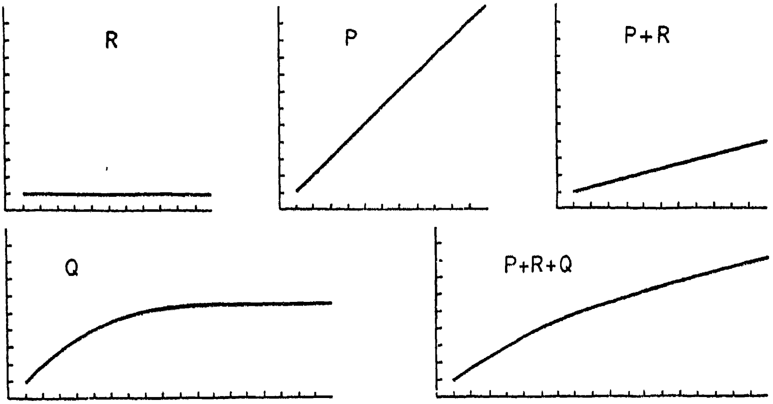


FIG. 12. Persistence curves $\sigma(h)$—abscissae σ, ordinates h—for random fluctuation (R), persistent periodicity (P), quasi-persistent functions (Q), and for combinations ($P+R$) and ($P+Q+R$)

and an asymptotic equivalent number S_r of repeated sets, as defined in (96 a). In this case
$$\zeta_r^2 = \omega_r^2 + \gamma_r^2 + \alpha_r^2, \tag{98}$$
and since (95 a) is valid for $\alpha_r(h)$ for large values of h,
$$\zeta_r^2(h) = \omega_r^2 + (\gamma_r^2 + \alpha_r^2 S_r)/h;$$
hence (96), after substituting for γ_r from (98), now becomes
$$\sigma_r(h) = 1 + (S_r - 1)(\alpha_r/\zeta_r)^2 + (h-1)(\omega_r/\zeta_r)^2. \tag{96 c}$$

A curve showing $\sigma_r(h)$ as a function of h may be called the *persistence curve* for periods of the length $r\Delta t$. The main types are illustrated in Fig. 12; other cases and combinations may occur.

By (44) (cf. also (47 b)) the standard deviation for the r ordinates of a single sine wave with period $r\Delta t$ and amplitude c is $c/\sqrt{2}$, and for the sum of a number of sine waves with periods $r\Delta t/n$, where n is an integer, is $\sqrt{(\frac{1}{2}\sum_n c_n^2)}$. This is therefore the value of ω_r in the above equations.

16.30. Persistence in relation to the periodogram. Suppose that the harmonic components of the various frequencies n, in the interval

$r\Delta t$, are calculated for each of B rows of r successive values of the function $w(t)+g(t)+a(t)$ considered on p. 589. Let the rows be numbered $\beta = 1, 2,..., B$, and let the coefficients for the harmonic component n, in the row β, be denoted by $a_n^{(\beta)}$ and $b_n^{(\beta)}$, the corresponding amplitude and phase being $c_n^{(\beta)}$ and $\epsilon_n^{(\beta)}$. Let this component be represented on a harmonic dial (for the frequency n) by a point $\mathrm{P}^{(\beta)}$, with coordinates $a_n^{(\beta)}$, $b_n^{(\beta)}$. The B rows will thus provide a *cloud of points* (19.2) on the dial.

Let the *centroid* of the cloud (that is to say, the point whose coordinates are $\sum_\beta a_n^{(\beta)}/B$, $\sum_\beta b_n^{(\beta)}/B$) be C, and let m_n denote the *root-mean-square radius* of the cloud, which is defined by the equation

$$m_n^2 = \sum_\beta (d_n^{(\beta)})^2/B, \tag{99}$$

where $d_n^{(\beta)}$ denotes the length of $\mathrm{CP}^{(\beta)}$. Let $E_2(c_n)$ denote the root-mean-square distance of the points $\mathrm{P}^{(\beta)}$ from the origin O, so that

$$E_2^2(c_n) = \sum_\beta (c_n^{(\beta)})^2/B. \tag{100}$$

Schuster, who first introduced statistical conceptions in order to test the significance of periodic (harmonic) components derived from series of geophysical or other data, called $E_2(c_n)$ the *expectancy* of the amplitude c_n.

In his studies of periodicity in such data as sunspot numbers, Schuster considered a large number of frequencies n, and determined the expectancy $E_2(c_n)$ for each. He plotted the values of $E_2(c_n)$ as ordinates, with abscissae n, and called the resulting graph a *periodogram*.

The abscissae n in the periodogram need not be confined to integer values; non-integral values n' may also be considered. It must be remembered that the periods of integer frequency n must relate to some particular number of values (r) in each row of the data analysed, so that they are integral submultiples of the 'standard' interval $r\Delta t$. By arranging the N values of the data in rows of other lengths r', and considering the nth harmonic component in each such row (n being an integer), we include in our analysis the components of period $r'\Delta t/n$; if this is equal to $r\Delta t/n'$, so that $n' = r'n/r$, the frequency of this component relative to the standard interval is n', which is in general a non-integer.

If the variation of the data analysed is due entirely to a number of pure sine waves of frequency n_1, n_2,... (relative to the chosen standard interval $r\Delta t$), which numbers need not be integers, the periodogram will have ordinates of magnitudes c_{n_1}, c_{n_2},..., if the numbers $n_1 r$, $n_2 r$,...

are integers, so that rows corresponding to some integral multiple of the period can be formed; otherwise the ordinates at n_1, n_2,... will approximate to these values. But the ordinates will not in general be zero for other abscissae, not corresponding to the true periods present in the data. It is not difficult to see that the component of frequency n_1, say, will make a contribution nearly equal to c_{n_1} to $E_2(c_n)$, for closely adjacent values of n, though there may be no true periodicity with such frequency n. Thus each true periodicity will be indicated by a maximum on the periodogram; the larger the value of N, the narrower will be the humps on the periodogram at the various maxima.

In the case of the function $w(t)+g(t)+a(t)$ considered in § 29, in which $w(t)$ is truly periodic with the period $r\Delta t$, and $g(t)$ is a random function with standard deviation μ (in the notation of § 25), while $a(t)$ is a function with quasi-persistence but with no true periodicity commensurate with the period $r\Delta t$, each part of the function will contribute to the amplitudes $c_n^{(\beta)}$ and to $E_2(c_n)$. By (82, 85), expressed in the present notation, the contribution of $g(t)$ to $E_2^2(c_n)$ will be $4\mu^2/r$, whatever the value of n; if $w(t)$ and $a(t)$ were not present, E_2 would be $2\mu/\surd r$, independent of n. However large may be the number (N) of available values of the data, the contribution of $g(t)$ to $E_2^2(c_n)$ cannot be reduced below the value $4\mu^2/r$. The joint contribution to $E_2^2(c_n)$, made by the nth component of $w(t)+g(t)$, will be $c_n^2+4\mu^2/r$, where c_n is the amplitude of the truly periodic nth component of $w(t)$.

If $a(t)$ were absent, the maxima of the periodogram would depend solely on the distribution of the truly periodic components, so far as integral values of n are concerned; the random function $g(t)$ would merely modify the relative heights of the maxima (reducing them relatively to the heights at values of n for which no true periodicity is present). If non-integral values of n are also considered, the corresponding modifications of r would somewhat affect the contribution of $g(t)$ to E_2, but ordinarily with only slight influence on the shapes of the maxima. Thus the presence of a truly random contribution in the data may only slightly reduce the usefulness of the periodogram.

In geophysical data, however, the non-periodic but also non-random variation $a(t)$ may seriously influence the form of the periodogram, because its contribution to E_2 may be highly dependent on n. Hence $a(t)$ needs serious consideration [22], though many writers, including even Schuster, have overlooked this.

The influence of $a(t)$ will here be considered, however, only in relation to the determination of the average harmonic coefficients, $a_n(h)$

and $b_n(h)$, from h successive rows of values of the original data. Because of the linearity of the harmonic-analysis formulae (45), $a_n(h)$ and $b_n(h)$ are the averages of the coefficients $a_n^{(\beta)}$ and $b_n^{(\beta)}$ for the h individual rows. Hence, on the harmonic dial, the point $a_n(h)$, $b_n(h)$ is simply the centroid of the h points $P^{(\beta)}$. If such averages are formed for each successive set of h rows, the original cloud of B points on the harmonic dial is reduced to one of B/h points.

If $a(t)$ were absent, the vectors from the origin O to any of these points, P, would be the vector sum of the vector OP_n, say, corresponding to the true harmonic component n, present in $w(t)$, and of the vector $P_n P$, due to the random part of the data, $g(t)$. If μ is the standard deviation of this random part, the distribution of the vectors $P_n P^{(\beta)}$ is as described in § 25; their root-mean-square length is $2\mu/\surd r$, by (81 b) and (85), whereas that of the vectors $P_n P$, joining P_n to the centroids of h points $P^{(\beta)}$, is $2\mu/\surd(rh)$. Thus the new cloud of points P has shrunk towards the point P_n in the ratio $1/\surd h$, as compared with the original cloud of points $P^{(\beta)}$. This is statistically true although, for any finite number of rows or points $P^{(\beta)}$, the position of P_n cannot be definitely ascertained.

We now consider the contribution of $a(t)$ to the vectors $OP^{(\beta)}$; the vector corresponding to $w(t)+g(t)$, for the row β, will be denoted by $OQ^{(\beta)}$, equal to OP_n combined with the random contribution $P_n Q^{(\beta)}$; thus the contribution of $a(t)$ is $Q^{(\beta)}P^{(\beta)}$. Since $a(t)$ by definition contains no true or persistent periodic component n, the mean of a very large number of vectors $Q^{(\beta)}P^{(\beta)}$ for successive or unselected rows will vanish. But owing to the property of conservation or quasi-persistence which we attribute to $a(t)$, and which distinguishes it from the random function $g(t)$, two or more successive vectors $Q^{(\beta)}P^{(\beta)}$ tend to lie in similar directions. This tendency could be numerically characterized by a procedure analogous to that of § 27, if we were dealing only with the function $w(t)+a(t)$, free from the random component $g(t)$. The vectors $Q^{(\beta)}P^{(\beta)}$ would then become $P_n P^{(\beta)}$; let $c_n'(1)$ be their root-mean-square magnitude; let $c_n'(h)$ be the corresponding root-mean-square magnitude of the vectors $P_n P$ determined by harmonic analysis of the average of h individual rows. Apart from the persistence of the variation of $a(t)$, $c_n'(h)$ would be $c_n'(1)/\surd h$; actually let

$$c_n'(h) = c_n'(1)\surd\{\sigma_{r,n}(h)/h\}.$$

This implies that the persistence reduces the ratio of shrinkage of the cloud of points $P^{(\beta)}$, towards P_n, from $1/\surd h$ to $\surd\sigma_{r,n}(h)$ times this amount.

When $g(t)$ is present as well as $a(t)$, their joint contribution to $OP^{(\beta)}$ is $P_nP^{(\beta)}$, which may be known with much accuracy if OP_n is well determined owing to the availability of many observations. If $\sigma_{r,n}(h)$ is calculated from the clouds of points $P^{(\beta)}$ and P, as if $g(t)$ were absent, the value of $\sigma_{r,n}(h)$ will not reach the value expressing the quasi-persistence of $a(t)$ alone (provided that $\sigma_{r,n}(h) > 1$, that is, if the conservation is positive); the random variation $g(t)$ may be said to weaken or dilute the persistency imparted to $w(t)+g(t)+a(t)$ by $a(t)$.

If $w(t)$ has no harmonic component n, the point P_n will coincide with the origin. In general the centroid of the B points $P^{(\beta)}$ will not coincide with O; the physical significance of its deviation from O can be tested by means of the formulae of § 25 and Table 1.

16.31. The summation-dial. Instead of the harmonic dial, with its clouds of points, which shrink with the number of rows combined, it is sometimes more convenient to use the *summation-dial*, which shows the *train of vectors* $OQ^{(1)}$, $Q^{(1)}Q^{(2)}$,..., produced by simple vectorial addition of the successive vectors $OP^{(\beta)}$ for $\beta = 1, 2, 3,..., B$. On this dial the vector for the sum of the h rows from $\beta = b+1$ to $\beta = b+h$ is indicated by the line $Q^{(b+1)}Q^{(b+h)}$ connecting the beginning of the vector for the row $(b+1)$ with the end of the vector for the row $(b+h)$; the vector for the *average* of the h rows is obtained by dividing the vector for the sum by h. If the vectors are derived by the harmonic analysis of a random function such as $g(t)$, the train of vectors is a true 'random-walk'; if they refer to a function $a(t)$ or $g(t)+a(t)$, which has the property of quasi-persistence, successive vectors of the train will 'prefer' the same direction, but later vectors will 'forget' this direction; if, however, the function analysed includes also a truly periodic or persistent part $w(t)$, the train will manifest a more or less distinct tendency to proceed in some definite direction. Various special cases or combinations of functions may be illustrated on the summation dial, for instance, similar directions for every third vector (*intermittent quasi-persistence* [22]), which would indicate that the actual length of the quasi-persistent period is $3r\Delta t$ (see § 33).

In practice it is sometimes better to use some other parameter instead of E_2; see 19.2. It may also be mentioned that quasi-persistence is '*infectious*' [22], that is, periods of not too different lengths will all show it, if one is affected by it.

16.32. The generalized harmonic dial. Using a $(2k)$-dimensional rectangular coordinate system with coordinates a_n and b_n $(n = 1,..., k)$, it is possible to represent a sum of k sine waves (associated with an approximating function $\psi_k(t)$ (given by (19)) by a single dot. All the considerations already discussed in §§ 30, 31 relative to plane clouds of points and summation-dials can be generalized to the case of $2k$ dimensions. The square of the length of the $2k$-dimensional vector is $c_1^2+...+c_k^2$, or $2\eta_k^2$, by (47 b). To give an exact representation of a set of r ordinates, $\psi_k(t)$ must have r harmonic coefficients, and in this case (cf. § 12) η_k is equal to the standard deviation ζ_r of the individual rows; the generalized harmonic dial can therefore be regarded as an illustration of the methods of § 29.

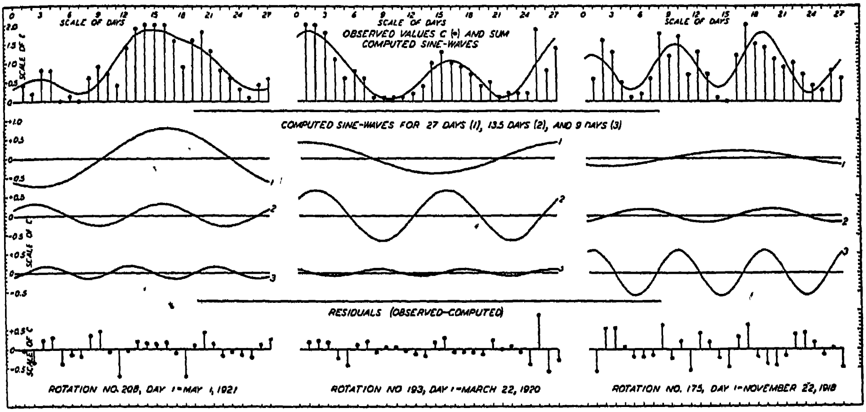


FIG. 13. Illustrating the harmonic analysis of the international daily magnetic character-figures C for three 27-day intervals (solar rotations), showing the observed values C, the sum of the computed sine waves having periods of 27, 13·5, and 9 days, the separate sine waves, and the residuals

16.33. Example: The 27-day recurrence tendency. We consider the series of the daily international magnetic character-figures C, comprising the 10,206 days between January 11, 1906, and December 20, 1933 [22]. We divide it into rows of 27 days, called '*rotations*', numbered 1001 (beginning January 11, 1906) to 1378 (in the original paper [22], these numbers were 1 to 378, and these numbers remain unaltered in some diagrams). In each rotation the days are numbered 1 to 27 (not 0 to 26, as in Table J of this book). For each rotation, the first three harmonics were computed (§ 19 (*b*)). Fig. 13 illustrates the harmonic analyses for the three rotations Nos. 1208, 1193, and 1175, which, in this order, were found to have the greatest amplitude c_1, c_2, and c_3, respectively; for each rotation, the first row gives the observed values C and the sum ψ_3 of the three sine waves, then the sine waves separately, and finally the residuals, or differences between the observed C and the sum of the sine waves. The standard deviations, for the rotation 1208 (including the intense magnetic disturbances of May 12–21, 1921), in units of C, are, in the notation of § 12, $m = 0{\cdot}66$ for the observed values, $\eta_k = 0{\cdot}59$ for the sum of the sine waves, and $\mu = 0{\cdot}29$ for the residuals.

Fig. 14 is the harmonic dial for the 378 sine waves of period 27 days; the largest amplitude, $c_1 = 0{\cdot}76$, for the rotation 1208, is seen near the outer circle; its time of maximum is at about day 16. Amplitudes less than 0·01 occur in the rotations 1061 and 1372. The average vector, indicated by a cross, falls close to the origin.

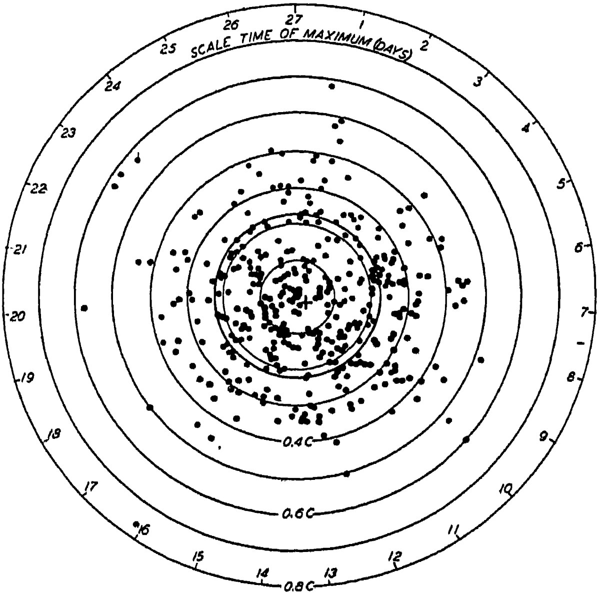


FIG. 14. Harmonic dial showing the dial-points for the 27-day sine wave in each of 378 27-day sets of values of the international daily magnetic character-figures C, for the period 1906–33 beginning with 1906 Jan. 11

We now examine whether the distribution of the cloud of 378 points in Fig. 14 is such that each dot on the dial can be regarded as the end-point of a random walk (§§ 23–4), all the random walks being supposed made under the same relevant conditions. The expectancy (the square root of the average of the squared amplitudes c_1) is found to be $E_2 = 0{\cdot}262$. In order to find the frequency-distribution, we count how many amplitudes c_1 fall in classes between equidistant limits, chosen according to the conventional statistical rule that about twenty classes should be occupied; for the present value of E_2 these are 0, 0·036, 0·072, etc.; they are marked by vertical lines in Fig. 15. The observed numbers of amplitudes in each class are entered as ordinates (black dots), and are compared with the theoretical distribution according to (86 a), for the expectancy E_2; the theoretical number that should lie between the limits $\kappa_1 E_2$ and $\kappa_2 E_2$ is $378\{\exp(-\kappa_1^2)-\exp(-\kappa_2^2)\}$. The observed distribution agrees fairly well with the theoretical curve.

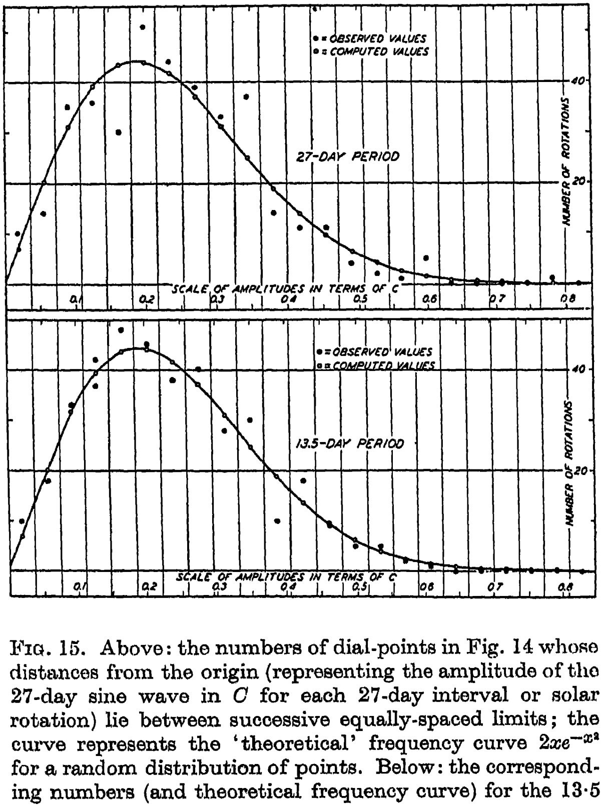


FIG. 15. Above: the numbers of dial-points in Fig. 14 whose distances from the origin (representing the amplitude of the 27-day sine wave in C for each 27-day interval or solar rotation) lie between successive equally-spaced limits; the curve represents the 'theoretical' frequency curve $2xe^{-x^2}$ for a random distribution of points. Below: the corresponding numbers (and theoretical frequency curve) for the 13·5 sine wave in the same 378 rotation-sets of C data

Only the isolated highest amplitude $c_1 = 0{\cdot}76$ may call for comment. However, if it is expressed as a multiple κE_2, we obtain $\kappa = 2{\cdot}90$, for which (86 a) gives $W \doteq 1/4500$, implying that, on the average, 1 out of 4,500 amplitudes should be equal to or greater than 0·76; it is therefore not strange that one should occur among the first 378 amplitudes observed, that is, in $\frac{1}{12}$ of the average number 4,500.

The lower part of Fig. 15 shows the same comparison for the waves of frequency 2, or period 13·5 days; their expectancy is 0·264. For the 9-day period, the expectancy is 0·236. A 'goodness of fit' test [11.1] confirms the view that the distribution of the dots in the cloud of Fig. 14 and in those for the other two frequencies is the same as if they were the end-points of random walks.

Fig. 16 shows the summation dial for the 27-day period. It may be hard to judge, from mere inspection, whether there is a systematic difference between this train of vectors and the random walk in Fig. 10. However, the degree of quasi-persistence in Fig. 16 is quite high as expressed by the number $\sigma(h)$ of equivalent sequences (formula 96); in the summation dial $\sigma(4)$, for instance, is determined by taking the average square of the distances $c_1(4)$ between every fourth successive point along the train, and dividing it by four times the average square of the distances c_1 between consecutive points (see remark at end of § 29). For the whole series 1906–33, the values of $\sigma(h)$ are as follows.

TABLE 4. *Equivalent number $\sigma(h)$ of repeated sets of sine waves of periods 27, 13·5, and 9 days, in the series of magnetic character-figures C.*

$h =$	*1*	*4*	*9*	*16*	*25 rotations*
Period 27 days . .	1·00	1·99	2·44	2·66	3·11
„ 13·5 days . .	1·00	2·17	3·00	2·95	3·03
„ 9 days . .	1·00	1·75	2·19	2·60	2·60

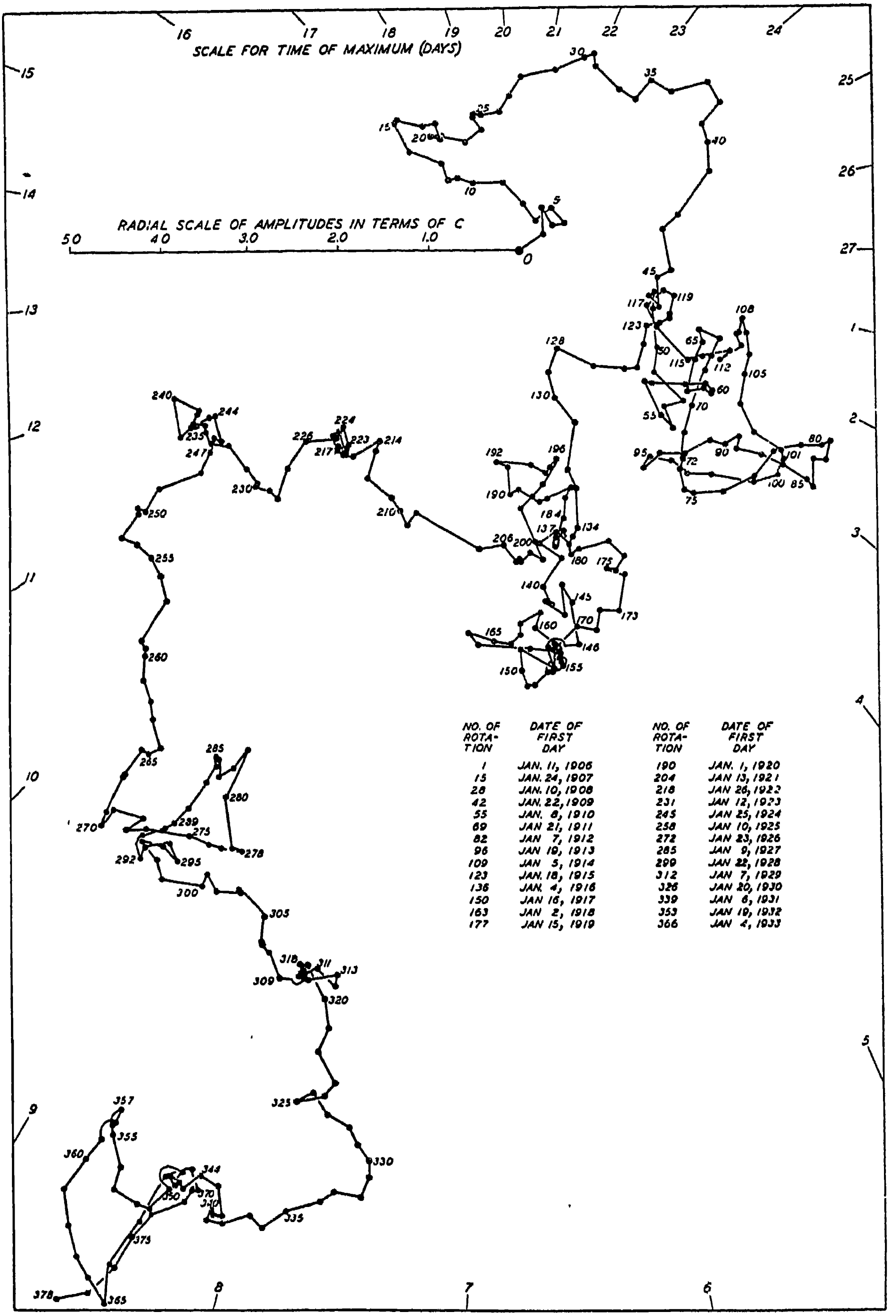
In order to test the strong quasi-persistence noticeable in the recurrence-diagram (Fig. 12.7) for the years 1928–33, the values of $\sigma(h)$ were also computed for these years alone (rotations 1299 to 1378, or 299 to 378 in the notation of Fig. 16) for $h = 2$, 4, and 9; they are:

for the 27-day period, 1·57, 2·06, and 2·76,

and for the 13·5-day period, 1·71, 2·88, and 4·61;

these are distinctly larger than the values in Table 4, but of course do not reach the values 2, 4, and 9 corresponding to persistent sine waves. The values of $\sigma(4)$ were also computed for pairs of years; the lowest value, 1·08, was found for 1916 and 1917, but some high values, approaching the persistent value 4·0, were found for the 13·5-day period, in the years 1930 and 1931 (3·24) and 1932 and 1933 (2·92). This strong persistence is evident in the summation-dials for these periods given elsewhere [22].

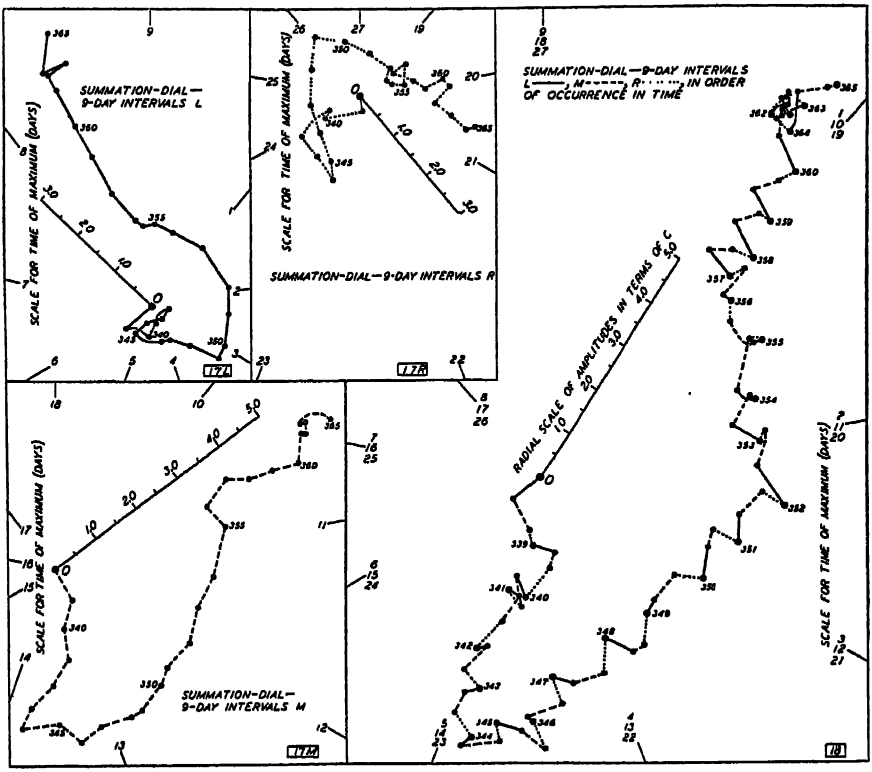
A more general kind of quasi-persistence may be imagined, such that, in the summation-dial, not the successive intervals, but perhaps the first, fourth, seventh, etc., exhibit a tendency to have the same phase. This type is actually met with in the case of the international magnetic character-figures C, if we break up our 378 rotations of 27 days into



NO. OF ROTA-TION	DATE OF FIRST DAY	NO. OF ROTA-TION	DATE OF FIRST DAY
1	JAN. 11, 1906	190	JAN. 1, 1920
15	JAN. 24, 1907	204	JAN 13, 1921
28	JAN. 10, 1908	218	JAN 26, 1922
42	JAN. 22, 1909	231	JAN 12, 1923
55	JAN. 8, 1910	245	JAN 25, 1924
69	JAN 21, 1911	258	JAN 10, 1925
82	JAN 7, 1912	272	JAN 23, 1926
96	JAN 19, 1913	285	JAN 9, 1927
109	JAN 5, 1914	299	JAN 22, 1928
123	JAN. 18, 1915	312	JAN 7, 1929
136	JAN. 4, 1916	326	JAN 20, 1930
150	JAN 16, 1917	339	JAN 6, 1931
163	JAN 2, 1918	353	JAN 19, 1932
177	JAN 15, 1919	366	JAN 4, 1933

Fig. 16. The summation-dial showing the successive addition of the dial-vectors of Fig. 14 in order of date. The number of the rotation-set is indicated at the end of the vector in many cases

(3×378) = 1,134 intervals of 9 days. We designate each of these 9-day intervals by the rotation-number with the addition of the letter *L*, *M*, or *R* (left, middle, or right) to distinguish the three thirds of each rotation. Figs. 17 and 18 show, in different arrangements, the summation-dials for the sine waves with 9-day periods for the years 1931 and 1932, comprising the 27 rotations Nos. 1339 to 1365, or 81 intervals of 9 days (in the figures, the rotation-numbers are diminished by 1,000). Fig. 18 shows the 81 vectors for the 9-day intervals as they follow each other in time, that is, for the intervals 339 *L*, 339 *M*, 339 *R*, etc., up to 365 *R*. The vectors for the intervals *L*, *M*, and *R* have been distinguished by drawing them as full, dashed, and dotted lines respectively. In Fig. 18 the number of each rotation is entered against the



FIGS. 17 and 18. Summation-dials for the dial-vectors, as in Fig. 14 but for the 9-day wave, in the series of international daily magnetic character-figures *C* for the two years 1931 and 1932, comprising the 27-day solar rotation sets Nos. 339–365. Each rotation set is divided into three 9-day sets *L* (left, 1–9), *M* (middle, 10–18), and *R* (right, 19–27). Fig. 17 is divided into 3 parts, *L*, *M*, *R*, showing the summation of the 9-day vectors from the sets *L*, *M*, and *R* respectively, and Fig. 18 (right) shows the summation of the vectors for all the 9-day sets in order of time end (which is marked boldly) of the third vector (R) for the rotation. (Note that if we retained only these marked points in Fig. 18, and omitted the intermediate points at the ends of the vectors L and M, we should obtain the summation-dial, on a threefold magnified scale, for the 9-day periods as calculated from *whole* rotations of 27 days, analogous to Fig. 16, discussed above.) The time of maximum is indicated by the scales around the borders of the dials, by the days 1 to 9 for L, 10 to 18 for M, and 19 to 27 for R, according to the numbering of the days in the rotations.

Calculations relative to Fig. 17, similar to those explained above with reference to Figs. 15 and 16, lead to the expectancy $E_2 = 0{\cdot}540$ for single vectors, and for $\sigma(h)$ we obtain the values 1·23 for $h = 3$, 1·87 for $h = 6$, and 2·78 for $h = 9$.

On inspection of Fig. 18, we note the feature mentioned above, namely, practical independence for the vectors immediately following each other, but every third vector—for instance, those for the M intervals, indicated by dashed lines—has a tendency to keep its phase. This is brought out more clearly in Figs. 17 L, 17 M, and 17 R, which show the trains of vectors for the L, M, and R intervals separately; these show considerable quasi-persistence. If we calculate $\sigma(h)$ for these three summation-dials of Fig. 17, we obtain, on the average, 1·54 for $h = 2$ and 1·94 for $h = 3$. The contrast between the values of $\sigma(3)$, namely 1·23 for Fig. 18, as against 1·94 for Figs. 17 L, 17 M, 17 R, is the numerical expression for this particular kind of quasi-persistence, which may be called *intermittent*. The three thirds of each rotation, L, M, and R, give nearly independent 9-day sine waves, but corresponding thirds, for instance, the L intervals alone, show strong quasi-persistence. This proves incidentally that the 9-day period has no self-existence, but is only a sub-period of the 27-day interval, which is the actual quasi-period.

The value $\sigma(3) = 1{\cdot}23$ for Fig. 17, small as it is, nevertheless exceeds unity, and reveals a weak degree of quasi-persistence; this, however, seems to be a general phenomenon in many cases in which ordinates with positive conservation are divided into sets, because, for instance, a group of high ordinates, split up by a division between two sets, adds in a similar way, in both sets, to the cosine coefficients a_n for the two sets.

The attempt to frame the most general definition of quasi-persistence as distinguished from random conditions raises the same fundamental difficulties as are encountered when we try to devise a satisfactory definition of the term 'accidental', in the theory of probability. These difficulties are discussed in books by Mises [11.4] and by Kamke [11.9] (see especially the definition of the 'fields of probabilities' discussed by Kamke [11.9]).

16.34. Example: The annual variation of magnetic activity. The monthly means for the u_1 measure of the magnetic activity (11.8) for the 59 years 1872–1930 give, for the computation of the average annual variation, ($h =$) 59 sets of ($r =$) 12 ordinates. The analysis described in § 29 gives $\zeta_{12}(1) = 16\cdot7$, $\zeta_{12}(59) = 4\cdot7$, $\sigma_{12}(59) = 4\cdot7$.

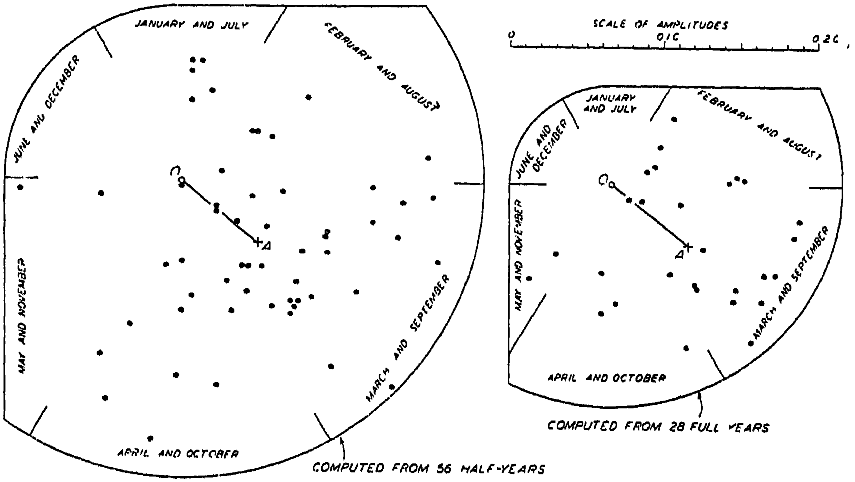


FIG. 19. Harmonic dials for the 6-monthly sine waves in the international daily magnetic character-figures C; on the left, as computed from 56 half-yearly sets of C data, and on the right, as computed from 28 yearly sets, 1906–33

We may regard $\zeta_{12}(1)$ as being caused by two independent influences, a regular annual variation (p. 366) with the standard deviation $4\cdot7$ (if $\zeta_{12}(59)$ may be taken as practically equivalent to $\zeta_{12}(\infty)$), and non-seasonal fluctuations of standard deviation ζ', to be computed (as in (97)), from $16\cdot7^2 = 4\cdot7^2+\zeta'^2$, so that $\zeta' = 16\cdot0$ (see 11.9).

16.35. Example: The semi-annual persistent wave in magnetic character-figures. From the series of monthly means of the international magnetic character-figures C, for 28 years, the sine-waves of 6 months period were computed. In Fig. 19 the harmonic dials have been plotted for this frequency or period, on the left as computed from the 56 half-years, and on the right as computed from the 28 calendar years; thus each dot $P^{(\beta)}$ in the right-hand diagram represents the point midway between the two corresponding dots $P^{(2\beta-1)}$, $P^{(2\beta)}$, on the left. The average wave for the whole 28 years has the amplitude $c = 0\cdot0675$, and its maxima occur at about the dates March 22 and

September 20, very near the equinoxes; it is represented by the average vector OA which, of course, is the same in both diagrams. The expectancy E_2 (cf. (100)) for the single waves is 0·111 on the left, and 0·096 on the right. For the average of 56 or 28 *accidental* waves we should therefore expect an amplitude $0{\cdot}111/\sqrt{56}$ or 0·0148 (Fig. 19, left), and $0{\cdot}096/\sqrt{28}$ or 0·0181 (Fig. 19, right). The average vector actually found, 0·0675, is respectively ($\kappa =$) 4·6 and 3·6 times as large as these expectancies. With these values of κ, the formula (86 a) for $W(\kappa)$ indicates probabilities of only about 10^{-9} and 10^{-6} that the waves are accidental.

The presence of the persistent semi-annual wave is suggested also by the mere comparison of the expectancies 0·111 and 0·096 found respectively from the half-years and the whole years. If the conditions were merely random, the expectancy for full years, to be inferred from that for half-years, would be only $0{\cdot}111/\sqrt{2}$ or 0·079, whereas actually the value found is 0·096. This explains why the values of κ, in the above calculation, are larger when the half-years are considered, making the proof for the reality of the semi-annual wave still more convincing.

For the 'accidental' or 'irregular' vectors $\mathrm{AP}^{(\beta)}$ the corresponding expectancies for the single vectors are 0·088 and 0·068, and for the averages of the 56 or 28 vectors (86 a) gives 0·0118 ($\kappa = 5{\cdot}7$) or 0·0129 ($\kappa = 5{\cdot}2$). This makes $W(\kappa)$ smaller than 10^{-12}.

In the corresponding summation-dial, Fig. 20, the decisive preponderance of the directions indicating maxima near the equinoxes excludes all similarity with a 'random-walk', and illustrates the 'reality' of the 6-monthly wave, which has just been quantitatively proved by the κ-test.

The 6-monthly wave is, of course, not restricted to C, but is also the significant physical characteristic of the annual variation of the u_1-measure of activity (§ 34); on the other hand, no reason to attribute special significance to the 12-monthly wave has so far been found. The 6-monthly sine wave in u_1 has a fairly constant phase, but its amplitude in disturbed years is more than twice as great as in quiet years. If the origin of the time-variable t (increasing by 360° per year) is chosen at the beginning of a sunspot-maximum year, the semi-annual wave with its changing amplitude can be roughly expressed by

$$\{6{\cdot}5+2{\cdot}6\cos(t/11)\}\sin(2t+261^\circ).$$

This expression is equivalent to the following sum of three terms with constant amplitude:

$$6{\cdot}5\sin(2t+261^\circ)+1{\cdot}3\sin\{(23/11)t+261^\circ\}+1{\cdot}3\sin\{(21/11)t+261^\circ\}.$$

Ordinary periodogram analysis would therefore indicate, besides the main 6-monthly wave, two waves of about $\frac{1}{8}$ of its amplitude, and periods of 5·74 and 6·29 months respectively; but this result differs only in form, not in physical content, from the (more simple) statement of a 6-monthly wave of constant phase, but variable amplitude.

16.36. Concluding remarks. This chapter has given an outline of the treatment of periodicities, or cycles, as part of a general morphology of geophysical time-curves. In each particular case the methods have to be adapted to the special conditions. The importance of conservation and quasi-persistence has been stressed because it represents a significant difference between geophysical data and the examples of 'random conditions' generally considered in ordinary text-books of statistics. The influence of conservation and quasi-persistence consists mainly in diminishing the 'effective number of independent cases'.

A last fictitious example may indicate the kind of wrong conclusion likely to be drawn if these considerations are disregarded. A quantity x is observed at 5-minute intervals during 24 hours, starting at sunrise. The observations are divided into 144 day-values and 144 night-values. The average for the day is found to be $x = 100$, that for the night as $x = 80$. Is the difference $100-80 = 20$ significant? The usual argument runs as follows. The standard deviations for the single observations are computed, and found to be 48 for the day, 36 for the night. The standard deviations of the day and night averages are 'therefore' by (88), 4 and 3 (obtained by division by 12, the square root of 144), and the difference between the two averages, namely 20, has the standard deviation 5 (namely the square root of 4^2+3^2). The difference 20 is therefore 4 times its

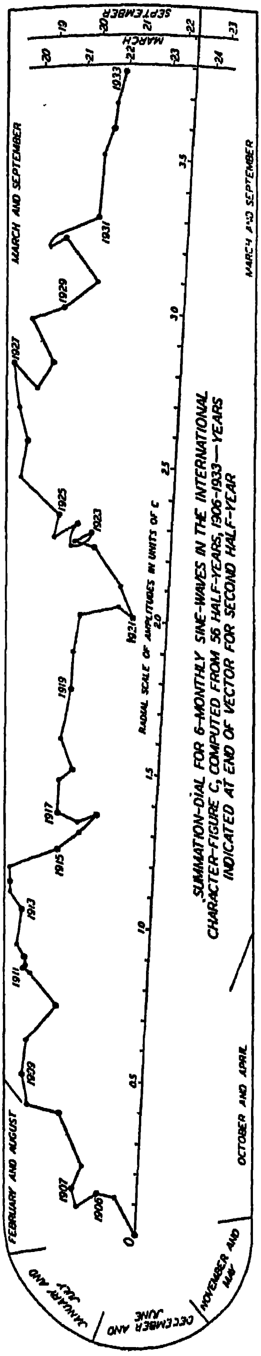


FIG. 20

standard deviation, a ratio which in random conditions should be surpassed only once in about 16,000 cases. The difference between the day and night values therefore appears significant. The fallacy of this conclusion lies in disregarding the possible conservation in the variation of x; for example, $\epsilon(144)$ might be as high as 16 (corresponding to equivalent repetitions over an interval of $16 \times 5 = 80$ minutes). If so, then the estimates for the standard deviations of the averages and their difference would have to be multiplied by the square root of 16, or 4; the difference between the day and night averages would then become of the same order of magnitude as its uncertainty. The error underlying the supposed argument becomes more obvious if the observations are supposed to be taken at smaller intervals, say every 12 seconds, giving 25 times as many observations, but not altering the day and night averages; this would *reduce* the (spurious) standard deviations for the averages by a factor $\frac{1}{5}$, and the difference $100-80 = 20$ could by such means be made to attain any fabulous degree of probability of significance.

This example, crude as it may look, has many counterparts in geophysical literature. The error here exposed sometimes leads to undue confidence in periodicities which are regarded as 'proved' by the smoothness of the curves obtained from calculations made in the manner of § 1; such smoothness, however, may not signify more than the presence of conservation in the single rows, which will naturally appear also in the average row. Since misconceptions arise even in such simple cases, still more caution is necessary in considering the results of more complicated analysis, such as periodogram tests, analyses of differences or sums, correlations between periodic phenomena, and so on. The methods here described are flexible enough to be useful as guides in such cases (see Chapter XIX, and the examples described elsewhere [23]). These methods have been developed to meet necessities arising in the study of important geophysical questions, in which the interest has been centred on the elucidation of problems of nature, rather than on the consideration of rigorous definitions or mathematical refinements; the problem of lending mathematical exactness, for instance, to the fundamental theorems of § 23, is left to students of the general theory of probability, for whom a field of geophysical importance is opened here.

The similarity of the 'random walk' and the summation-dial to the *Brownian movement* of particles in a liquid, or to the zigzag paths of molecules in a gas, is obvious. A very light particle among many heavy particles would make a random flight, but a heavy particle among many light particles would show 'quasi-persistence'; however, its successive positions after sufficiently large intervals would appear random, although the 'mean free path' would be actually smaller than that calculated, on the assumption of randomness, from these positions at large time-intervals.

16.19 (*a*), *continued from p.* 567. For the conversion of the harmonic coefficients a_n and b_n to the amplitude c_n and phase-angle ϵ_n, use may be made either of the convenient tables published by Miller [25 *b*] for this purpose, or of the following method. (For this and other practical details regarding harmonic analysis, see [25 *a*].)

If the numerical value $|a|$ of a is less than $|b|$, an auxiliary angle κ, of magnitude between 0° and 45°, is calculated from

$$\tan\kappa = |a/b|;$$

according as the signs of a and b are $(+,+)$, $(+,-)$, $(-,+)$, $(-,-)$, the values of ϵ are respectively as follows: κ, $180°-\kappa$, $-\kappa$, and $180°+\kappa$. The value of c is given by $c = |b|+|a|\tan\frac{1}{2}\kappa$.

If $|a| > |b|$, we calculate κ' from $\tan\kappa' = |b/a|$; the value of ϵ is then given, in the above four cases, by $90°-\kappa'$, $90°+\kappa'$, $\kappa'-90°$, and $270°-\kappa'$. The value of c is given by $c = |a|+|b|\tan\frac{1}{2}\kappa'$.

XVII

SPHERICAL HARMONIC ANALYSIS IN GEOPHYSICS

17.1. The representation of arbitrary functions of position on a sphere. Spherical harmonic functions are of value in geophysics because they afford an analytical representation of an arbitrary function of position on a sphere, in a form which facilitates many types of physical investigation, as may be seen in Chapters XVIII, XX, and XXII.

In ordinary harmonic analysis (Ch. XVI) a function $f(t)$, defined in the interval $0 \leqslant t \leqslant 2\pi$, is represented as a combination of sine waves which in this interval complete 0, 1, 2, 3,..., whole periods. Let t be regarded as the angle between a variable radius of a circle and a fixed radius. Then every point of the circumference of the circle can be associated with the corresponding value of $f(t)$, and the development of $f(t)$ in a trigonometrical series can be regarded as the analysis of this arbitrary distribution of values of $f(t)$ round the circumference, into a sum of specially simple distributions, which correspond to the 'circular' functions $\sin nt$, $\cos nt$, or $\sin(nt+\epsilon_n)$.

The spherical harmonic functions now to be considered serve the same purpose for an arbitrary distribution of values over the surface of a sphere.

Spherical polar coordinates, analogous to the ordinary geographical coordinates of position on the earth, are first defined. We take an axis Z′OZ (with the direction indicated by the order of these letters) through the centre O of the sphere, and an initial meridian half-plane bounded by this axis. The position of any point P is then determined by (*a*) its distance r (always positive) from the centre O, (*b*) the polar angle or 'distance' θ between OZ and OP, and (*c*) the easterly longitude ϕ, which is the angle between the initial meridian half-plane, and the meridian plane POZ; clearly $0^\circ \leqslant \theta \leqslant 180^\circ$, $0^\circ \leqslant \phi \leqslant 360^\circ$. The rectangular coordinates x, y, z of P, with respect to orthogonal axes OX, OY, OZ (the first two being in longitudes 0° and 90°), are related to r, θ, ϕ by the equations (Fig. 1)

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta. \tag{1}$$

Any function $f(\theta,\phi)$ may be regarded as a function of position on a sphere (of any radius, say a) centred at O. Its distribution along any circle of latitude, defined by the polar distance θ_0, is represented by the function $f(\theta_0, \phi)$, which can be expressed, by ordinary harmonic analysis, as a trigonometrical series of terms, $a_m \cos m\phi$, $b_m \sin m\phi$; the coefficients a_m, b_m will be functions of θ_0. Dropping the suffix for θ_0, it follows that $f(\theta, \phi)$ can be represented in the form

$$f(\theta,\phi) = \sum_{m=0}^{\infty} \{a_m(\theta)\cos m\phi + b_m(\theta)\sin m\phi\}. \tag{2}$$

The dependence of $f(\theta, \phi)$ on θ is now involved in the harmonic coefficients a_m, b_m; we consider what functions of θ are most appropriate for the expression of a_m and b_m.

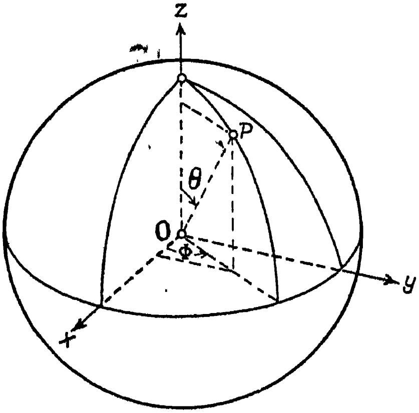


FIG. 1. Illustrating spherical polar coordinates

17.2. Zonal harmonics. The simplest case is that of a *zonal distribution*, in which the function f is independent of ϕ, so that $f = a_0(\theta)$. We seek suitable orthogonal functions, $P_n(\theta)$, say, in which to express $a_0(\theta)$. As in 16.8 we write

$$\alpha_k(\theta) = C_{0,0}P_0(\theta) + C_{1,0}P_1(\theta) + \dots + C_{k,0}P_k(\theta), \tag{3}$$

and require that the squared error $\{a_0(\theta) - \alpha_k(\theta)\}^2$, now averaged over the whole spherical surface (of area $4\pi a^2$), shall be as small as possible. Since the radius of the circle of latitude θ is $a \sin\theta$, the area of the zone between θ and $\theta + d\theta$ is equal to $2\pi a^2 \sin\theta\, d\theta$, so that this average is

$$\frac{1}{4\pi a^2}\int_0^{\pi} 2\pi a^2 \{a_0(\theta) - \alpha_k(\theta)\}^2 \sin\theta\, d\theta, \tag{4}$$

or, taking new variables $\mu = \cos\theta$, $d\mu = -\sin\theta\, d\theta$,

$$\tfrac{1}{2}\int_{-1}^{1} \{a_0(\mu) - \alpha_k(\mu)\}^2\, d\mu. \tag{5}$$

This expression is analogous to 16 (24), and the minimum condition is satisfied (cf. 16.9) if the functions $P_n(\theta)$, now regarded as functions of μ and written $P_n(\mu)$, are orthogonal functions (16.7) relative to the interval $-1 \leqslant \mu \leqslant 1$. For this interval we already know a system of orthogonal functions $P_n(\mu)$, namely the zonal or Legendre functions (16.11). In the formulae 16 (33) P_n is represented as a sum of powers of μ ($= \cos\theta$); another form, obtained by using formulae of the type (39), p. 619, is in terms of cosines of multiples of θ; thus

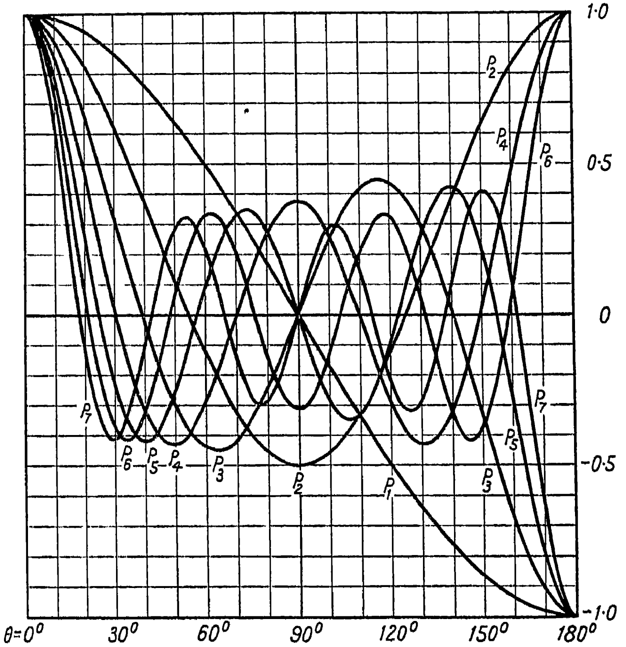


FIG. 2. The zonal harmonics $P_1, P_2,\ldots, P_7$ as functions of θ

$$\left.\begin{aligned} &P_0 = 1, \qquad P_1 = \cos\theta, \qquad P_2 = \tfrac{1}{4}(3\cos 2\theta+1),\\ &P_3 = \tfrac{1}{8}(5\cos 3\theta+3\cos\theta), \qquad P_4 = \tfrac{1}{64}(35\cos 4\theta+20\cos 2\theta+9),\\ &P_5 = \tfrac{1}{128}(63\cos 5\theta+35\cos 3\theta+30\cos\theta),\ldots, \end{aligned}\right\} \tag{6}$$

or, in general [2],

$$P_n(\theta) = \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}\left\{2\cos n\theta+2\frac{1.n}{1.(2n-1)}\cos(n-2)\theta+\right.$$
$$+2\frac{1.3.n(n-1)}{1.2(2n-1)(2n-3)}\cos(n-4)\theta+$$
$$\left.+2\frac{1.3.5}{1.2.3}\frac{n(n-1)(n-2)}{(2n-1)(2n-3)(2n-5)}\cos(n-6)\theta+\ldots\right\}.$$

If n is odd the series ends with a multiple of $\cos\theta$; if n is even, the last term is constant, and does not contain the factor 2; including the factor before the bracket it is $\{1.3.5\ldots(n-1)/2.4.6\ldots n\}^2$.

Fig. 2 shows graphs of the first seven zonal functions. Each P_n has n zeros *between* $\theta = 0°$ and $\theta = 180°$; according as n is even or odd, $P_n(\theta)$ is symmetrical or antisymmetrical with respect to the equator $\theta = 90°$. By 16 (36) the integral of the square of $P_n(\mu)$ between $\mu = -1$ and $+1$ is $2/(2n+1)$, so that (cf. 16.7) the functions

$$[(2n+1)/2]^{\frac{1}{2}}P_n(\mu)$$

are normalized, and their coefficients, in a sum of several such functions, would indicate the average relative importance of the various terms. But such normalization of the zonal functions is not usual, and is hardly necessary, since the magnitude of the factor $[(2n+1)/2]^{\frac{1}{2}}$ changes only slowly with n, e.g. in the ratio 1 : 4·1 from $n = 0$ to $n = 8$.

The average value of $\{P_n(\mu)\}^2$ over the surface of the sphere is, by 16 (36)

$$\int\limits_0^\pi \{P_n(\mu)\}^2 2\pi a^2 \sin\theta\, d\theta/4\pi a^2 = \int\limits_{-1}^{+1} \{P_n(\mu)\}^2\, d\mu/2 = 1/(2n+1). \tag{7}$$

The decrease in the average value of P_n, as n increases, is not very obvious in Fig. 2, because P_n is there plotted as a function of θ; in a diagram of P_n as a function of μ ($= \cos\theta$), the equatorial region around $\theta = 90°$ would be widened, and the polar regions more condensed (as in Fig. 7, p. 634).

17.3. Associated functions. The zonal functions are not the only orthogonal system of functions of μ ($= \cos\theta$) in the interval $-1 \leqslant \mu \leqslant 1$ (or $0 \leqslant \theta \leqslant 180°$). The functions $a_m(\theta)$ and $b_m(\theta)$, the coefficients of $\cos m\phi$ and $\sin m\phi$ in (2), will be expressed in terms of other functions, called *associated Legendre functions* or *spherical functions*, and denoted by $P_{n,m}(\theta)$; here n is called the *degree*, and m the *order* of $P_{n,m}$; $P_{n,m}$, in the form introduced by Neumann, is given by

$$P_{n,m}(\cos\theta) = \sin^m\theta \frac{d^m P_n(\cos\theta)}{d(\cos\theta)^m}, \tag{8}$$

or, in terms of μ, by

$$P_{n,m}(\mu) = (1-\mu^2)^{\frac{1}{2}m} \frac{d^m P_n(\mu)}{d\mu^m}. \tag{8 a}$$

From 16 (34) (p. 555) it follows that

$$P_{n,m}(\mu) = 0 \quad \text{for } m > n. \tag{9}$$

In this notation the zonal functions P_n would be written as $P_{n,0}$.

(It should be added that some authors write P_n^m or Q_n^m for what is here denoted by $P_{n,m}$; sometimes the significance of n and m is interchanged.) By a generalization of 16 (34) $P_{n,m}$ can be expressed as a finite series:

$$P_{n,m}(\theta) = \frac{(2n)!}{2^n.n!(n-m)!}\sin^m\theta\left\{\cos^{n-m}\theta-\frac{(n-m)(n-m-1)}{2(2n-1)}\cos^{n-m-2}\theta+\right.$$
$$\left.+\frac{(n-m)(n-m-1)(n-m-2)(n-m-3)}{2.4(2n-1)(2n-3)}\cos^{n-m-4}\theta-...\right\}. \quad (10)$$

When $(n-m)$ is even, the last term inside the bracket is a constant; when $(n-m)$ is odd, the last term is a multiple of $\cos\theta$. The following are some important special cases of (10), obtained and expressed like the formulae (6) for $P_{n,0}$ (see also p. 689).

$$\left.\begin{aligned} P_{1,1} &= \sin\theta, \quad P_{2,1} = \tfrac{3}{2}\sin 2\theta, \quad P_{3,1} = \tfrac{3}{4}\sin\theta(5\cos 2\theta+3),\\ P_{2,2} &= 3\sin^2\theta, \quad P_{3,2} = \tfrac{15}{2}\sin\theta\sin 2\theta,\\ P_{3,3} &= 15\sin^3\theta. \end{aligned}\right\} \quad (10\text{ a})$$

Laplace and Gauss used functions (which Schmidt denotes by $P^{n,m}$), obtained by replacing the first numerical factor in (10) by unity, so that

$$P_{n,m}(\theta)/P^{n,m}(\theta) = (2n-1)!!/(n-m)!, \quad (11)$$

in the notation $(2n-1)!! = 1.3.5...(2n-1)$,

introduced by Schuster (by analogy with $n!$).

17.4. Normalized spherical functions. The whole infinite set of spherical functions of order m, that is, $P_{m,m}$, $P_{(m+1),m}$,..., is orthogonal; in fact it may be proved [2], by repeated partial integration, using formula (8), and substituting from 16 (35), that

$$\int\limits_{-1}^{1} P_{n,m}(\mu)P_{n',m}(\mu)\,d\mu = \begin{cases} 0 & \text{for } n \neq n', \\ \dfrac{2}{2n+1}\dfrac{(n+m)!}{(n-m)!} & \text{for } n = n'. \end{cases} \quad (12)$$

The second part of this equation shows that the functions $P_{n,m}$ are of very diverse order of magnitude; for example, the mean values of the squares of $P_{4,1}$ and $P_{4,4}$ are in the ratio $(5!/3!)/(8!/1) = 1 : 2016$. The functions $P_{n,m}$ will therefore later be replaced by other functions, denoted by P_n^m, which are merely numerical multiples of $P_{n,m}$ (cf. (20)), and which are more nearly normal.

Developing $a_m(\theta)$, $b_m(\theta)$ in terms of $P_{n,m}$ for various values of n, (2) becomes

$$f(\theta,\phi) = \sum_{n=0}^{\infty}\{C_n P_{n,0}(\theta)+\sum_{m=1}^{n}(A_{n,m}\cos m\phi+B_{n,m}\sin m\phi)P_{n,m}(\theta)\}, \quad (13)$$

or (cf. 16 (10), p. 550)

$$f(\theta,\phi) = \sum_{n=0}^{\infty}\{C_n P_{n,0}(\theta)+\sum_{m=1}^{n} C_{n,m} P_{n,m}(\theta)\sin(m\phi+\epsilon_{n,m})\}. \tag{14}$$

The corresponding formulae in terms of the functions P_n^m are (cf. (19, 20))

$$f(\theta,\phi) = \sum_{n=0}^{\infty}\{C_n P_n^0(\theta)+\sum_{m=1}^{n}(A_n^m\cos m\phi+B_n^m\sin m\phi)P_n^m(\theta)\}$$

$$= \sum_{n=0}^{\infty}\{C_n P_n^0(\theta)+\sum_{m=1}^{n} C_n^m P_n^m(\theta)\sin(m\phi+\epsilon_n^m)\}. \tag{14 a}$$

Thus in (13, 14) $f(\theta,\phi)$ is represented as a linear combination of the functions

$$P_{n,m}(\theta)\cos m\phi, \qquad P_{n,m}(\theta)\sin m\phi, \tag{15}$$

or of

$$P_{n,m}(\theta)\sin(m\phi+\epsilon_{n,m}). \tag{16}$$

The functions (15) will be called (spherical) surface *harmonics*; when $m = 0$ they are zonal or Legendre functions. They are orthogonal to one another on the spherical surface, that is to say, the product of any two such functions, averaged over the sphere, is zero unless they are identical; thus, remembering that the area of a surface element is $a^2\sin\theta\,d\theta\,d\phi$, and that the whole area is $4\pi a^2$,

$$\frac{1}{4\pi}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} P_{n,m}(\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}m\phi\, P_{n',m'}(\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}m'\phi\,.\sin\theta\,d\theta d\phi = 0 \tag{17}$$

unless $n = n'$ and $m = m'$. For $m \neq m'$ this follows from the orthogonality of the trigonometrical functions of ϕ, and for $m = m'$, $n \neq n'$ it follows from (12). The mean square of (15) over the surface of the sphere is given for $m = 0$ by (7), and for $m > 0$ (by (12)) by

$$\frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi}\left[P_{n,m}(\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}m\phi\right]^2\sin\theta\,d\theta d\phi = \frac{1}{2(2n+1)}\frac{(n+m)!}{(n-m)!}. \tag{18}$$

Schmidt defined new orthogonal functions $P_n^m(\theta)$, numerical multiples of $P_{n,m}(\theta)$, as follows:

$$P_n^m(\theta) = P_{n,m}(\theta) \quad \text{when} \quad m = 0 \tag{19}$$

and

$$P_n^m(\theta) = \left\{2\frac{(n-m)!}{(n+m)!}\right\}^{\frac{1}{2}} P_{n,m}(\theta) \quad \text{when} \quad m > 0. \tag{20}$$

In the symbol P_n^m, m is merely an affix, not an index or exponent.

The first few factors $P_n^m/P_{n,m}$ are

	$m = 1$	$m = 2$	$m = 3$
$n = 1$	1		
$n = 2$	$\frac{1}{3}\sqrt{3} = 0{\cdot}5774$	$\frac{1}{6}\sqrt{3} = 0{\cdot}2887$	
$n = 3$	$\frac{1}{6}\sqrt{6} = 0{\cdot}4082$	$\frac{1}{30}\sqrt{15} = 0{\cdot}1291$	$\frac{1}{60}\sqrt{10} = 0{\cdot}0527$

(21)

Clearly $P_1^1 = P_{1,1}$.

The factors in (20) are chosen so that the surface harmonics $P_n^m(\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}m\phi$ shall have the same order of magnitude as the zonal functions of the same degree; this follows from (18), since their mean square over the spherical surface is given by

$$\frac{1}{4\pi}\int_0^\pi\int_0^{2\pi}\left[P_n^m(\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}m\phi\right]^2\sin\theta\,d\theta d\phi = \frac{1}{2n+1}, \tag{22}$$

the same as in (7).

The functions P_n and P_n^m therefore do not correspond to completely normalized harmonics; but they are advantageous in numerical calculations, because the coefficients A_n^m, B_n^m, and C_n^m, in (14 a), indicate at once the approximate orders of magnitude of the corresponding terms.

The functions

$$R_n^m \equiv (2n+1)^{\frac{1}{2}}P_n^m \tag{22 a}$$

are completely normalized in the sense that the average square value of $R_n^m\cos m\phi$ or $R_n^m\sin m\phi$ over the sphere is 1; introduced by Schmidt [7], they were used for a time by Schuster, but were given up later.

The functions $P_{n,m}$ were used by Neumann [4] and Maxwell [19]; some authors denoted them by Q_n^m or T_n^m. Laplace and Gauss used the functions (denoted $P^{n,m}$ by Schmidt) mentioned at the end of § 3.

Schmidt's functions P_n^m are very convenient in geophysical work and are used here exclusively. The reader is referred [1] to Schmidt's valuable numerical tables of these functions, and his condensed collection of formulae relating to them.

17.5. Developments in series of spherical harmonics. The coefficients A_n^m, B_n^m in (14 a) are found (as in 16.8) from the condition that the square of the difference between $f(\theta,\phi)$, and a *finite* series of the form (14 a), averaged over the sphere, shall be a minimum. On carrying out the calculation, using (17) and (22), it is found that

$$\begin{Bmatrix}A_n^m\\ B_n^m\end{Bmatrix} = \frac{2n+1}{4\pi}\int_0^\pi\int_0^{2\pi} f(\theta,\phi)P_n^m(\theta)\begin{Bmatrix}\cos m\phi\\ \sin m\phi\end{Bmatrix}\sin\theta\,d\theta d\phi, \tag{23}$$

however many terms the series contains.

In actual calculations, when $f(\theta,\phi)$ is given by a table of values, or by a set of curves, for various values of θ and ϕ, it is convenient first to determine the coefficients $a_m(\theta)$ and $b_m(\theta)$ of the formula (2), by ordinary harmonic analysis (formula 16 (45)), for a number of values of θ. By using the notation

$$\delta_0 = 2, \qquad \delta_1 = \delta_2 = \dots = 1, \tag{24 a}$$

we may obtain formulae valid both for $m = 0$ and for $m > 0$, namely

$$\begin{Bmatrix} a_m(\theta) \\ b_m(\theta) \end{Bmatrix} = \frac{1}{\pi\delta_m} \int\limits_0^{2\pi} f(\theta,\phi) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} d\phi, \tag{24 b}$$

so that (23) becomes

$$\begin{Bmatrix} A_n^m \\ B_n^m \end{Bmatrix} = \delta_m \frac{2n+1}{4} \int\limits_0^{\pi} \begin{Bmatrix} a_m(\theta) \\ b_m(\theta) \end{Bmatrix} P_n^m(\theta) \sin\theta \, d\theta$$

$$= \delta_m \frac{2n+1}{4} \int\limits_{-1}^{+1} \begin{Bmatrix} a_m(\mu) \\ b_m(\mu) \end{Bmatrix} P_n^m(\mu) \, d\mu. \tag{24 c}$$

These integrals can be evaluated numerically or graphically.

Schmidt [11] has developed special methods of calculating the coefficients A_n^m, B_n^m for use when many different functions $f(\theta,\phi)$, to be expressed in terms of surface harmonics, are all given for the same special set of points (stations or observatories) on the terrestrial sphere.

17.6. The functions P_n^m expressed in terms of their roots. In Fig. 3 P_7^0, P_7^1,..., P_7^7 are represented as functions of θ. These curves illustrate the following general theorems (for the proofs the reader is referred to the text-books on spherical harmonics cited in the bibliography for this chapter).

(*a*) Every spherical function $P_n^m(\theta)$ has $n-m$ real different zeros *between* $\theta = 0$ and $\theta = \pi$. The zonal functions $P_n^0(\theta)$ have the value $+1$ at the north pole ($\theta = 0°$), and the value $(-1)^n$ at the south pole ($\theta = \pi$); every other function $P_n^m(\theta)$, $m > 0$, is zero at both poles as well as at $(n-m)$ values of θ between the poles.

(*b*) According as $n-m$ is even or odd, P_n^m is symmetrical or antisymmetrical with respect to the equator, i.e.

$$P_n^m(\theta) = (-1)^{(n-m)} P_n^m(\pi-\theta). \tag{25}$$

When $n-m$ is odd, one of the $n-m$ zeros of P_n^m occurs at the equator $\theta = \frac{1}{2}\pi$.

(*c*) The functions P_n^n, given according to (10) and (20) by

$$P_n^n = \frac{\{2.(2n)!\}^{\frac{1}{2}}}{2^n.n!}\sin^n\theta, \tag{26}$$

are very small over an extensive region round the poles (see P_7^7, Fig. 3).

The roots of the equation $P_n^m(\theta) = 0$, other than those at $\theta = 0$ and $\theta = 180°$, may be called $\theta_1, \theta_2,..., \theta_{n-m}$. According to (10) and (20),

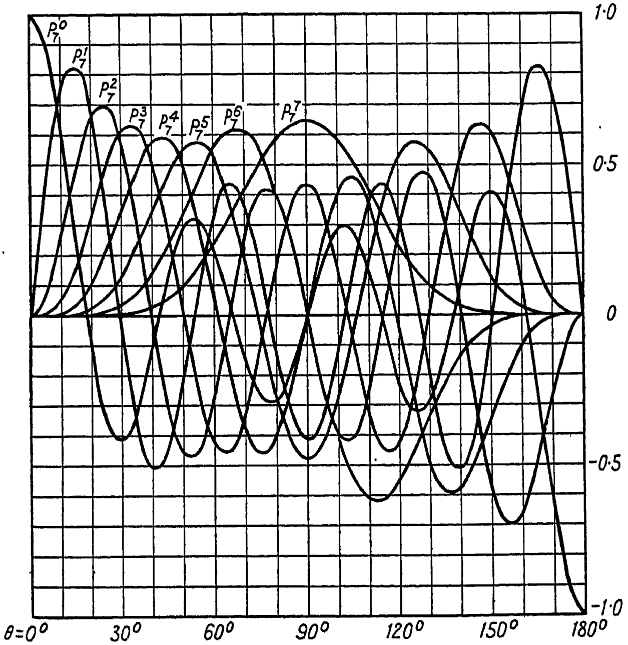


FIG. 3. The associated harmonics $P_7^0, P_7^1,..., P_7^7$ as functions of θ

and a well-known theorem of algebra concerning the linear factors of an equation,

$$P_n^m(\theta) = p_n^m \sin^m\theta(\cos\theta-\cos\theta_1)(\cos\theta-\cos\theta_2)...(\cos\theta-\cos\theta_{n-m}),$$

where

$$p_n^0 = 1.3.5...(2n-1)/n!,$$
$$p_n^m = 1.3.5...(2n-1)\{2/(n-m)!(n+m)!\}^{\frac{1}{2}} \quad \text{for } m > 0. \tag{27 a}$$

Because of the symmetry conditions (25),

$$\theta_1 = 180°-\theta_{n-m}, \qquad \theta_2 = 180°-\theta_{n-m-1}, \quad \text{etc.,}$$

and if $n-m$ is odd, $\theta_{(n-m+1)/2} = 90°$. Consequently

$$(\cos\theta-\cos\theta_1)(\cos\theta-\cos\theta_{n-m}) = (\cos\theta-\cos\theta_1)(\cos\theta+\cos\theta_1)$$
$$= -\sin(\theta+\theta_1)\sin(\theta-\theta_1) = \sin(\theta+\theta_1)\sin(\theta+\theta_{n-m}),$$

and so on, so that

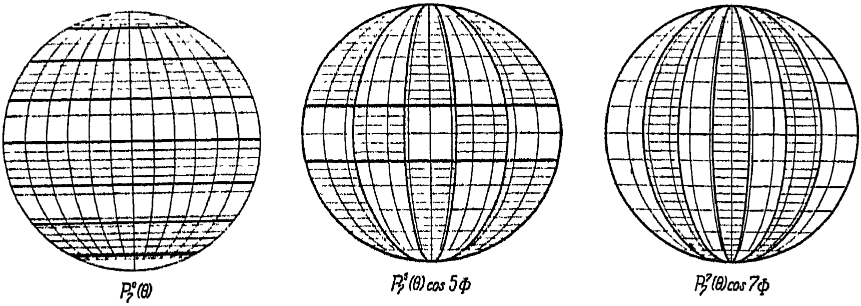
$$P_n^m(\theta) = p_n^m \sin^m\theta\sin(\theta+\theta_1)\sin(\theta+\theta_2)...\sin(\theta+\theta_{n-m}). \tag{27 b}$$

The coefficients p_n^m and the roots, given by Schmidt [1], are reproduced in Table D at the end of the book; the product (27 b) is convenient for calculating $P_n^m(\theta)$.

17.7. Tesseral and sectorial surface harmonics. The surface harmonics

$$P_n^m(\theta)\begin{Bmatrix}\cos m\phi\\ \sin m\phi\end{Bmatrix}$$

vanish along $(n-m)$ circles of latitude, corresponding to the zeros of P_n^m, and also, because of the factors $\cos m\phi$ or $\sin m\phi$, along $2m$ meridians at equal intervals π/m. These zero-lines divide the surface of the sphere into four-cornered (or, at the poles, three-cornered) regions in each of which the sign of the surface harmonic is constant, while it is reversed on crossing the boundary between two adjacent regions. Consequently, for $n > m > 0$, the surface harmonics are called *tesseral* surface harmonics. When $m = 0$ the functions are called *zonal* surface harmonics. When $m = n$ the functions are called *sectorial* surface harmonics, because there are no circles of latitude along which the function vanishes; the regions of constant sign are therefore sectors of the sphere. Fig. 4 shows equal-area diagrams of hemispheres, as divided up by the zero-lines of a zonal, a tesseral, and a sectorial function; within the domains of positive sign the circles of latitude are drawn at 5° intervals, while in the regions of negative sign their interval, as for the meridians, is 15°.



$P_7^0(\theta)$ $P_7^5(\theta)\cos 5\phi$ $P_7^7(\theta)\cos 7\phi$

FIG. 4. Equal-area map of the zero-lines (the thick lines in the diagram), over one hemisphere, of a zonal harmonic (P_7^0), a tesseral harmonic ($P_7^5\cos 5\phi$), and a sectorial harmonic ($P_7^7\cos 7\phi$): the central meridian corresponds to $\phi = 0$. The areas in which the sign of the function is positive are indicated by drawing the (thin-line) circles of latitude at closer intervals

17.8. Spherical harmonic functions as solutions of Laplace's equation. The spherical functions $P_n^m(\theta)$ and the spherical surface harmonics have so far been considered merely as a means of representing an arbitrary function of position over a spherical surface. But just as trigonometrical series owe their importance in geophysics not only to the formal usefulness of Fourier's method of expansion, but also to their association with all problems involving oscillations or fluctuations, so also the spherical functions and surface harmonics have close connexions with physical problems, and particularly with the magnetic and other potentials, which in free space satisfy Laplace's equation, namely (in terms of rectangular coordinates)

$$\nabla^2 V \equiv \frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2} = 0. \tag{28}$$

Every solution V of this equation is called a *harmonic function.* Many such functions can be written down at sight, viz.

$$x,\quad y,\quad z,\quad xy,\quad yz,\quad zx,\quad x^2-y^2,\quad y^2-z^2,\quad z^2-x^2,\quad xyz, \tag{29}$$

while a less obvious but very important one is $1/r$, where $r^2 = x^2+y^2+z^2$ (see 1.7). All these functions are of homogeneous degree in x, y, z; of course the general solution V involves terms of all degrees.

Let D or D' denote any space-differential operator, that is, any combination or sum of combinations of $\partial/\partial x$, $\partial/\partial y$, and $\partial/\partial z$; ∇^2 is one example of such an operator. The successive application of two such operators is commutative, that is to say, $DD' = D'D$, and, in particular, $D\nabla^2 = \nabla^2 D$. Hence if $\nabla^2 f(x,y,z) = 0$, so that also $D\nabla^2 f(x,y,z) = 0$, it follows that $\nabla^2 Df(x,y,z) = 0$. Hence any derivative, $Df(x,y,z)$, of a function which is a solution of Laplace's equation, is also a solution. This gives a powerful means of generating new solutions from any known particular solution. For example,

$$\frac{\partial}{\partial z}\left(\frac{1}{r}\right) = -\frac{z}{r^3} = -\frac{\cos\theta}{r^2} \tag{30}$$

(since $z = r\cos\theta$), which is the potential of a magnetic or electric doublet (1.9). This method of generating harmonic functions was developed by Maxwell [19] in his 'theory of poles'.

17.9. Development for the reciprocal radius. In the theory of the Figure of the Earth it is found necessary to express the potential at a point P of a fluid body, due to a distant body or particle at A, as a function of the coordinates r and θ of P, relative to the centre O of the fluid body, and the line OA (Fig. 5). Taking OA as the unit of distance, and the particle at A as of unit mass, for simplicity, it is necessary to expand the potential $1/R$, where $R =$ AP, as a function of r, θ; now

$$R^2 = 1-2\mu r+r^2; \qquad \mu = \cos\theta. \tag{31}$$

Clearly $1/R$ can be expanded as a series in r (convergent for $r < 1$), the coefficients being functions of μ. This was done in 1784 by Legendre, after whom the coefficients of the powers r^n are called *Legendre* functions or *Legendre* polynomials; they turn out to be the *zonal* functions $P_n(\mu)$. Thus

$$\frac{1}{R}=\frac{1}{(1-2\mu r+r^2)^{\frac{1}{2}}}=\sum_{n=0}^{\infty}P_n(\mu)r^n. \tag{32}$$

This may be proved [2] by expanding $(1-2\mu r+r^2)^{-\frac{1}{2}}$ or $\{1-r(2\mu-r)\}^{-\frac{1}{2}}$ according to the binomial theorem, and collecting all the multiples of r^n; the coefficient of r^n is found to be $P_n(\mu)$ in the form 16 (34).

More generally, if OA $= a$, so that

$$R^2=a^2-2\mu ar+r^2=a^2\{1-2\mu(r/a)+(r/a)^2\},$$

(32) becomes

$$\frac{1}{R}=\frac{1}{a}\sum_{n=0}^{\infty}\left(\frac{r}{a}\right)^n P_n(\mu), \tag{32 a}$$

which is convergent for $r<a$; for $r>a$, we write

$$R^2=r^2\{1-2\mu(a/r)+(a/r)^2\},$$

and get

$$\frac{1}{R}=\frac{1}{a}\sum_{n=0}^{\infty}\left(\frac{a}{r}\right)^{n+1}P_n(\mu), \tag{32 b}$$

which is convergent for $r>a$.

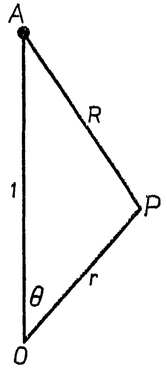


FIG. 5

17.10. The general spherical surface harmonic function S_n. In discussing particular problems, especially in geophysics, in which solutions of Laplace's equation (or harmonic functions, cf. § 8) are involved, certain boundary conditions have usually to be imposed on the solution; these conditions can often be expressed most simply in terms of coordinates other than Cartesian. For example, if V has to satisfy a certain condition over the surface of a sphere ($r=a$), it is convenient to express V in terms not of x, y, z, but of spherical polar coordinates r, θ, ϕ. A harmonic function of homogeneous degree n in x, y, z can evidently be expressed, in terms of the coordinates r, θ, ϕ by means of (1), as $r^n F(\theta,\phi)$; such a function is called a *spherical harmonic function* of degree n; n may be positive or negative; we shall consider only integral values of n.

The potential function which may be regarded as the most fundamental of all, namely $1/r$, of degree $n=-1$, is obviously a very special spherical harmonic function, since it involves only one of the three variables r, θ, ϕ.

While there are in general many harmonic functions [illegible]gned degree (e.g. six have been given in (29) for the case n[illegible]*al* homogeneous expression of degree n is, of course[illegible] $\nabla^2 V=0$; this equation imposes certain limitati[illegible] function, and therefore also on the function $F(\theta,\phi)$ in its expression $r^nF(\theta,\phi)$ in terms of r, θ, ϕ. These limitations are expressible most readily by transforming the differential equation itself, $\nabla^2V = 0$, in terms of the new independent variables r, θ, ϕ. A simple method of doing this, not only for the coordinates r, θ, ϕ, but for any set of orthogonal coordinates, was discovered by Kelvin; it was founded on a hydrodynamical interpretation of ∇^2V. In the present case it may be shown [2] that ∇^2V becomes

$$\nabla^2V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2V}{\partial\phi^2}; \tag{33}$$

on substituting $r^nF(\theta,\phi)$ in this expression the result is

$$r^{n-2}\left[n(n+1)F+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial F}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2F}{\partial\phi^2}\right], \tag{34}$$

or, in terms of μ $(=\cos\theta)$ and ϕ,

$$r^{n-2}\left[n(n+1)F+\frac{\partial}{\partial\mu}\left\{(1-\mu^2)\frac{\partial F}{\partial\mu}\right\}+\frac{1}{1-\mu^2}\frac{\partial^2F}{\partial\phi^2}\right], \tag{35}$$

or

$$r^{n-2}\left[(1-\mu^2)\frac{\partial^2F}{\partial\mu^2}-2\mu\frac{\partial F}{\partial\mu}+n(n+1)F+\frac{1}{1-\mu^2}\frac{\partial^2F}{\partial\phi^2}\right]. \tag{36}$$

If $r^nF(\theta,\phi)$ is a harmonic function, the expression (35) or (36) must be zero; it is convenient in this case to denote $F(\theta,\phi)$ by $S_n(\theta,\phi)$; omitting the factor r^{n-2}, the equation for S_n is thus obtained as

$$(1-\mu^2)\frac{\partial^2S_n}{\partial\mu^2}-2\mu\frac{\partial S_n}{\partial\mu}+n(n+1)S_n+\frac{1}{1-\mu^2}\frac{\partial^2S_n}{\partial\phi^2}=0. \tag{37}$$

Any function S_n satisfying the equation (37) is called a spherical *surface* harmonic of degree n, because it indicates the nature of the variation of the complete harmonic function $r^nS_n(\theta,\phi)$ over any sphere r = constant.

The degree n of the harmonic affects the differential equation (37) for $S_n(\theta,\phi)$ in one term only, i.e. $n(n+1)S_n$, arising from the term

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)$$

in ∇^2V. Since

$$n(n+1) = \{-(n+1)\}\{-(n+1)+1\} = n'(n'+1), \qquad n' = -(n+1),$$

it follows that, if $r^nS_n(\theta,\phi)$ is a harmonic function, so also is

$$\frac{1}{r^{n+1}}S_n(\theta,\phi). \tag{38}$$

17.11. Expression of S_n in terms of the functions P_n^m. Since $r^n S_n$ is a rational homogeneous function of degree n in x, y, and z, the equations (1) of direct transformation show that $S_n(\theta, \phi)$ must involve $\sin\theta$ and $\cos\theta$ (jointly) to the degree n at most, and $\sin\phi$ and $\cos\phi$ likewise.

For any angle χ, $\sin^n\chi$ and $\cos^n\chi$ can be expressed as follows (cf. E. W. Hobson, *Trigonometry*, p. 52)

$$\left.\begin{aligned}
(-1)^{\frac{1}{2}n}2^{n-1}\sin^n\chi &= \cos n\chi - n\cos(n-2)\chi + \\
&\quad + \frac{n(n-1)}{2!}\cos(n-4)\chi - \ldots + (-1)^{\frac{1}{2}n}\frac{1}{2}\frac{n!}{[(\frac{1}{2}n)!]^2} \quad (n \text{ even}),\\
(-1)^{\frac{1}{2}(n-1)}2^{n-1}\sin^n\chi &= \sin n\chi - n\sin(n-2)\chi + \frac{n(n-1)}{2!}\sin(n-4)\chi \\
&\quad - \ldots + (-1)^{\frac{1}{2}(n-1)}\frac{n!\sin\chi}{[\frac{1}{2}(n-1)]![\frac{1}{2}(n+1)]!} \quad (n \text{ odd}),\\
2^{n-1}\cos^n\chi &= \cos n\chi + n\cos(n-2)\chi + \frac{n(n-1)}{2!}\cos(n-4)\chi + \ldots.
\end{aligned}\right\} \qquad (39)$$

When n is even, the last line ends with the term $n!/2[(\frac{1}{2}n)!]^2$.

Taking $\chi = \phi$, these results may be used to transform the powers of $\sin\phi$ and $\cos\phi$ which occur in $S_n(\theta, \phi)$, when this is obtained by means of (1), into sines and cosines of integral multiples of ϕ; collecting the terms involving $m\phi$, we may express S_n in the form

$$S_n = \sum_{m=0}^{n} S_{n,m}(\theta)\cos(m\phi + \epsilon_n^m), \qquad (40)$$

where the coefficients $S_{n,m}$ are naturally functions of θ. On substituting from (40) in the differential equation (37), we get

$$\sum_{m=0}^{n}\left[(1-\mu^2)\frac{\partial^2 S_{n,m}}{\partial\mu^2} - 2\mu\frac{\partial S_{n,m}}{\partial\mu} + \right.$$
$$\left. + \left\{n(n+1) - \frac{m^2}{1-\mu^2}\right\}S_{n,m}\right]\cos(m\phi + \epsilon_n^m) = 0. \qquad (41)$$

Since this is to be true for all values of ϕ, the factor of each term periodic in ϕ must separately vanish (cf. the remark which follows 16 (21)). Hence in S_n the factors $S_{n,m}$, functions of θ, must satisfy the equation

$$(1-\mu^2)\frac{\partial^2 S_{n,m}}{\partial\mu^2} - 2\mu\frac{\partial S_{n,m}}{\partial\mu} + \left\{n(n+1) - \frac{m^2}{1-\mu^2}\right\}S_{n,m} = 0; \qquad (42)$$

of the two solutions of this equation, the one expressible in positive powers of μ, which alone concerns us, is our function $P_n^m(\mu)$, or a merely numerical multiple of it by an arbitrary factor $C_{n,m}$ say. (The proof that P_n^m is a solution of (42) is given in text-books [2] on spherical harmonics; it is briefly as follows: differentiate both sides of the identity

$$(\mu^2-1)\frac{d}{d\mu}(\mu^2-1)^n = 2n\mu(\mu^2-1)^n$$

$n+1$ times with respect to μ; it then follows by 16 (35) that the zonal function $P_n^0(\mu)$ satisfies (42) in the special case $m = 0$. Further m-fold differentiation with respect to μ then shows, by (8), that P_n^m is a solution of the general equation (42).)

Another form of (42), with θ instead of $\mu = \cos\theta$ as independent variable, is

$$\frac{d^2P_n^m}{d\theta^2}+\cot\theta\frac{dP_n^m}{d\theta}+\left\{n(n+1)-\frac{m^2}{\sin^2\theta}\right\}P_n^m = 0. \tag{42 a}$$

Thus the general surface harmonic of degree n is

$$\sum_{m=0}^{n} C_n^m P_n^m(\mu)\cos(m\phi+\epsilon_n^m), \tag{43}$$

where C_n^m and ϵ_n^m are arbitrary. The most general harmonic function expressible in integral powers of x, y, z is a sum of harmonics $r^nS_n(\theta,\phi)$ of all degrees n, namely

$$\sum_{n=0}^{\infty} r^n \sum_{m=0}^{n} C_n^m P_n^m(\mu)\cos(m\phi+\epsilon_n^m); \tag{44}$$

similarly the most general harmonic function expressible in a series of integral powers of $1/r$ is (cf. (38))

$$\sum_{n=0}^{\infty} r^{-(n+1)} \sum_{m=0}^{n} C_n^m P_n^m(\mu)\cos(m\phi+\epsilon_n^m). \tag{45}$$

17.12. Transformation and other formulae. The expression of an arbitrary function $f(\theta,\phi)$ as an infinite series of spherical surface harmonics is unique, provided that the function has continuous second derivatives over the whole sphere; moreover the series is absolutely and uniformly convergent. At discontinuities, Gibbs's phenomenon (16.16) appears [17].

A change of coordinate system involves much more complicated *transformation formulae* than are needed for developments in trigonometrical series. But the simplest case, a change of the initial meridian, merely alters the phase angles, ϵ_n^m, in (14). The next simplest case refers to a change from one axis, OZ_0, to another, OZ (Fig. 6). If Q is any point on the sphere, given by θ, ϕ relative to OZ, and by θ', ϕ' relative to OZ_0, it follows from spherical trigonometry that, if θ_0, ϕ_0 are the coordinates of OZ_0 relative to OZ,

$$\cos\theta' = \cos\theta\cos\theta_0+\sin\theta\sin\theta_0\cos(\phi-\phi_0). \tag{46}$$

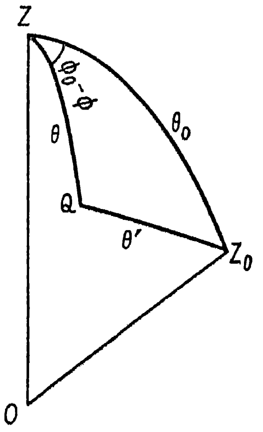


FIG. 6

In order to express a surface harmonic which, relative to the axis OZ_0, is zonal, say $P_n(\cos\theta')$, in terms of surface spherical harmonics relative to the axis OZ, we remember that, as in (43), every surface harmonic in the new expression for $P_n(\cos\theta')$ must be of the same degree n. Therefore

$$P_n(\cos\theta') = A_n^0 P_n^0(\cos\theta)+$$
$$+\sum_{m=1}^{n}(A_n^m\cos m\phi+B_n^m\sin m\phi)P_n^m(\cos\theta). \tag{47}$$

The coefficients may be calculated by means of (23); for instance,

$$A_n^m = \frac{2n+1}{4\pi}\int\limits_0^\pi\int\limits_0^{2\pi} P_n(\cos\theta')P_n^m(\cos\theta)\cos m\phi\sin\theta\, d\theta d\phi. \tag{48}$$

In order to evaluate these and similar surface integrals, we transform the variables to the system θ', ϕ'. For this transformation, we again use (43) to express the surface harmonic $P_n^m(\cos\theta)\cos m\phi$, namely

$$P_n^m(\cos\theta)\cos m\phi$$
$$= a_0 P_n^0(\cos\theta')+\sum_{m=1}^{n}(a_m\cos m\phi'+b_m\sin m\phi')P_n^m(\cos\theta'). \tag{49}$$

If this is inserted in (48) the integral over the sphere ($0 \leqslant \theta' \leqslant \pi$, $0 \leqslant \phi' \leqslant 2\pi$) reduces to $4\pi a_0/(2n+1)$, by (17) and (22), so that

$$A_n^m = a_0.$$

Now a_0 can be obtained from (49) by putting $\theta = \theta_0$, $\phi = \phi_0$, since that makes $\theta' = 0$, and by § 6 (a), $P_n^m(\cos 0)$ is 1 for $m = 0$, and 0 for $m > 0$; consequently

$$A_n^m = P_n^m(\cos\theta_0)\cos m\phi_0,$$

and (47) becomes

$$P_n(\cos\theta') = P_n(\cos\theta\cos\theta_0+\sin\theta\sin\theta_0\cos(\phi-\phi_0))$$
$$= \sum_{m=0}^{n} P_n^m(\cos\theta)P_n^m(\cos\theta_0)\cos m(\phi-\phi_0). \tag{50}$$

The absence of constant factors in (50) is due to the use of Schmidt's functions P_n^m. It may be noted that the special case of (50), given by $n = 1$, namely,

$$P_1^0(\cos\theta') = P_1^0(\cos\theta)P_1^0(\cos\theta_0)+P_1^1(\cos\theta)P_1^1(\cos\theta_0)\cos(\phi-\phi_0) \quad (50\,\text{a})$$

is identical with (46), since $P_1(\theta) = \cos\theta$, $P_1^1(\theta) = \sin\theta$.

Formulae connecting the functions P_n^m with other such functions having different indices n, m, or with their differential coefficients with respect to θ, are very numerous. The following is a summary of a few of the principal formulae of this kind; for brevity the argument θ of each function P_n^m is omitted.

The formula

$$(2n+1)\cos\theta\, P_n^m = \{(n+1)^2-m^2\}^{\frac{1}{2}}P_{n+1}^m+(n^2-m^2)^{\frac{1}{2}}P_{n-1}^m \quad (51\,\text{a})$$

connects functions of the same order m, but successive degrees, and

$$2m\cos\theta\, P_n^m = \{(n-m)(n+m+1)\}^{\frac{1}{2}}\sin\theta\, P_n^{m+1}$$
$$+\{\delta_{m-1}(n+m)(n-m+1)\}^{\frac{1}{2}}\sin\theta\, P_n^{m-1} \quad (\text{for } m>0) \quad (51\,\text{b})$$

connects functions of the same degree n, but successive orders (see (24 a) for the definition of δ_m). Further useful formulae are

$$(2n+1)\sin\theta\, P_n^m = -\{\delta_{m-1}(n-m+1)(n-m+2)\}^{\frac{1}{2}}P_{n+1}^{m-1}+$$
$$+\{\delta_{m-1}(n+m)(n+m-1)\}^{\frac{1}{2}}P_{n-1}^{m-1} \quad (m>0), \quad (52)$$

$$(2n+1)\sin\theta\, P_n^m = \{(n+m+1)(n+m+2)/\delta_m\}^{\frac{1}{2}}P_{n+1}^{m+1}-$$
$$-\{(n-m)(n-m-1)/\delta_m\}^{\frac{1}{2}}P_{n-1}^{m+1}, \quad (53)$$

$$\frac{dP_n^m}{d\theta} = \tfrac{1}{2}\{\delta_{m-1}(n+m)(n-m+1)\}^{\frac{1}{2}}P_n^{m-1}-\tfrac{1}{2}\{(n+m+1)(n-m)\}^{\frac{1}{2}}P_n^{m+1}$$
$$(m>0), \quad (54)$$

$$\frac{dP_n^0}{d\theta} = -\{\tfrac{1}{2}n(n+1)\}^{\frac{1}{2}}P_n^1, \quad (55)$$

$$\frac{(2n+1)}{n(n+1)}\sin\theta\frac{dP_n^m}{d\theta} = \frac{\{(n+1)^2-m^2\}^{\frac{1}{2}}}{n+1}P_{n+1}^m-\frac{(n^2-m^2)^{\frac{1}{2}}}{n}P_{n-1}^m, \quad (56)$$

$$2mP_n^m/\sin\theta = \{\delta_{m-1}(n+m)(n+m-1)\}^{\frac{1}{2}}P_{n-1}^{m-1}+$$
$$+\{(n-m)(n-m-1)\}^{\frac{1}{2}}P_{n-1}^{m+1} \quad (m>0), \quad (57\,\text{a})$$

$$2mP_n^m/\sin\theta = \{\delta_{m-1}(n-m+1)(n-m+2)\}^{\frac{1}{2}}P_{n+1}^{m-1}+$$
$$+\{(n+m+1)(n+m+2)\}^{\frac{1}{2}}P_{n+1}^{m+1} \quad (m>0). \quad (57\,\text{b})$$

In the form given, these formulae are convenient for numerical work, for successive calculation, or for checking purposes. The proofs, however, are somewhat less laborious if Neumann's functions $P_{n,m}$ are used (cf. (19, 20) which may together be written as

$$P_n^m = \{2(n-m)!/\delta_m(n+m)!\}^{\frac{1}{2}}P_{n,m});$$

these functions are identical with the functions Q_n^m used by Adams [6], to whose *proofs* the reader is referred. Thus the above formulae may be written also as follows:

$$(2n+1)\cos\theta\, P_{n,m} = (n-m+1)P_{n+1,m}+(n+m)P_{n-1,m}, \qquad (51\,\text{aa})$$

proved by Adams [6] as his formula (8), p. 248, or (15), p. 255:

$$\begin{aligned}(2n+1)\sin\theta\, P_{n,m} = &-(n-m+1)(n-m+2)P_{n+1,m-1}+\\ &+(n+m)(n+m-1)P_{n-1,m-1} \quad (m>0), \qquad (52\,\text{a})\end{aligned}$$

which is Adams's formula (11), p. 252;

$$(2n+1)\sin\theta\, P_{n,m} = P_{n+1,m+1}-P_{n-1,m+1}, \qquad (53\,\text{a})$$

which is Adams's formula (19), p. 257;

$$dP_{n,m}/d\theta = \tfrac{1}{2}(n+m)(n-m+1)P_{n,m-1}-\tfrac{1}{2}P_{n,m+1} \quad (m>0), \qquad (54\,\text{a})$$

which is Adams's formula (13), p. 254. If we remember that (since $\mu = \cos\theta$) $d/d\theta = -\sin\theta\, d/d\mu$, we recognize our formula (8) in

$$P_{n,1} = -dP_n^0/d\theta. \qquad (55\,\text{a})$$

The formulae (56, 57) are due to Schmidt [11 and G 10] and Fanselau [18]; (56) may be derived from the preceding formulae as follows:

$$(2n+1)\sin\theta\, dP_{n,m}/d\theta = n(n-m+1)P_{n+1,m}-(n+1)(n+m)P_{n-1,m} \qquad (56\,\text{a})$$

is obtained by multiplying equation (54 a) by $\sin\theta$; then, on the right-hand side, $\sin\theta\, P_{n,m-1}$ is transformed according to (53 a), and $\sin\theta\, P_{n,m+1}$ according to (52 a). The formulae

$$2mP_{n,m}/\sin\theta = (n+m)(n+m-1)P_{n-1,m-1}+P_{n-1,m+1} \quad (m>0), \qquad (57\,\text{aa})$$

$$2mP_{n,m}/\sin\theta = (n-m+1)(n-m+2)P_{n+1,m-1}+P_{n+1,m+1} \quad (m>0) \qquad (57\,\text{ba})$$

are similarly verified: we multiply by $\sin\theta$ and use (53 a) and (54 a). Finally

$$2m\cos\theta\, P_{n,m} = \sin\theta\, P_{n,m+1}+(n+m)(n-m+1)\sin\theta\, P_{n,m-1} \qquad (51\,\text{bb})$$

is verified by substitutions, on the right, according to (52 a) and (53 a); this leads to (51 aa).

For later use we here note an expression, due to Schuster, for

$$-m^2\cot\theta\, P_{n,m}+\frac{d}{d\theta}\left(\sin\theta\cos\theta\frac{dP_{n,m}}{d\theta}\right). \qquad (58\,\text{a})$$

By means of (42 a) (which, of course, is also valid for $P_{n,m}$) multiplied by $\sin\theta\cos\theta$, we may show that (58 a) is equal to

$$-n(n+1)\sin\theta\cos\theta\, P_{n,m}-\sin^2\theta\frac{dP_{n,m}}{d\theta}.$$

According to (51 aa) and (56 a)

$$-n(n+1)\cos\theta\, P_{n,m} = \frac{-n(n+1)}{2n+1}\{(n-m+1)P_{n+1,m}+(n+m)P_{n-1,m}\},$$

$$-\sin\theta\frac{dP_{n,m}}{d\theta} = \frac{-1}{2n+1}\{n(n-m+1)P_{n+1,m}-(n+1)(n+m)P_{n-1,m}\},$$

so that (58 a) is also equal to

$$-\frac{\sin\theta}{2n+1}\{(n+2)n(n-m+1)P_{n+1,m}+(n-1)(n+1)(n+m)P_{n-1,m}\}. \quad (58\,\mathrm{b})$$

In terms of Schmidt's functions P_n^m, the equality (58 a) = (58 b) may be expressed more symmetrically thus:

$$-m^2\cot\theta\, P_n^m+\frac{d}{d\theta}\left(\sin\theta\cos\theta\frac{dP_n^m}{d\theta}\right)$$

$$= -\frac{\sin\theta}{2n+1}\{[(n+1)^2-1][(n+1)^2-m^2]^{\frac{1}{2}}P_{n+1}^m+[n^2-1][n^2-m^2]^{\frac{1}{2}}P_{n-1}^m\}. \quad (58\,\mathrm{c})$$

These and many other useful formulae are to be found in Schuster's various memoirs in the *Phil. Trans. London* (A), **189, 200,** and **208.**

17.13. Separation of a field of force, known over a spherical surface, into parts of external or internal origin. Suppose that we have a distribution of gravitating, electric or magnetic matter, and that we know the potential $V = f(\theta,\phi)$ over the surface of a sphere of radius a and centre O, not passing through any of the matter; on and near the sphere, in regions not occupied by the attracting matter, $\nabla^2 V = 0$, so that $f(\theta,\phi)$ is a harmonic function, and can be expressed as a series (43) of harmonic functions. This expansion subdivides $f(\theta,\phi)$ into parts $S_n(\theta,\phi)$ whose variation with respect to r must be proportional either to r^n, as in (44), or to $r^{-(n+1)}$, as in (45), or to a linear combination of these two factors.

If there is no matter outside the sphere, then we know that the force, including its radial component $-\partial V/\partial r$, must vanish at infinity, and therefore no positive powers of r can occur in V; hence we may express V, everywhere over and outside the sphere, as

$$V = a\sum_{n=0}^{\infty}(a/r)^{n+1}S_n(\theta,\phi) \quad (r > a); \quad (59)$$

if, on the contrary, all the matter is outside the sphere, and none inside, then $-\partial V/\partial r$ must be finite throughout the interior, and no negative power of r can appear in V; consequently in this case

$$V = a\sum_{n=0}^{\infty}(r/a)^{n}S_n(\theta,\phi) \quad (r < a). \quad (60)$$

If, however, neither of these conditions is fulfilled, so that there is matter both inside and outside the sphere, and the field proceeds from both without and within, we may combine (59) and (60) into the most general harmonic function, in the convenient form

$$V = a\sum_{n=0}^{\infty}\sum_{m=0}^{n} P_n^m(\cos\theta)\left[\left\{c_n^m\left(\frac{r}{a}\right)^n+(1-c_n^m)\left(\frac{a}{r}\right)^{n+1}\right\}A_n^m\cos m\phi+\right.$$
$$\left.+\left\{s_n^m\left(\frac{r}{a}\right)^n+(1-s_n^m)\left(\frac{a}{r}\right)^{n+1}\right\}B_n^m\sin m\phi\right], \qquad (61)$$

where c_n^m and s_n^m are numbers lying between 0 and 1; they represent the fractions of the harmonic term $P_n^m\cos m\phi$ or $P_n^m\sin m\phi$ in V which, at $r = a$, are due to matter *outside* the sphere. The factor a (the radius of the sphere) on the right-hand side of (61) is inserted in order to give to the coefficients A_n^m and B_n^m the physical dimensions of force.

Over the sphere itself ($r = a$), $V = a\sum S_n(\theta,\phi)$ in all the three cases (59)–(61); but (61) contains an infinity of unknown numbers c_n^m and s_n^m associated with V, which are not determined by the mere knowledge of V itself over the sphere $r = a$. They can be determined, however, if we know, besides V, its radial gradient, which is in fact the radial component of force over the sphere (taken downward, in keeping with the convention as to the magnetic vertical intensity Z); for if we expand the known value of $\partial V/\partial r$ (at $r = a$), say $F(\theta,\phi)$, in a series like that found for $f(\theta,\phi)$, determining the coefficients by the method already given (§ 5), we obtain an expression which may conveniently be written in the form

$$\frac{\partial V}{\partial r} = \sum_{n=0}^{\infty}\sum_{m=0}^{n} P_n^m(\cos\theta)(\alpha_n^m\cos m\phi+\beta_n^m\sin m\phi) \quad (\text{over } r = a); \qquad (62)$$

this may be compared with the expression obtained by differentiating the above formula (61) with respect to r, and putting $r = a$ after the differentiation, namely, with

$$\frac{\partial V}{\partial r} = \sum_{n=0}^{\infty}\sum_{m=0}^{n} P_n^m(\cos\theta)[\{nc_n^m-(n+1)(1-c_n^m)\}A_n^m\cos m\phi+$$
$$+\{ns_n^m-(n+1)(1-s_n^m)\}B_n^m\sin m\phi]; \qquad (63)$$

the coefficients of each separate harmonic term $P_n^m\cos m\phi$ or $P_n^m\sin m\phi$ for each n and m must be equal in the two expressions, so that

$$\alpha_n^m = \{nc_n^m-(n+1)(1-c_n^m)\}A_n^m, \qquad (64)$$

$$\beta_n^m = \{ns_n^m-(n+1)(1-s_n^m)\}B_n^m; \qquad (65)$$

from these, since α, β, A, and B are known, c_n^m and s_n^m can be determined. Thus a knowledge of V and of its radial gradient, over the surface of a sphere (which does not pass through any of the matter that produces the field), enables us to ascertain how much of each separate harmonic term in V is due respectively to inside and outside causes. In the case of a magnetic field, no electric currents must cross the sphere, since such currents are equivalent, as regards their magnetic field, to magnetic shells—cf. 1.13.

17.14. Application to terrestrial magnetism. Spherical harmonic functions were introduced by Legendre and Laplace in the course of researches on dynamical and gravitational problems, such as those of the earth's figure, and the tides. Their first application to magnetism was made by Gauss [22] in 1839, in his great memoir on the earth's magnetic field. Gilbert, in 1600, by means of experiments on spherical pieces of loadstone, had reached the conclusion that the earth's magnetic field proceeds from within, and is substantially that of a uniformly magnetized sphere, though with many minor irregularities. Between his time and that of Gauss, the direction and intensity of the magnetic force had been measured at many points scattered widely over the earth's surface, and had been represented on maps by isomagnetic lines (1.3); but no progress had been made in representing the field in a mathematical form. Gauss assumed, what is very nearly correct, that there is no magnetic matter near the ground and no electric current passing from the atmosphere to the ground, so that the magnetic field there must possess a potential function V satisfying Laplace's equation, and representable by a series of spherical harmonics as in (61).

This potential cannot be measured directly, however; what can be measured are the three components of force, X horizontal and northwards ($= \partial V/r\,\partial\theta$), Y horizontal and eastwards ($= -\partial V/r\sin\theta\,\partial\phi$), and Z vertical and downwards ($= \partial V/\partial r$). These are determined at the earth's surface $r = a$, where a denotes the earth's radius. Thus, by analysis of the actual observations of Z, we can find, to a degree of completeness and accuracy conditioned by the observations, the coefficients α_n^m and β_n^m in (62); likewise, from the observations of X and Y, we can find the coefficients in the two following series derived from (61):

$$Y_{r=a} = \left(\frac{-\partial V}{r\sin\theta\,\partial\phi}\right)_{r=a} = \frac{1}{\sin\theta}\sum\sum P_n^m\{mA_n^m\sin m\phi - mB_n^m\cos m\phi\}, \qquad (66)$$

$$X_{r=a} = \left(\frac{\partial V}{r\,\partial\theta}\right)_{r=a} = \sum\sum\frac{dP_n^m}{d\theta}\{A_n^m\cos m\phi + B_n^m\sin m\phi\}. \qquad (67)$$

The coefficients in (66) are found by expanding $Y \sin\theta$ in the form found for $f(\theta,\phi)$ in (13) and (23); those in (67) are less easily found from the observations of X, because the expansion is not directly in a series of surface harmonics, but of functions $dP_n^m/d\theta$; these, however, may themselves be expressed in harmonic form (54). Schmidt [1] has given convenient tables for the numerical development of the series (66) and (67); his tables include numerical values not only of P_n^m, but also of the functions

$$X_n^m = \frac{1}{n}\frac{dP_n^m}{d\theta}, \qquad Y_n^m = \frac{m}{n}\frac{P_n^m}{\sin\theta}. \tag{68}$$

The factors $(1/n)$ and (m/n) are chosen in order to keep the numerical values of X_n^m, Y_n^m, and P_n^m similar in order of magnitude. Some details as to the actual computations will be given in §§ 19–22, and some formulae for X_n^m and Y_n^m will be derived, from (54) and (57), (58) in 20.4, p. 689.

It may be noted that (66) and (67) both contain A_n^m and B_n^m, and, as Gauss pointed out, if we know X all over the earth, the value of Y can be deduced from it, by substituting for A and B from (67) in (66); if the results did not agree, it would show that the field was not derivable completely from a potential V, i.e. that the atmosphere must either be magnetic or convey electric currents. Actually Gauss showed that, to the accuracy permitted by the observations of Y and X available in his time, the values of Y were consistent with the A's and B's derived from X. These indicate the form of V itself (apart from an arbitrary constant), so that the data for Z or $\partial V/\partial r$ can be used to determine from (64) and (65) whether the earth's field is of internal or external origin. Gauss showed, as nearly as his data for Z could indicate, that they agreed with Gilbert's conclusion that the cause is internal, so that V is of the form $\sum (a/r)^{n+1}S_n(\theta,\phi)$, and $c_n^m = s_n^m = 0$ for every n and m; actually by far the most important term in V is the first, proportional to $P_1(\cos\theta)/r^2$ or $(\cos\theta)/r^2$; this corresponds (1.9, 1.15) to the field of a uniformly magnetized sphere (or, what is equivalent at outside points, to that of an infinitesimal magnet at the earth's centre).

17.15. An integral theorem on spherical harmonics. Let $S_n(\theta,\phi)$, given by

$$\sum_{m=0}^{n} P_n^m\{A_n^m \cos m\phi + B_n^m \sin m\phi\}, \tag{69}$$

be any spherical surface harmonic of degree n; also let $P_{n'}(\cos\psi)$ be a zonal spherical function in relation to an axis in the direction (θ_0,ϕ_0).

17.17. The potential of a magnetic double-layer or shell. Consider two concentric spheres with nearly equal radii $(a+\delta a)$ and a. Let magnetic matter be distributed over their surfaces, with surface-density $-f(\theta,\phi)$ over the inner sphere, and $f(\theta,\phi)a^2/(a+\delta a)^2$ over the outer one. Clearly the total magnetic matter is equal in amount, but opposite in sign, for the two spheres. Such a distribution is called a spherical magnetic double-layer or shell. At the point (θ,ϕ) the magnetic moment of the shell, per unit area, is (to the first order in δa)

$$M(\theta,\phi) = f(\theta,\phi)\,\delta a = \sum S_n(\theta,\phi)\,\delta a \equiv \sum M_n(\theta,\phi), \quad \text{say.} \tag{75}$$

Maxwell calls M the strength of the magnetic shell. At the point (r,θ_0,ϕ_0) the potential W of the whole shell can be expressed in terms of V, the potential of a spherical layer with density $f(\theta,\phi)$ and radius a, as follows:

$$W = \frac{a^2}{(a+\delta a)^2}V(a+\delta a)-V(a) = a^2\left\{\frac{V(a+\delta a)}{(a+\delta a)^2}-\frac{V(a)}{a^2}\right\} = a^2\frac{\partial}{\partial a}\left(\frac{V}{a^2}\right)\delta a, \tag{76}$$

or, by (73) and (74),

$$W = -4\pi\sum_{n=0}^{\infty}\frac{n+1}{2n+1}M_n(\theta_0,\phi_0)\left(\frac{r}{a}\right)^n \quad \text{for } r<a, \tag{77}$$

$$W = 4\pi\sum_{n=0}^{\infty}\frac{n}{2n+1}M_n(\theta_0,\phi_0)\left(\frac{a}{r}\right)^{n+1} \quad \text{for } r>a. \tag{78}$$

This can also be derived from a general theorem in potential theory; the values of the potential on the two sides of a magnetic shell of any form, of moment M per unit area, differ by $4\pi M$, while the normal component of force is the same on the two sides. That is, in the case of a spherical shell (indicating the outer and inner sides of the sphere $r = a$ by the suffixes $+$ and $-$),

$$4\pi M = W_+-W_-, \qquad \partial W_+/\partial r = \partial W_-/\partial r \quad (\text{for } r=a). \tag{79}$$

We express W within (w) and outside (o) the sphere in the general forms (59) and (60), writing T_n for the spherical surface function of degree n: thus

$$W = \sum (r/a)^n T_n^{(w)}(\theta,\phi) \qquad (\text{within, } r<a),$$

$$W = \sum (a/r)^{n+1} T_n^{(o)}(\theta,\phi) \quad (\text{outside, } r>a).$$

Substituting from (75) in (79), and equating corresponding terms, we get

$$4\pi M_n = T_n^{(o)}-T_n^{(w)}, \quad nT_n^{(w)} = -(n+1)T_n^{(o)},$$

so that $$T_n^{(o)} = 4\pi\frac{n}{2n+1}M_n, \qquad T_n^{(w)} = -4\pi\frac{n+1}{2n+1}M_n, \tag{80}$$
which is equivalent to (77) and (78).

17.18. The magnetic potential of a steady current-distribution in a thin spherical shell. Consider a very thin spherical shell of conducting material, covered within and without by insulating matter, so that no current can leave or enter the shell. From any chosen fixed point A on the shell draw a line along the surface to any other point P. Let J denote the amount of electricity, in electromagnetic units, that crosses the line AP per unit time, from left to right as viewed by an observer looking along AP from outside the sphere. If the conditions are steady, so that there is no continual accumulation of electricity in any area, as much electricity must flow into, as out of, any closed area, in any time-interval. In this case J must be the same for all lines joining AP along the surface, because any two such lines enclose an area. Hence, if A is fixed, J depends only on the position of P; it is called the *current-function*. Evidently no electricity crosses the lines $J =$ constant; thus these lines must be along the direction of flow, and so they are called *current-lines*. Between two adjacent current-lines $J = J_0$ and $J = J_0+dJ$, the flow is dJ; if their distance apart at any point along them is ds, the *current-intensity*, or rate of flow of charge per unit breadth across the stream, is therefore dJ/ds, inversely proportional to the distance between the current-lines.

It is physically obvious that the geometrical form of the current-lines is independent of the choice of the initial point A; a change to another initial point B only alters the value of the current-function by a constant difference, namely, the value of J at B, reckoned from A.

Different current-lines can never intersect each other, and all the current-lines form closed curves on the sphere, because otherwise at the end-points electric charge would continually be accumulated or dissipated, contrary to the supposition of a steady state.

From the definition of J it follows that the flow round a point of minimum J is clockwise, viewed from outside; round a maximum it is counter-clockwise.

Consider a closed infinitesimal current-element between the current lines J_0 and J_0+dJ. By Ampère's law (1.13) the magnetic field of this current is identical with that of a magnetic shell whose boundary coincides with the current-element, and whose magnetic moment per unit area is equal to dJ. We choose the shell so as to extend over that part of the spherical surface in which $J > J_0$. Proceeding from the point of maximum J on the sphere, and replacing all the current-elements round it, in succession, by magnetic shells, we obtain a resultant magnetic shell whose magnetic moment per unit area, apart from an unessential additive constant, is everywhere equal to the current-function J. If this function is expressed in spherical functions, $J = \sum J_n(\theta, \phi)$; then the magnetic potential is given by (77) and (78), on substituting J_n for M_n. Since the electromagnetic unit of current is 10 amperes, the magnetic potential at the point (r, θ, ϕ) is given by

$$W = -\frac{4\pi}{10}\sum_{n=0}^{\infty}\frac{n+1}{2n+1}J_n(\theta,\phi)\left(\frac{r}{a}\right)^n \quad (r < a), \tag{81}$$

$$W = \frac{4\pi}{10}\sum_{n=0}^{\infty}\frac{n}{2n+1}J_n(\theta,\phi)\left(\frac{a}{r}\right)^{n+1} \quad (r > a), \tag{82}$$

when the current and current-function are supposed expressed in amperes.

Conversely, if the magnetic potential $W = \sum W_n(\theta, \phi)$ over the sphere of radius r is produced by a steady surface distribution of current over a sphere of radius a, then the current-function in amperes is

$$J = \sum J_n(\theta, \phi),$$

where, by (81) and (82),

$$J_n = -\frac{10}{4\pi}\frac{2n+1}{n+1}\left(\frac{a}{r}\right)^n W_n \quad (r < a), \tag{83}$$

$$J_n = \frac{10}{4\pi}\frac{2n+1}{n}\left(\frac{r}{a}\right)^{n+1} W_n \quad (r > a). \tag{84}$$

17.19. Numerical calculation of the spherical harmonic coefficients. Gauss [22] calculated the coefficients in the spherical harmonic expression for the potential of the earth's magnetic field from the values of X, Y, Z derived from the isomagnetic charts at 84 points, spaced at 30° intervals of longitude along 7 circles of latitude. The values at the 12 points on each circle of latitude were harmonically analysed as in (2), up to $m = 4$, giving 9 coefficients $a_m(\theta)$, $b_m(\theta)$, for 7 values of θ for each of X, Y, and Z. Those for Z were substituted in equations (63 in all) of the form

$$a_0(\theta) = \sum_{n=1}^{4}\gamma_n P_n(\theta), \quad a_m(\theta) = \sum \alpha_n^m P_n^m(\theta), \quad b_m(\theta) = \sum \beta_n^m P_n^m(\theta), \tag{85}$$

including only values of n up to 4; the X and Y equations (66) and (67) involve $P_n^m/\sin\theta$ and $dP_n^m/d\theta$ instead of P_n^m. Thus for each element 24 coefficients (such as γ_n, α_n^m, β_n^m) were connected by 63 equations. Gauss assumed from the outset, since his data were incomplete, that the field is purely of internal origin; thus the factors c_n^m, s_n^m of § 13 are all zero, and the three sets of 24 coefficients in the equations relating to X, Y, Z can be simply expressed in terms of a single set C_n, A_n^m, B_n^m. Gauss inserted these relations and used all the 189 equations in combination to find these 24 coefficients, applying the method of least squares.

Schmidt [7, 8] developed the general formulae for each element, taking account of the possible existence of an external part of the field, and a part also not possessing a potential. Instead of considering X and Y, which are discontinuous at the poles (1.2), he used the continuous functions $X \sin\theta$ and $Y \sin\theta$. From these he deduced the 'pseudo-potentials'

$$U = a\int\limits_0^\theta X\,d\theta, \qquad U' = -a\sin\theta\int\limits_0^\phi Y\,d\phi + U_{\phi=0}, \tag{86}$$

which he supposed to be developed in series of surface harmonics. If no electric currents traverse the field, so that, by 1 (24a), p. 16,

$$\operatorname{curl} F = 0, \tag{87}$$

the field will possess a potential V, and $U = U' = V$. But in general U and U', as determined from the observations of X and Y, are unequal, and it is necessary to consider whether this is due merely to imperfect data, or whether the field really possesses a part N not having a potential. It is always possible to represent the distribution of the *horizontal* component of the field as the sum of two parts, one derived from a potential V, the other corresponding to a part N not having a potential; this division is to a large extent arbitrary, though N cannot be zero if $U \neq U'$. In practice the largest possible part of the field is referred to a potential, by taking

$$V = \tfrac{1}{2}(U+U'). \tag{88}$$

The non-potential part of the field then has the horizontal components $X-\partial V/a\,\partial\theta$, $Y+\partial V/a\sin\theta\,\partial\phi$, while the vertical component of N is taken to be zero; the physical assumption underlying this procedure is that the electric currents near and above the ground are purely vertical. The separation of V into parts of internal and external origin then follows in the manner described in § 13. The vertical electric current-density i (positive upwards) is given in electromagnetic units by the radial component of curl F, so that

$$4\pi i = \operatorname{curl}_r F = \frac{1}{a\sin\theta}\left\{\frac{\partial(Y\sin\theta)}{\partial\theta}+\frac{\partial X}{\partial\phi}\right\}, \tag{89}$$

using a well-known formula of vector analysis (cf. (14), p. 735).

Schmidt has also taken into account the ellipticity of the earth, treating it as a spheroid, and using elliptic polar coordinates. The formulae obtained are, in general, similar to those for the sphere, and the convergence of the series is unaffected; the coefficients of the spherical harmonics are affected only by a fraction of the order $\frac{1}{300}$ of their values calculated without this refinement—that is, by about the same amount as the fractional difference between the earth's polar and equatorial radii (18.2). The methods developed by Schmidt were applied by Bauer to the data on the 1922 world isomagnetic charts.

Fanselau [18] has calculated values of the spherical harmonics and of their derivatives, for a number of magnetic observatories.

17.20. The mutual dependence of the spherical harmonics. If a function $f(\theta, \phi)$, which is to be developed in terms of surface harmonics, is known all over the sphere, the coefficients are uniquely given by (23), on account of the orthogonality of the harmonics (§ 4). This property, which is true also for the harmonic analysis of a fully known periodic function of a *single* variable, is in that case retained, at least to a certain extent (but cf. 16.14) when the function is known only for a series of equidistant values of the independent variable (16.12). But in the case of spherical harmonic analysis, when $f(\theta, \phi)$ is known only at a limited number of points (θ, ϕ), this property is lost, at least as regards the polar distance θ; the solution of equations of the type (85) for individual values of θ leads to coefficients which are *dependent* upon the number of surface harmonic terms included in the equations. Thus, in the computation by Schmidt, the value of the first and largest coefficient A_1^0 was found to be 0·3242, 0·3313, or 0·3327, according as the expansion terminated at $n = 3$, $n = 5$, or $n = 7$ respectively.

A similar ambiguity occurs (16.14) in the harmonic analysis of a series of values of a simple periodic function, even in the relatively favourable case of equally spaced values. In the spherical case, when the points θ, ϕ are irregularly distributed—as they are for the magnetic observatories—the ambiguity of the coefficients is enhanced. Fig. 7 refers to an attempt [3.20c] to represent the secular magnetic variation, for the period 1902–20, by a series of spherical harmonics, using the values derived from fourteen observatories (in Fig. 7 the polar distance of these stations is indicated by short lines at the upper and lower borders). The development was extended to terms of the degree $n = 2$. It is evident from the figure that where numerous observations are available (e.g. the eleven stations between $\theta = 0°$ and 63·4°, and between $\theta = 90°$ and 126·6°), the zonal terms P_2 and P_3 affect the X

development (67) in very similar ways; likewise for the terms P_2^2 and P_3^2 in the Z development (62). Hence in this case any terms P_3 and P_3^2 actually present will appreciably add to the apparent values of P_2 and P_2^2, respectively, in a calculation not extending beyond second degree terms. The orthogonality of the various terms, which would prevent such errors arising if the function were *everywhere* known, is here of little avail because of the unfavourable distribution of the known values. No analytical means can overcome this drawback attaching to the faulty distribution of the magnetic observatories.

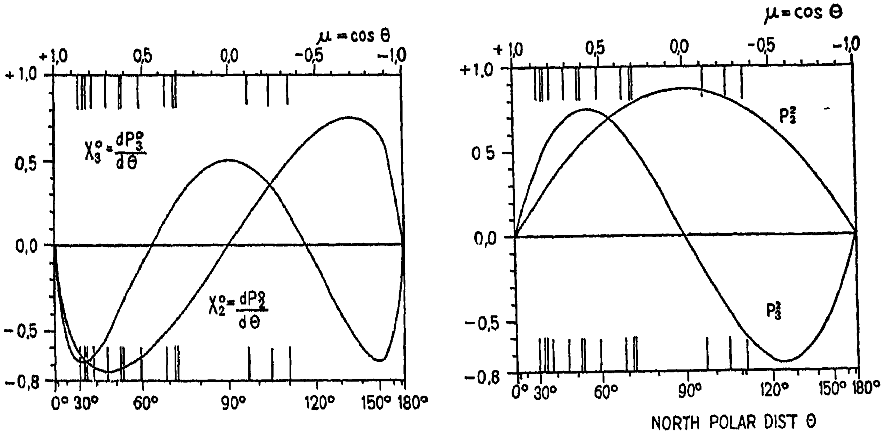


FIG. 7

17.21. Schuster's method of development.

17.21. Schuster's method of development. Schuster developed formulae and tables to facilitate spherical harmonic analysis by reducing it to a series of harmonic analyses. The first stage is the harmonic analysis of $f(\theta, \phi)$ along the circles of latitude, giving $a_m(\theta)$, $b_m(\theta)$, as in (2), for a series of values of θ. If the function is continuous at the poles ($\theta = 0$ and $\theta = 180°$) every a_m and b_m except a_0 must there vanish. Thus for $m > 0$ the data determine $a_m(\theta)$ and $b_m(\theta)$ at points distributed over the range $0 \leqslant \theta \leqslant \pi$. Within this range it is possible to represent them by Fourier series involving either cosine or sine terms only; in the former case we define $a_m(\theta)$ for $\pi < \theta < 2\pi$ by $a_m(2\pi-\theta)$, and in the latter by $-a_m(2\pi-\theta)$. For reasons made apparent later, Schuster used cosine series for odd values of m, and sine series for even values of m, thus:

$$\left.\begin{aligned} a_m(\theta) &= \alpha_{m,0}+\alpha_{m,1}\cos\theta+\alpha_{m,2}\cos 2\theta+\ldots, \\ b_m(\theta) &= \beta_{m,0}+\beta_{m,1}\cos\theta+\beta_{m,2}\cos 2\theta+\ldots, \end{aligned}\right\} \quad (m \text{ odd}) \qquad (90)$$

$$\left.\begin{aligned} a_m(\theta) &= \alpha_{m,1}\sin\theta+\alpha_{m,2}\sin 2\theta+\ldots, \\ b_m(\theta) &= \beta_{m,1}\sin\theta+\beta_{m,2}\sin 2\theta+\ldots. \end{aligned}\right\} \quad (m \text{ even}) \qquad (91)$$

The coefficients $\alpha_{m,p}$ and $\beta_{m,p}$ are obtained by ordinary harmonic analysis (16 (9)), and therefore, using the definitions already given for the range $\pi \leqslant \theta \leqslant 2\pi$,

$$\alpha_{m,p} = \frac{2}{\pi}\int\limits_0^\pi a_m(\theta)\cos p\theta\, d\theta \quad (p > 0); \tag{92}$$

this integral is to be evaluated as described in Chapter XVI, either numerically or mechanically, using graphs of $a_m(\theta)$, or individual values as in 16.12.

It now suffices to substitute in (90) and (91) the spherical harmonic developments of $\cos p\theta$ and $\sin p\theta$ in terms of spherical functions of constant *order* m, i.e.

$$\cos p\theta = u^m_{m,p}\, P^m_m(\theta) + u^m_{m+1,p}\, P^m_{m+1}(\theta) + \dots \quad (m \text{ odd}), \tag{93}$$

$$\sin p\theta = v^m_{m,p}\, P^m_m(\theta) + v^m_{m+1,p}\, P^m_{m+1}(\theta) + \dots \quad (m \text{ even}). \tag{94}$$

Clearly, by (24 c), with the notation δ_m of (24 a),

$$u^m_{n,p} = \delta_m \frac{2n+1}{4}\int\limits_{-1}^{1} P^m_n(\theta)\cos p\theta\, d\cos\theta,$$

and similarly for $v^m_{n,p}$. Schuster showed that many of the coefficients vanish, i.e.

$$\left.\begin{array}{ll} u^m_{n,p} = 0 & (m \text{ odd}) \\ v^m_{n,p} = 0 & (m \text{ even}) \end{array} \quad (n \leqslant p-2;\, n+p \text{ even}).\right\} \tag{95}$$

The formulae (93) and (94) therefore begin as follows:

for m odd, p even, $m > p-1$,

$$\cos p\theta = u^m_{m,p}\, P^m_m + u^m_{m+2,p}\, P^m_{m+2} + \dots,$$

for m odd, p even, $m \leqslant p-1$,

$$\cos p\theta = u^m_{p-1,p}\, P^m_{p-1} + u^m_{p+1,p}\, P^m_{p+1} + \dots,$$

etc., as Table C shows.

For $n \geqslant p-1$ Schuster calculated the coefficients $u^m_{n,p}$ (m odd) and $v^m_{n,p}$ (m even) up to $n = 12$, $p = 12$. Table C is a modified extract from his tables. The relations (95) explain the choice of the cosine or sine series in the developments of $a_m(\theta)$ and $b_m(\theta)$, according as m is odd or even.

On substituting from (93), (94) in (90), (91), and collecting the terms $P^m_n(\theta)$ in order, we get

$$f(\theta, \phi) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} (A^m_n \cos m\phi + B^m_n \sin m\phi) P^m_n(\cos\theta), \tag{96}$$

where, by reason of (95),

$$A_n^m = \sum_{p=0}^{n+1} \alpha_{m,p}\, u_{n,p}^m, \qquad B_n^m = \sum_{p=0}^{n+1} \beta_{m,p}\, u_{n,p}^m \quad \text{(for } m \text{ odd)}, \tag{97}$$

$$A_n^m = \sum_{p=1}^{n+1} \alpha_{m,p}\, v_{n,p}^m, \qquad B_n^m = \sum_{p=1}^{n+1} \beta_{m,p}\, v_{n,p}^m \quad \text{(for } m \text{ even)}. \tag{98}$$

The coefficients u and v are given in the horizontal rows of Table C.

Thus from the table, using only two places of decimals,

$A_0^0 = 0{\cdot}79\alpha_{0,0}$

$A_1^0 = 1{\cdot}18\alpha_{0,2}$

$A_1^1 = 1{\cdot}18\alpha_{1,0} - 0{\cdot}59\alpha_{1,2}$

$A_2^0 = -0{\cdot}49\alpha_{0,1} + 1{\cdot}47\alpha_{0,3}$

$A_2^1 = 0{\cdot}85\alpha_{1,1} - 0{\cdot}85\alpha_{1,3}$

$A_2^2 = 1{\cdot}28\alpha_{2,1} - 0{\cdot}43\alpha_{2,3}$

$A_3^0 = -0{\cdot}69\alpha_{0,2} + 1{\cdot}72\alpha_{0,4}$

$A_3^1 = 0{\cdot}42\alpha_{1,0} + 0{\cdot}84\alpha_{1,2} - 1{\cdot}05\alpha_{1,4}$

$A_3^2 = 1{\cdot}33\alpha_{2,2} - 0{\cdot}67\alpha_{2,4}$

$A_3^3 = 1{\cdot}63\alpha_{3,0} - 1{\cdot}09\alpha_{3,2} + 0{\cdot}27\alpha_{3,4}$

The formulae for B_n^m are obtained from these merely by substituting $\beta_{m,p}$ for $\alpha_{m,p}$.

In geomagnetic calculations X is first developed in terms of ϕ, for various circles of latitude, so that (cf. (2))

$$X = X_0^{(c)}(\theta) + X_1^{(c)}(\theta)\cos\phi + X_2^{(c)}(\theta)\cos 2\phi + \dots + X_1^{(s)}(\theta)\sin\phi + X_2^{(s)}(\theta)\sin 2\phi + \dots. \tag{99}$$

Then the integral functions

$$V_m^{(c)}(\theta) = a\int_0^\theta X_m^{(c)}(\theta)\, d\theta$$

and $V_m^{(s)}(\theta)$ are formed by numerical or graphical integration, starting from $V_m^{(c)}(0) = V_m^{(s)}(0) = 0$; the integration should give, as far as the accuracy of the observation permits, $V_m^{(c)}(180°) = V_m^{(s)}(180°) = 0$ for $m > 0$. Since $X = \partial V/a\partial\theta$, this gives V/a as a series similar to (99). The V_m functions are then expressed in the form (90) and (91), and the form (96) is obtained by the substitutions (93), (94), or by (97), (98). (Alternatively the improvement described in § 22 might be adopted, developing $X\sin\theta$.) For $Y\sin\theta$, which is $\partial V/a\partial\phi$, the integration with respect to ϕ can be made after the harmonic analysis in the form (2).

As Schmidt [23] has pointed out recently, some explanation is necessary to justify Schuster's choice of the *infinite* series (93) and (94) instead of the *finite* series, which for $m = 0$, for instance, would give, merely by algebraic elimination from the equations (6),

$$\cos\theta = P_1^0, \qquad \cos 2\theta = \tfrac{4}{3}P_2 - \tfrac{1}{3}, \qquad \cos 3\theta = \tfrac{8}{5}P_3 - \tfrac{3}{5}P_1, \text{ etc.} \tag{100}$$

(see the general formula for $\cos p\theta$ and $\sin p\theta$ in Art. 97 of Byerly's book [16.15]). The reason is the different effect of an *extension* of the series (90) or (91): with the infinite series (93) and (94), $\cos p\theta$ and $\sin p\theta$ affect only the higher harmonics, of degrees $n \geqslant p-1$ (as can best be seen in Table C), while in the case of the finite series (100), each $\cos p\theta$ or $\sin p\theta$ contains spherical harmonics of the lowest degrees. This apparent constancy of the first terms in the final series (96), when the number of terms taken in (90) or (91) is increased, is similar to that mentioned in 16.9, but is of little *practical* value, as Schmidt has shown. In fact, Schuster's method should only be applied if the points for which the data are given are evenly distributed over the globe.

17.22. Schmidt's and other methods. As already indicated in § 19, Schmidt started with developments for $X\sin\theta$ (removing the discontinuity at the poles):

$$X\sin\theta = \sum P_n^m(e_n^m \cos m\phi + f_n^m \sin m\phi). \tag{101}$$

If the potential V/a is

$$V/a = \sum P_n^m(g_n^m \cos m\phi + h_n^m \sin m\phi), \tag{102}$$

then

$$X = \sum \frac{dP_n^m}{d\theta}(g_n^m \cos m\phi + h_n^m \sin m\phi). \tag{103}$$

Using (56), the formula (103), after multiplication by $\sin\theta$, can easily be transformed into (101), so that, by comparison of corresponding terms, the coefficients g_n^m can be expressed by a combination of coefficients e_n^m, and, similarly, h_n^m by f_n^m.

As to $Y\sin\theta$, there is no difficulty in the transformation of the series for $Y\sin\theta$ into that for V, since $Y\sin\theta = \partial V/a\partial\phi$.

The method of Neumann [4, 13] uses the values on selected circles of latitude; in one special case, these latitudes correspond to the roots of the zonal harmonics, $P_n^0(\theta) = 0$. This method has been further developed by Seeliger [14] and applied by Prey [16], and Schmidt discusses it in his book [1].

If the values of X, Y, Z are given only for a limited number of observatories, the development can be made by the method of least squares. A convenient method has been devised by Schmidt [11], and has been applied by Bartels [26] in an example worked out in detail.

17.23. Concluding remarks. Gauss's memoir of 1839 [22], making use of spherical harmonics, not only gave mathematical form and precision to Gilbert's conclusion (§ 14); it also enabled the larger irregularities in the earth's field to be expressed analytically, and extended to these irregularities the conclusion, which Gilbert's methods could scarcely have established, that the field is of internal origin.

Subsequent analyses with better data will be described in Chapter XVIII.

After Gauss, Schuster was the first to make a new application of spherical harmonic analysis to the earth's magnetism; in 1889, fifty years after Gauss, he analysed the daily variation of the earth's magnetic field (Chapter XX).

In general, spherical harmonics occur wherever an equation is involved in which Laplace's equation (28) appears, and in which spherical polar coordinates are appropriate. In geomagnetism they give the only definite means of separating interior and exterior parts of the fields. On the other hand, it unfortunately proves very laborious to deal even with spherical harmonic functions when many terms need to be taken. But to say that in difficult problems the use of spherical harmonics is laborious is not to slight the method, because any other accurate treatment would be still more difficult.

Workers in this field are advised to use the partly normalized functions P_n^m of (19) and (20), introduced by Adolf Schmidt. They have the advantage that the mean square value of P_n^m is the same as that for P_n^0, so that the coefficients in the series indicate the relative importance of the various terms. This choice would also remedy, for the future, the regrettable confusion of notation which has existed in geomagnetic literature in the past. A resolution recommending the use of Schmidt's functions [1] was adopted by the International Association of Terrestrial Magnetism and Electricity at its Washington Assembly in 1939.

XVIII

THE SPHERICAL HARMONIC ANALYSIS OF THE MAIN FIELD

18.1. Gauss's analysis and its repetitions. In this book the spherical harmonics used are those of the partly normalized form, P_n^m, introduced by Schmidt (17.4); we recall that the square of $P_n^m(\theta)\sin(m\phi+\epsilon_m)$ averaged over the sphere is $1/(2n+1)$.

The geomagnetic potential will be written in the form

$$V = a\sum_{n=1}^{\infty}\{(r/a)^n T_n^e + (a/r)^{n+1} T_n^i\} = V^e + V^i, \tag{1}$$

where a denotes the earth's radius,

$$T_n \equiv \sum_{m=0}^{n} (g_n^m \cos m\lambda + h_n^m \sin m\lambda) P_n^m(\theta), \tag{2}$$

and the affixes e and i refer to the parts which are respectively of external and internal origin. The coefficients g_n^m, h_n^m were called 'elements of the theory of terrestrial magnetism' by Gauss in his famous memoir; now they are usually referred to as Gauss coefficients. They have the same physical dimensions as the magnetic force, and can therefore be expressed in γ. (Note the (unimportant) difference between the definitions given in 2.21, and in (1), (2); the 'parameters' of the earth magnet, in the notation of 2.21, would be a^{n+2} times the 'elements of the theory'.) Comparing (1) and (2) with 17 (61) we find

$$\left.\begin{aligned} g_n^{me} &= c_n^m A_n^m, & h_n^{me} &= s_n^m B_n^m, \\ g_n^{mi} &= (1-c_n^m) A_n^m, & h_n^{mi} &= (1-s_n^m) B_n^m, \\ c_n^m &= g_n^{me}/(g_n^{mi}+g_n^{me}), & s_n^m &= h_n^{me}/(h_n^{mi}+h_n^{me}). \end{aligned}\right\} \tag{3}$$

We give the first eight Gauss coefficients (external plus internal) found in the most important spherical harmonic analyses of the earth's main field.

TABLE 1. *The first eight Gauss coefficients of the earth's magnetic potential V. Unit $10\gamma = 10^{-4}$ c.g.s.*

Author	*Epoch*	g_1^0	g_1^1	h_1^1	g_2^0	g_2^1	h_2^1	g_2^2	h_2^2
Gauss [17.22]	1835	−3235	−311	+625	+51	+292	+ 12	− 2	+157
Erman-Peterson [2, 3]	1829	−3201	−284	+601	− 8	+257	− 4	− 14	+146
Adams [4, 5]	1845	−3219	−278	+578	+ 9	+284	− 10	+ 4	+135
Adams [4, 5]	1880	−3168	−243	+603	−49	+297	− 75	+ 61	+149
Fritsche [6, 8]	1885	−3164	−241	+591	−35	+286	− 75	+ 68	+142
Schmidt [9]	1885	−3168	−222	+595	−50	+278	− 71	+ 65	+149
Dyson and Furner [3.9]	1922	−3095	−226	+592	−89	+299	−124	+144	+ 84

These authors based their calculations on different observational material. It is likely that the change of the coefficients from one epoch to the next indicates not so much the actual secular change, but rather the increase in our knowledge of the magnetic field as well as of its secular variation—the latter being necessary for the reduction of field observations, taken in different years, to a common epoch.

Gauss [1] had long wished to apply his theory to actual observed data, but it was not until 1838 that he obtained observational material which seemed adequate for a first trial, namely Sabine's total intensity (1837) chart [12], Barlow's isogones (1833), and Horner's isoclines (1836). He computed the 24 coefficients of the first 4 orders (17.19) from values taken from 84 points on the charts, and compared the observations at 91 stations with the values computed from his series.

Erman and Petersen [2] used values for 90 points (9 on each of 10 circles of latitude); they devoted special attention to the reduction for secular variation.

Adams [4, 5] calculated 48 terms (up to the order $n = 6$); for the epoch 1845 he used Sabine's chart [12], and for 1880 the Admiralty charts; from these charts he took the values at every 10° in longitude and 5° in latitude.

Fritsche's calculations [6–8] were criticized by Schmidt as giving too much weight to the scanty data in polar regions. Both authors used Neumayer's charts [G 25].

18.2. The analysis by Schmidt. Schmidt, as described in 17.19, analysed each component of magnetic force separately, and considered the possible existence of three parts of the field, two of them having potentials V^i and V^e, the third part being a non-potential field N. He also developed general formulae in which account was taken of the deviation of the earth's surface from a true sphere. Its form was supposed to be spheroidal, with the equatorial radius a and the polar radius b; the ellipticity is measured by the parameter

$$\epsilon^2 = (a^2-b^2)/b^2, \tag{4}$$

for which he took Bessel's numerical value 0·006719, corresponding to the oblateness $(a-b)/a = 1/299$. He developed V/b, and instead of using the geographical polar distance θ, he introduced the reduced polar distance v, which is given by

$$\tan v = (1+\epsilon^2)^{\frac{1}{2}} \tan\theta. \tag{5}$$

Also, instead of analysing the distribution of X, Y, and Z, he considered the following multiples of them:

$$X(1+\epsilon^2\cos^2 v)^{\frac{1}{2}}, \qquad Y(1+\epsilon^2)^{\frac{1}{2}}, \qquad Z[(1+\epsilon^2\cos^2 v)/(1+\epsilon^2)]^{\frac{1}{2}}. \tag{6}$$

In other respects the computations were the same as in the spherical case up to the stage of the separation of the external and internal parts of the potential. In the *spherical* case, if A and B (with appropriate suffixes and affixes) are the coefficients for a particular term in V/b, obtained from the horizontal components, and if α and β are the corresponding coefficients in the development of Z (17 (62)), these satisfy the formulae 17 (64) and 17 (65), which read, in the notation (3) (but omitting the affixes n, m),

$$\alpha = ng^e-(n+1)g^i, \qquad \beta = nh^e-(n+1)h^i, \tag{7}$$

$$A = g^e+g^i, \qquad B = h^e+h^i. \tag{8}$$

Consequently,

$$\left.\begin{aligned} g^e &= \frac{(n+1)A+\alpha}{2n+1}, & h^e &= \frac{(n+1)B+\beta}{2n+1}, \\ g^i &= \frac{nA-\alpha}{2n+1}, & h^i &= \frac{nB-\beta}{2n+1}. \end{aligned}\right\} \tag{9}$$

In the case of the spheroidal earth the corresponding formulae are given below in (10); they contain additional factors π_n^m, p_n^m, κ_n^m, and q_n^m, which depend on ϵ^2 and differ little from unity. Using the abbreviations

$$\delta_n^m = 1\bigg/\left\{n\frac{\pi_n^m}{p_n^m}+(n+1)\frac{\kappa_n^m}{q_n^m}\right\}, \qquad \epsilon_n^m = n\frac{\pi_n^m}{p_n^m}\delta_n^m, \tag{10 a}$$

Schmidt showed that

$$\left.\begin{aligned} g^i &= \epsilon A-\delta\alpha, & h^i &= \epsilon B-\delta\beta, \\ g^e &= A-g^i, & h^e &= B-h^i. \end{aligned}\right\} \tag{10}$$

Table 2 gives the values of ϵ_n^m and δ_n^m, calculated by Schmidt.

While it is of considerable theoretical interest to consider the influence of the earth's oblate form, it must be admitted that this refinement has not added much to our actual knowledge of the field. It makes no serious difference to the estimates of either the exterior part, the interior part, or the non-potential part of the results. No significant physical property of the geomagnetic field can therefore be attributed to the earth's oblateness.

Bauer [3.5] repeated Schmidt's analysis for the data of the epoch 1922, but published only the coefficients for the zonal harmonics and g_1^1, h_1^1.

TABLE 2. *The numerical values of the coefficients* δ_n^m *and* ϵ_n^m *which occur in Schmidt's formulae* (10) *for the analysis of the magnetic field at the surface of a spheroidal earth.*

δ_n^m

$n =$	*1*	*2*	*3*	*4*	*5*	*6*
$m = 0$	0·33423	0·20064	0·14332	0·11148	0·09121	0·07718
1	0·33512	0·20077	0·14337	0·11150	0·09122	0·07719
2	..	0·20115	0·14350	0·11156	0·09125	0·07720
3	..	..	0·14371	0·11165	0·09130	0·07724
4	..	..	..	0·11179	0·09138	0·07728
5	..	..	..	..	0·09147	0·07734
6	..	..	..	..	..	0·07740
Spherical earth	0·33333	0·20000	0·14286	0·11111	0·09091	0·07692

ϵ_n^m

$n =$	*1*	*2*	*3*	*4*	*5*	*6*
$m = 0$	0·33423	0·40038	0·42882	0·44464	0·45470	0·46167
1	0·33288	0·40019	0·42876	0·44461	0·45469	0·46166
2	..	0·39962	0·42857	0·44452	0·45464	0·46163
3	..	..	0·42825	0·44438	0·45456	0·46159
4	..	..	..	0·44417	0·45445	0·46152
5	..	..	..	..	0·45431	0·46144
6	..	..	..	..	..	0·46133
Spherical earth	0·33333	0·40000	0·42857	0·44444	0·45454	0·46154

18.3. The first-order harmonic.

The first-order terms in the potential V/a, at the surface $r = a$, are as follows (if the affix e or i is added to V, g, and h, the corresponding equation for the part of respectively external or internal origin is obtained):

$$\begin{aligned} V_1/a &= g_1^0 P_1^0 + (g_1^1 \cos\phi + h_1^1 \sin\phi) P_1^1 \\ &= g_1^0 \cos\theta + (g_1^1 \cos\phi + h_1^1 \sin\phi)\sin\theta. \end{aligned} \tag{11}$$

This expression can be transformed as follows. Let

$$H_0^2 = g_1^{0\,2} + g_1^{1\,2} + h_1^{1\,2}, \quad \cos\theta_0 = g_1^0/H_0, \quad \tan\phi_0 = h_1^1/g_1^1 \quad (H_0 > 0), \tag{12}$$

so that

$$g_1^0 = H_0 \cos\theta_0, \quad g_1^1 = H_0 \sin\theta_0 \cos\phi_0, \quad h_1^1 = H_0 \sin\theta_0 \sin\phi_0. \tag{13}$$

Also let
$$\cos\Theta = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\phi-\phi_0), \tag{14}$$

so that Θ is the angle between the (general) direction θ, ϕ, and the special direction θ_0, ϕ_0 determined by (12) from the coefficients in (11). From (11), (13), (14) it follows that

$$V_1 = H_0 \cos\Theta. \tag{15}$$

This is the same as the *potential* (at $r = a$) of the field of a sphere of radius a, uniformly magnetized along the direction $(-\theta_0, -\phi_0)$, with an intensity such that at 90° from this axis the horizontal magnetic force is H_0 (1.15). But the *field* at $r = a$ is not necessarily the same as that due to such uniform magnetization, because part of it will be of external origin, unless the coefficients g and h, with affix e, are zero. By applying the above transformation to the equations of type (11), with affixes e and i, referring to the external and internal parts of the field, we obtain two equations, analogous to (15), as follows:

$$V_1^i/a = H_0^i(a/r)^2\cos\Theta^i, \qquad V_1^e/a = H_0^e(r/a)\cos\Theta^e, \tag{16}$$

where Θ^i, Θ^e denote the angles between the direction (θ, ϕ) and directions (θ_0^i, ϕ_0^i), (θ_0^e, ϕ_0^e) determined by equations analogous to (12), in which g and h are replaced by g^i, h^i or g^e, h^e.

The components of the magnetic field due to V_1^i at the surface $r = a$ are a vertical (downward) component

$$Z_1^i = (\partial V_1^i/\partial r)_{r=a} = -2H_0^i \cos\Theta^i, \tag{17}$$

and a horizontal component, directed away from the pole θ_0, ϕ_0,

$$H_1^{(i)} = -(\partial V_1^i/a\,\partial\Theta^i)_{r=a} = H_0^i \sin\Theta^i. \tag{18}$$

This field is therefore properly identified with the field of a uniformly magnetized sphere (1.15), or with that of a dipole (1.9) of moment

$$M = H_0^i a^3. \tag{18a}$$

The direction $\Theta^i = 0$, or (θ_0^i, ϕ_0^i), is that of the magnetic pole with *northern* magnetization, for there Z_1^i is negative, that is, directed outwards.

From (17) and (18) it follows that (omitting the index i)

$$H_0^2 = H_1^2 + (\tfrac{1}{2}Z_1)^2. \tag{18b}$$

The moment $M = H_0 a^3$ of a uniformly magnetized sphere can therefore easily be computed, by (18b), from the horizontal and vertical components H_1, Z_1 observed at any point on the surface. This led Bauer [15] to define a quantity G, the '*local magnetic constant*', by the equation

$$G^2 = H^2 + (\tfrac{1}{2}Z)^2 = X^2 + Y^2 + (\tfrac{1}{2}Z)^2, \qquad G = \tfrac{1}{2}H(4+\tan^2 I)^{\frac{1}{2}}; \tag{18c}$$

hence Ga^3 is the hypothetical moment of the earth inferred from the magnetic field components at a given locality on the assumption that this field is due to a homogeneous magnetization of the earth; this assumption limits the physical significance of G, which, however, is sometimes discussed.

The equipotential surfaces for V_1^e have the equation $r\cos\Theta^e = \text{const.}$, and since $r\cos\Theta^e$ is the projection of r on the direction (θ_0^e, ϕ_0^e), these surfaces are *planes*, perpendicular to that direction, and the field has a uniform intensity H_0^e, along the direction opposite to (θ_0^e, ϕ_0^e).

Another possible transformation of (11) represents V_1/a as corresponding to a combination of the fields of two dipoles, one directed along the geographical axis, and having its maximum north component, of amount g_1^0, on the geographical equator, and the other with its axis in the plane of the equator pointing towards the longitude ϕ_0, and having its maximum east or west intensity $(g_1^{1\,2}+h_1^{1\,2})^{\frac{1}{2}}$ in its own equatorial plane, which is identical with the two geographical meridian planes $\phi_0 \pm 90°$; yet a third representation of the field is by a combination of three mutually orthogonal dipoles, at the earth's centre, namely g_1^0 along the axis $\theta = 0$, g_1^1 in the plane of the equator and pointing towards $\phi = 0$, and h_1^1 in the same plane, and pointing in the direction $\phi = 90°$.

Similar transformations may be made for the higher terms in V, using Maxwell's conceptions of quadrupoles, octupoles, etc. (§ 12).

The following table gives the data for the main internal component V_1^i as determined by Schmidt and Bauer for different epochs; instead of (θ_0^i, ϕ_0^i), which refer to the magnetic pole with *northern* magnetization, lying in the *southern* hemisphere, we give θ_1 $(= 180-\theta_0^i)$, ϕ_1 $(= 180+\phi_0^i)$, which indicate the position of the magnetic pole with southern magnetization (which attracts the northern end of the compass-needle); this is situated in the northern hemisphere.

Author	*Epoch*	H_0	θ_1	ϕ_1
Schmidt [9]	1885	$0{\cdot}3238\,\Gamma$	11° 28′	−68° 30′
Bauer [3.5]	1922	$0{\cdot}3109\,\Gamma$	11° 28′	−69° 8′

Because the hypothetical conception of the uniformly magnetized sphere gives a fairly good approximation to the actual field, we consider it in some detail. The geomagnetic axis (that is, the axis of uniform magnetization) goes through the earth's centre O, and intersects the earth's surface in two antipodal points, A in the southern hemisphere (here taken to be at latitude 78·5° S., longitude 111° E.) and B in the northern hemisphere (latitude 78·5° N., longitude

69° W.). The intensity of the magnetization, if it is assumed uniform throughout the earth, is $3H_0/4\pi$, or, if we assume $H_0 = 0{\cdot}315$ gauss, about 0·08 gauss; since cobalt-steel, in bars, can easily be given a remanent magnetization 5,000 times as high (400 gauss), the earth's field could be produced by a uniform distribution of such parallel, highly magnetized bars, embedded in non-magnetic material filling up the remaining volume, if the magnetized bars occupied a volume, per cubic metre, of 200 c.c., corresponding to 1·5 kilogram of cobalt-steel per cubic metre.

The internal cause of the field whose potential is V_1^i cannot, however, be inferred from observations of the field at and above the surface of the earth; the same field could be produced by an unlimited number of alternative internal distributions of magnetization or of electric-current systems. For instance, 17 (84) shows that V_1^i could be produced by a surface current-system, the flow being from east to west, with a current-function $J_1 = (30/4\pi)aH_0^i \cos\Theta^i$ amperes, or a current-density

$$-\partial J_1/a\,\partial\Theta = 0{\cdot}8\sin\Theta \text{ amp./cm.},$$

the total current amounting to 1,000 million amperes. Alternatively the field might be ascribed to uniform magnetization confined to one or more concentric shells, or to a strong dipole at the earth's centre, of moment

$$M = H_0^i a^3 = 8{\cdot}1\times 10^{25} \text{ gauss cm.}^3$$

For the latter reason, we may call the field of the uniform magnetization that of the *centred dipole.*

18.4. Geomagnetic coordinates and time. The axis AB of the centred dipole is almost as fundamental for geomagnetism as the axis of rotation (SN) is for geography. Hence Schmidt [16] introduced *geomagnetic coordinates*, namely the geomagnetic polar distance Θ or the geomagnetic latitude $90°-\Theta$, measured from the direction of OB, and the geomagnetic longitude Λ, measured eastwards from the meridian half-plane bounded by AB and containing the geographical south pole S. The magnetic declination corresponding to the centred dipole field, at any point P, is the angle NPB or ψ, measured eastwards from the geographical north direction PN (Fig. 1).

The geomagnetic coordinates Φ, Λ of a point P, whose geographical coordinates are θ, ϕ, are calculated from the spherical triangle NBP (Fig. 1), where $\text{NB} = \theta_1$, $\text{NP} = \theta$, $\text{BP} = \Theta$, $\text{NBP} = 180°-\Lambda$, $\text{NPB} = -\psi$, $\text{BNP} = \phi-\phi_1$. We solve for Θ, Λ, and ψ by the ordinary formulae of spherical trigonometry. For logarithmic work, it is convenient to draw the perpendicular PD from P to the line NB, and to introduce the auxiliary angle $\kappa = \mathrm{ND}$. Then

$$\left.\begin{aligned} \tan\kappa &= \tan\theta\cos(\phi-\phi_1), \\ \tan\Lambda = \tan(\phi-\phi_1)\sin\kappa/\sin(\kappa-\theta_1), &\qquad \cot\Theta = \cos\Lambda\cot(\kappa-\theta_1), \\ \sin\psi = -\sin\theta_1\sin(\phi-\phi_1)/\sin\Theta &= -\sin\theta_1\sin\Lambda/\sin\theta. \end{aligned}\right\} \quad (19)$$

Schmidt [16] has published tables of the geomagnetic coordinates for many observatories, taking $\theta_1 = 11{\cdot}5°$, $\phi_1 = 69°$ W.; McNish [17] has given nomographs, from which such coordinates can be found easily for *any* terrestrial point. Our Table A (p. T 1) gives Θ, Λ, and Ψ for past and present magnetic observatories.

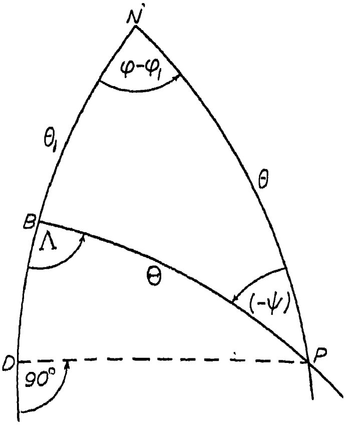


Fig. 1. Illustrating the calculation of geomagnetic coordinates; N is the north geographical pole, and B the northern end of the geomagnetic axis

Vegard, in a description of the daily variations in polar aurora, introduced the conception 'magnetic midnight'. Geomagnetic noon and midnight, for any station P, are the moments when the sun is on the geomagnetic meridian of the station, or on the opposite meridian. More generally, *geomagnetic time* is defined by the angle between the geomagnetic meridian through the station and the one opposite to that through the sun; it can therefore be found by computing the geomagnetic longitudes of the station and of the sun at any instant. At stations in middle and low latitudes the geomagnetic time differs only slightly from the geographic local time [14.13].

18.5. The field of the centred dipole. The distribution of the horizontal magnetic force and its components X and Y, computed for the field of the centred dipole, or the field of uniform magnetization, is shown in Fig. 2 by vectors of appropriate length and direction, at a network of points. The projection [19] chosen for this figure is not the usual Mercator projection, which over-emphasizes the polar regions; it is also not one of the many equal-area projections, because the curvature of the meridians which they involve might give a wrong impression of the actual distribution of the vectors. Instead, a simple rectangular net is chosen, with the ratio 4 to 5 for one degree of longitude to one of latitude; thus there is no distortion near the latitude 37°, but the equatorial belt is compressed, and the polar zones are expanded

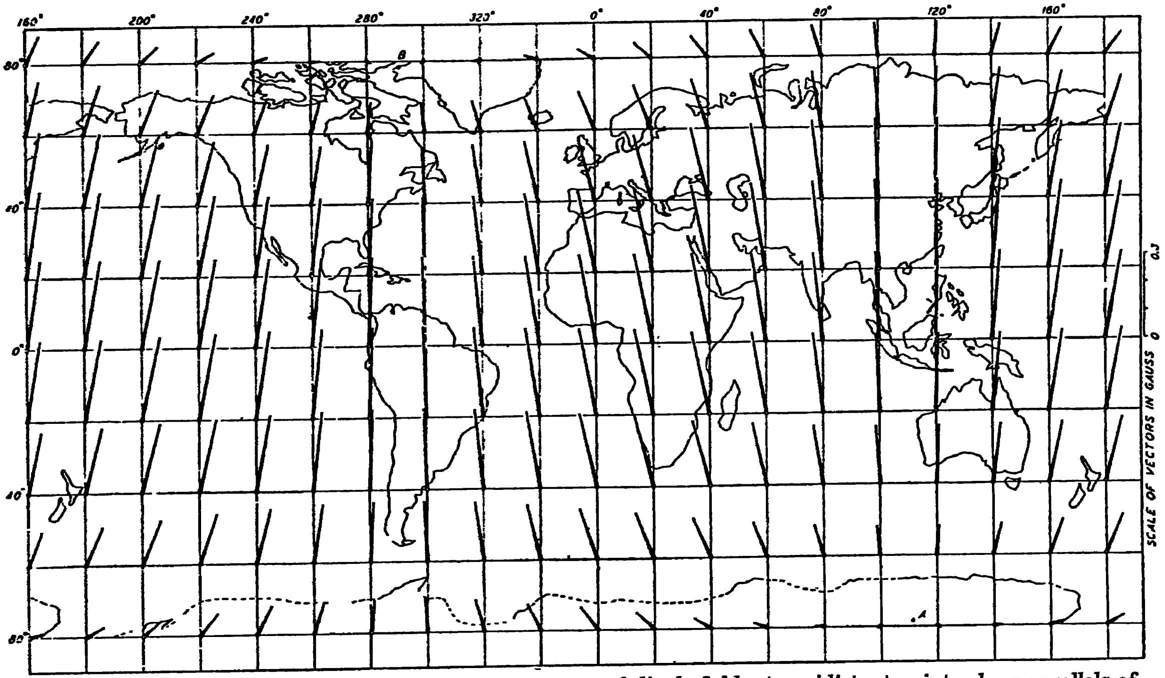


FIG. 2. The horizontal magnetic force vectors for the centred dipole field, at equidistant points along parallels of latitude, 20° apart

in the east-west direction. The vectors are, of course, drawn so that the scale for the north and east components, X and Y, and for the horizontal intensity H, is the same everywhere; the angle between the direction of the vector and the meridian is therefore equal to the declination, that is, the angle ψ (§ 4).

Two noteworthy features of Fig. 2 are that Y has the same value at all points of the same meridian, and X has the same value at all points along the equator. These properties may be inferred from the fact that the vector sum of the fields of a set of dipoles whose vector moments are $\mathbf{M}_1$, $\mathbf{M}_2$,..., all situated at the same point O, is the field of the single dipole at O, with the vector moment $\mathbf{M} = \mathbf{M}_1+\mathbf{M}_2+...$; at any point P, therefore, the sum of any particular component (e.g. X or Y) for all the separate fields is equal to that component of the field of the dipole $\mathbf{M}$. In considering the values of Y at points on the meridian with the longitude ϕ, the earth's centred dipole may be supposed resolved into component dipoles lying in and perpendicular to the plane of this meridian; the former component makes no contribution to Y on this meridian, and the latter makes the same contribution at all points on this and the opposite meridian ($\phi\pm 180°$). In considering X at points on the equator, the dipole is resolved into components along the geographical axis SN, and in the plane of the equator; the latter makes no contribution to X at points along the equator.

Fig. 3 shows the isogonic lines (or lines of equal declination D) for the field of the centred dipole. The lines of equal H or equal I are not given, since they are merely the circles of geomagnetic latitude ($= 90-\Theta$). The geomagnetic equator (of the dipole field), shown in Fig. 3, is the line $\Theta = 90°$, this intersects the geographic equator in longitudes 21° and 201° east; its northernmost point, in latitude 11·5°, has the longitude 111° east, and its southernmost point is in longitude 291° east.

18.6. The magnetic centre C and the eccentric dipole. Suppose that the origin of the polar coordinate system is moved from the earth's centre O to another point O′ (without changing the *directions* of the polar axis and initial meridian). Let (r, θ, ϕ) be the polar coordinates of a point P relative to O, and (x, y, z) its corresponding Cartesian coordinates, given by 17 (1); let O′ be the point (x_0, y_0, z_0), and let the polar and Cartesian coordinates of P relative to O′ be (r', θ', ϕ') and (x', y', z'). Then

$$x' = x-x_0, \qquad y' = y-y_0, \qquad z' = z-z_0,$$

or

$$x = x_0+x', \qquad y = y_0+y', \qquad z = z_0+z'.$$

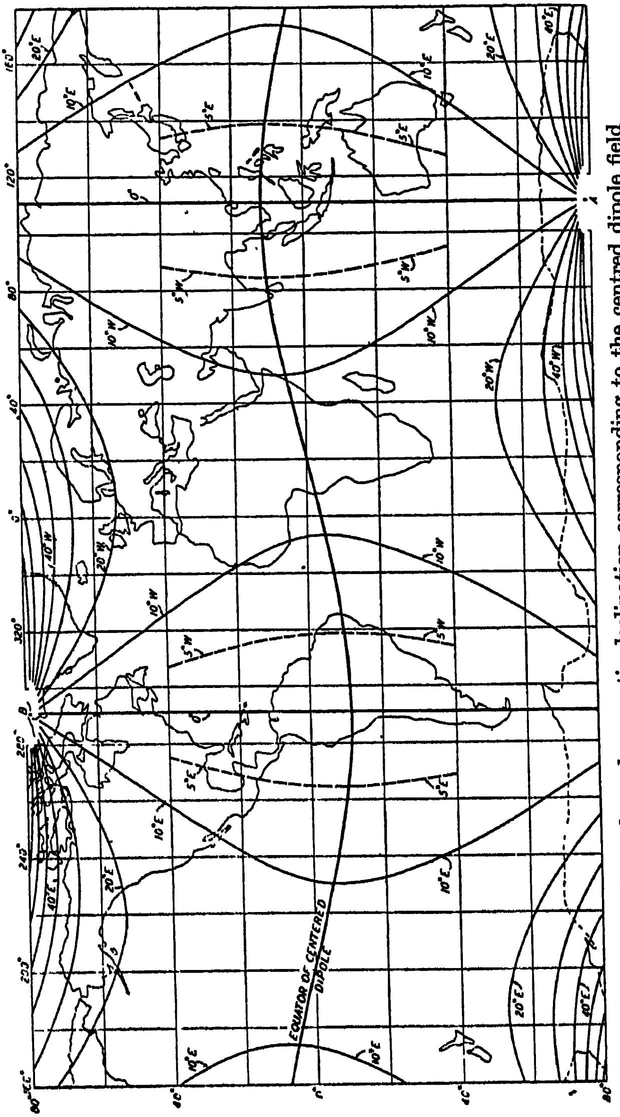


FIG. 3. Lines of equal magnetic declination corresponding to the centred dipole field

These equations suffice to determine r', θ', ϕ' in terms of r, θ, ϕ and x_0, y_0, z_0, using also relations analogous to those of 17 (1), for x', y', z'. Clearly

$$r^2 = x_0^2+y_0^2+z_0^2+2(x_0x'+y_0y'+z_0z')+r'^2 \qquad (20)$$
$$= r_0^2+2r_0r'\cos\psi'+r'^2,$$

where
$$\cos\psi' = \cos\theta_0\cos\theta'+\sin\theta_0\sin\theta'\cos(\phi'-\phi_0).$$

Any spherical harmonic term of degree n, in V^i, can be expressed in the form $S_n(\theta,\phi)/r^{n+1}$ or $f_n(x,y,z)/r^{2n+1}$, where $f_n(x,y,z)$ is a homogeneous (harmonic) polynomial of degree n in x, y, z. By the above substitution f_n can be expressed as a sum $\sum_{n'=0}^{n} F_{n'}(x',y',z')$, where $F_{n'}$ is a homogeneous polynomial of degree n' in x', y', z', whose coefficients involve x_0, y_0, and z_0; evidently the term of highest degree, $F_n(x',y',z')$, is of the same form as f_n, so that it is equal to $r'^nS_n(\theta',\phi')$; it is not difficult to prove that the other terms are also, separately, harmonic functions, satisfying Laplace's equation, so that they can be written as $r'^{n'}S_{n'}(\theta',\phi')$. Hence

$$f_n(x,y,z) = \sum_{n'=0}^{n} r'^{n'}S_{n'}(\theta',\phi') = r'^n\sum_{n'=0}^{n} r'^{n'-n}S_{n'}(\theta',\phi'). \qquad (21)$$

The factor $1/r^{2n+1}$ in $f_n(x,y,z)/r^{2n+1}$ can be expanded as a series of inverse powers of r' (cf. (20)), the first term being $1/r'^{2n+1}$; the coefficients of the other terms are functions of θ' and ϕ'. Thus the expression of f_n/r^{2n+1}, or $r^nS_n(\theta,\phi)$, in terms of r', θ', ϕ', obtained by multiplying this expansion of $1/r^{2n+1}$ by (21), is a series of inverse powers of r', the first term being $S_n(\theta',\phi')/r'^{n+1}$, of precisely the same form as $r^nS_n(\theta,\phi)$.

Consequently when V^i is expressed in terms of r', θ', and ϕ', the terms of degree -2, namely $S_1(\theta,\phi)/r^2$, which correspond to the field of the centred dipole, are transformed to a series of terms of which the first is $S_1(\theta',\phi')/r'^2$, the later terms being proportional to $1/r'^3$, $1/r'^4$,...; the coefficients of these terms are surface harmonics of degrees 2, 3,... The further terms in V^i, when expressed in terms of r', θ', ϕ', all correspond to harmonics of the form $S_{m-1}(\theta, \phi)/r^m$, where $m > 2$, so that the first term in the new expression (V'^i) for V^i is $S_1(\theta',\phi')/r'^2$, which corresponds to the field of a dipole with the same vector moment as the centred dipole, but with its centre at O′. Thus this vector moment is an invariant feature of the field; it is called the vector moment of *the* field dipole, without reference to any particular origin. But the choice of origin affects the form of the remaining harmonics in V^i; in the case of the second harmonic in V'^i, this consists of a part $S_2(\theta',\phi')/r'^3$ identical in form with the second harmonic in V^i, involving $g_2^0, g_2^1, h_2^1, g_2^2, h_2^2$, together with a contribution, $T_2(\theta',\phi')/r'^3$, say, which comes from the transformation of the first term in V^i, namely

$$(g_1^0 \cos\theta + g_1^1 \sin\theta \cos\phi + h_1^1 \sin\theta \sin\phi)/r^2,$$

so that it involves these factors g_1^0, g_1^1, and h_1^1, each multiplied by one or other of the coordinates x_0, y_0, z_0 of O′. It is easy to see that this contribution to $V_2'^i$ from V_1^i does not involve $\cos 2\phi'$ or $\sin 2\phi'$, so that in $V_2'^i$ the factors of $P_2^2(\cos\theta')\cos 2\phi'$ and $P_2^2(\cos\theta')\sin 2\phi'$ are g_2^2 and h_2^2, as in V_2^i.

As Kelvin first pointed out, the terms in $V_2'^i$ involving P_2 and P_2^1, with coefficients $g_2^{0\prime}$, $g_2^{1\prime}$, and $h_2^{1\prime}$, can be made to vanish by suitable choice of x_0, y_0, z_0; the point O′ so chosen is called the magnetic centre C of the field. With this choice of origin, the potential V'^i is that of the field dipole centred at C (this is called the eccentric dipole of the field, since C is not at the earth's centre), together with a term $V_2'^i$ of the special form $P_2^2(\cos\theta')\sin(2\phi'+\epsilon_2')$, and higher terms $V_n'^i$ $(n > 2)$.

Schmidt found the following formulae for the coordinates of C; he also showed that the choice of C, as the origin O′, made the square of $V_2'^i$, averaged over any sphere centred at O′, a minimum; it is, indeed, easy to see, from 17 (17, 22), that this averaged square of $V_2'^i$ is a multiple of $(g_2^{0\prime})^2+(g_2^{1\prime})^2+(h_2^{1\prime})^2+(g_2^{2\prime})^2+(h_2^{2\prime})^2$; the last two terms, as stated above, are independent of the position of O′, so that the expression is a minimum when $g_2^{0\prime}$, $g_2^{1\prime}$, $h_2^{1\prime}$ are all zero; these conditions require that O′ is at C, as follows:

$$x_0 = a(L_0 - g_1^0 E)/3H_0^2, \quad y_0 = a(L_1 - g_1^1 E)/3H_0^2, \quad z_0 = a(L_2 - h_1^1 E)/3H_0^2, \tag{22}$$

where

$$H_0^2 = g_1^0 g_1^0 + g_1^1 g_1^1 + h_1^1 h_1^1, \tag{23}$$

$$\left.\begin{aligned} L_0 &= 2g_1^0 g_2^0 + (g_1^1 g_2^1 + h_1^1 h_2^1)\surd 3, \\ L_1 &= -g_1^1 g_2^0 + (g_0^1 g_2^1 + g_1^1 g_2^2 + h_1^1 h_2^2)\surd 3, \\ L_2 &= -h_1^1 g_2^0 + (g_1^0 h_2^1 - h_1^1 g_2^2 + g_1^1 h_2^2)\surd 3, \\ E &= (L_0 g_1^0 + L_1 g_1^1 + L_2 h_1^1)/4H_0^2. \end{aligned}\right\} \tag{24}$$

For the epoch 1922, in terms of the unit $10\gamma = 10^{-4}$ c.g.s. Schmidt found the values

$$g_2^0 = -67, \quad g_2^1 = +292, \quad h_2^1 = -122, \quad g_2^2 = +143, \quad h_2^2 = +113, \tag{25}$$

and so obtained

$$x_0/a = +0{\cdot}0061, \qquad y_0/a = -0{\cdot}0509, \qquad z_0/a = +0{\cdot}0168. \tag{26}$$

The distance OC is 0·0536a or 342 km.; and C is in latitude 6·5° north and longitude 161·8° east. Let C′ be the point where OC meets the earth's surface.

The plane of the equator of the centred dipole, that is, the plane through the earth's centre O perpendicular to the magnetic axis AB, all but contains C, which lies only 10 km. south of it. This fact, that the dipole is shifted in a direction practically perpendicular to its axis, makes it less difficult to visualize the field of the eccentric dipole (Fig. 4).

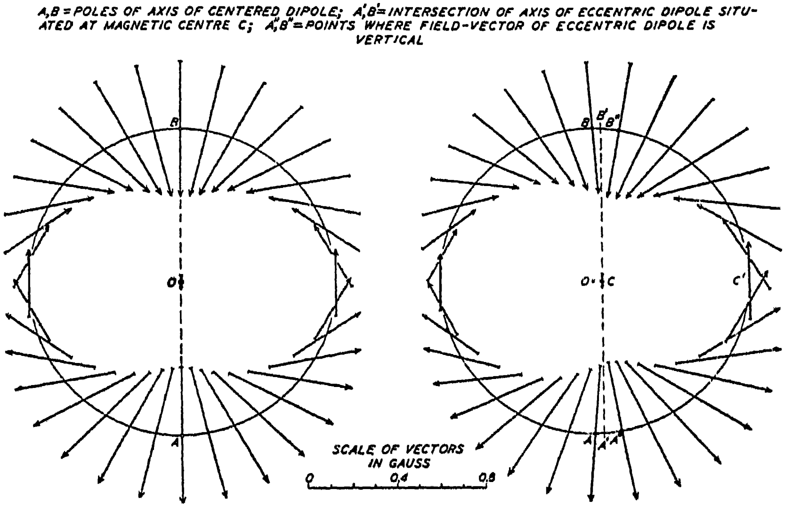

A,B = POLES OF AXIS OF CENTERED DIPOLE; A′,B′= INTERSECTION OF AXIS OF ECCENTRIC DIPOLE SITUATED AT MAGNETIC CENTRE C; A″,B″=POINTS WHERE FIELD-VECTOR OF ECCENTRIC DIPOLE IS VERTICAL
B
O
A
B
B′
B″
O
C
C′
A
A′
A″
SCALE OF VECTORS IN GAUSS
0
0,4
0,8

Fig. 4. Magnetic vectors of the fields of the centred dipole (left) and of the eccentric dipole (right). The distance of the eccentric dipole, at C, from the centre O, is 0·0536 times the radius of the circle, as in the case of the earth

The highest value of the horizontal intensity on the surface of the earth (computed for the eccentric dipole), occurs near C′, being 0·371 gauss, while near its antipode C″ it is only 0·270 gauss, these values having the rather considerable ratio 1·37: 1. From the surface to 100 km. altitude above the ground, *H* decreases, near C′, from 0·371 to 0·353 gauss, but near C″ the decrease is only from 0·270 to 0·258 gauss.

Figs. 5–11 show the field of the eccentric dipole; Bartels [19] has given extensive tables for this field. The difference between the eccentric-dipole field over the geographic north and south polar areas is noticeable.

The extension of the axis of the eccentric dipole, that is, a line parallel to AB drawn through C, pierces the earth's surface in two points A′ and B′, whose positions are: A′, 76·3° south, 121·2° east; B′, 80·1° north, 277·3° east.

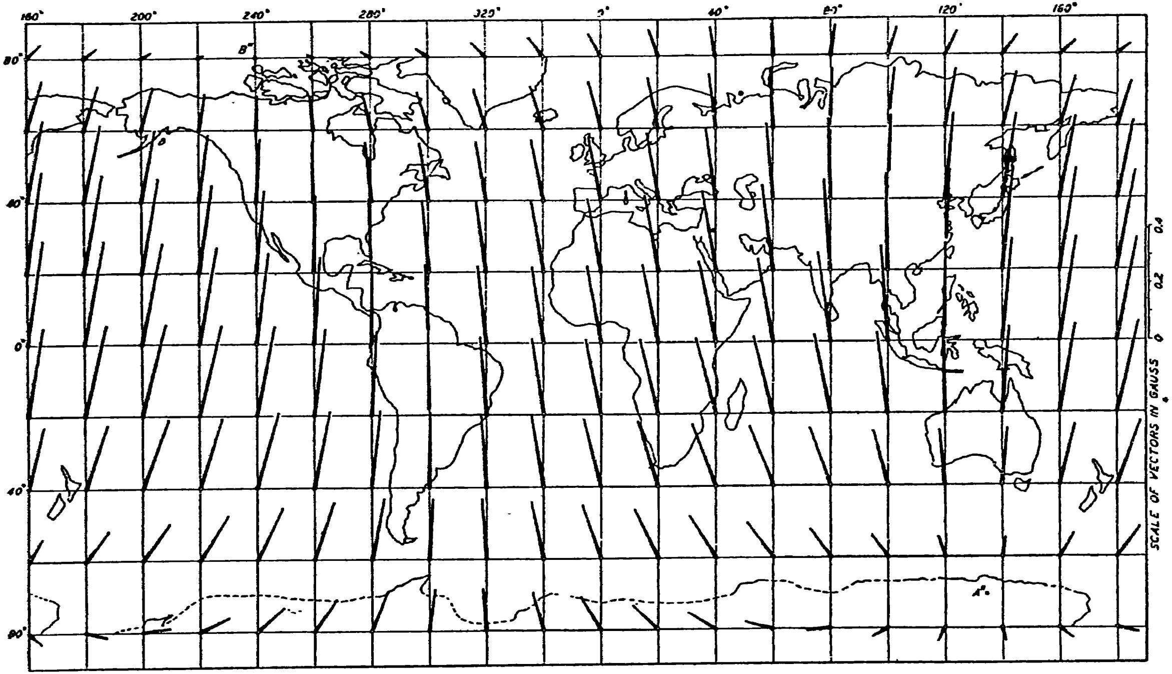
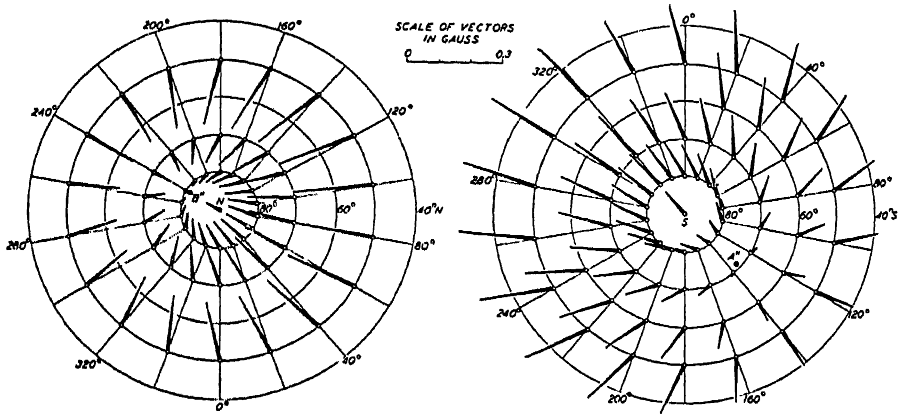


FIG. 5. The horizontal magnetic force-vectors for the eccentric dipole field, at equidistant points along parallels of latitude, 20° apart

At these points the magnetic vector is along the axis, but because of the eccentricity of C, the vector is not vertical there, so that the charts show no special features at A′ and B′. The magnetic vector in the field of the eccentric dipole is vertical at two points A″ and B″ which have the character of poles for this field. They are situated (Fig. 4) in the plane through OC and AB, on the extensions of the great circles AA′ and BB′ beyond A′ and B′, at 9·2° angular distance from A or B. Their positions are: A″, 71·2° south, 133·6° east; B″, 81·0° north, 239·4° east.



NORTH POLAR AREA

FIG. 6

SOUTH POLAR AREA

FIG. 7

FIGS. 6 and 7. The horizontal magnetic vectors over the north and south polar areas, corresponding to the eccentric dipole field

It is of interest to compare the positions of these points with those of the magnetic dip-poles P_S and P_N as actually observed (Figs. 12 and 13); we assume that their coordinates are: P_S, 71·2° south, 150·8° east; P_N, 70·8° north, 264·0° east. The angular distances are: $AP_S = 12{\cdot}2°$; $BP_N = 10{\cdot}4°$; $A''P_S = 5{\cdot}5°$; $B''P_N = 11{\cdot}6°$; in the north B and B″ are about equally far from P_N, but in the south A″ is distinctly nearer to P_S than to A. This shows the better approximation to the field, which we obtain by using the eccentric dipole (although the positions of P_S and P_N are not very fundamental characteristics of the actual field, because their locations may, and probably do, depend on regional anomalies in the polar zone). The line $P_S P_N$ is even more eccentric than the axis of the eccentric dipole; the perpendicular drawn from the earth's centre to this line $P_S P_N$ measures 1,140 km., more than one-sixth of the earth's radius, and is directed towards a surface-point on the equator in about longitude 208° east.

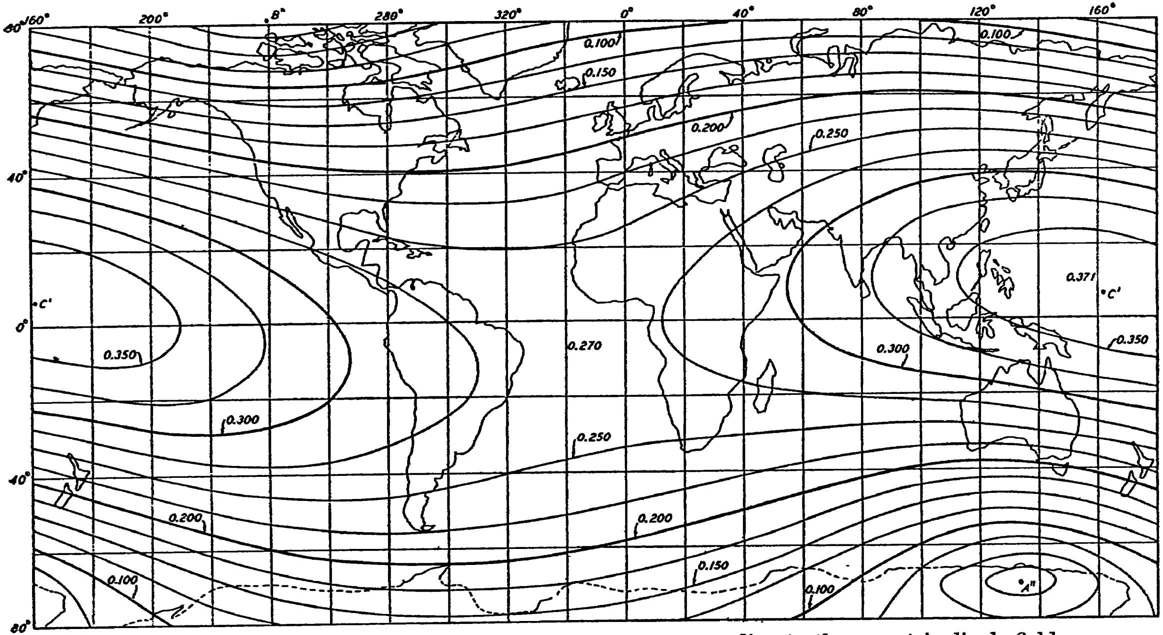


FIG. 8. Lines of equal horizontal magnetic intensity corresponding to the eccentric dipole field

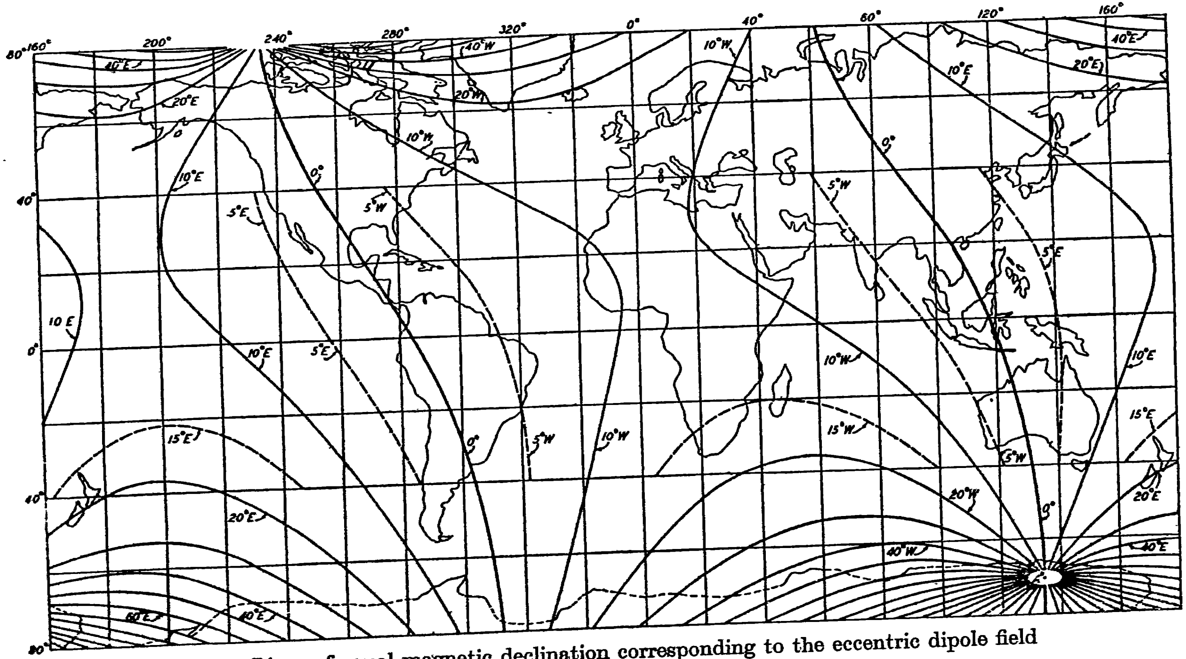


Fig. 9. Lines of equal magnetic declination corresponding to the eccentric dipole field

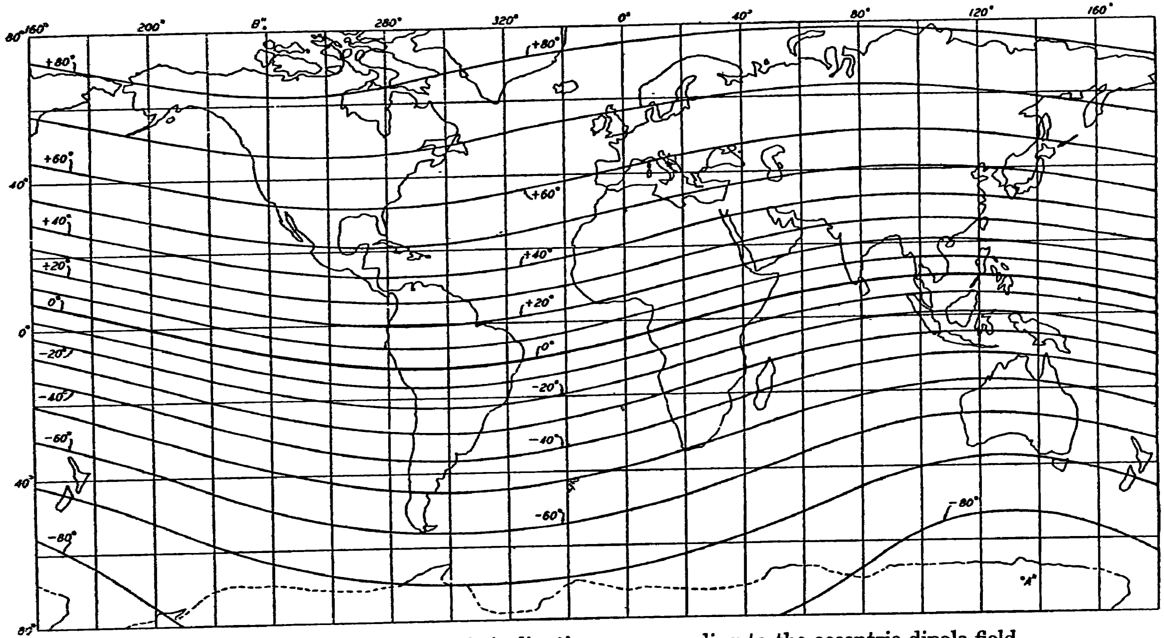


FIG. 10. Lines of equal magnetic inclination corresponding to the eccentric dipole field

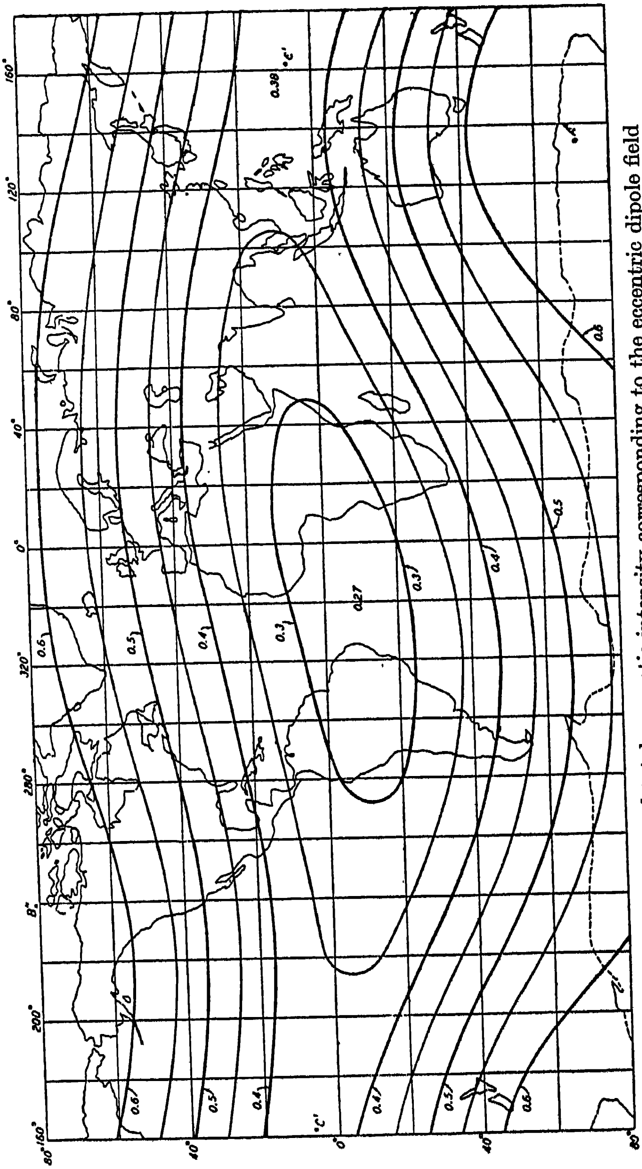


FIG. 11. Lines of equal total magnetic intensity corresponding to the eccentric dipole field

18.7. The terms of higher order. The relative importance of the terms of the third and higher orders in V may be estimated by forming the differences between F or its components (at $r = a$) for the observed field, and for the field of the dipole, either centred (V_1^i) or eccentric ($V_1'^i$). Figs. 14 and 15 give the horizontal vectors [19] for these differences. Average numerical values for the differences in the three force components, taken without regard to sign, are given in the following table, for the northern and southern zones 25° to 65°, for the intervening zone 25° S. to 25° N., and for the whole zone 65° S. to 65° N.; the average numerical values of the actual force components for the centred dipole are also given, in column (1), for comparison (those for the eccentric dipole are only slightly different).

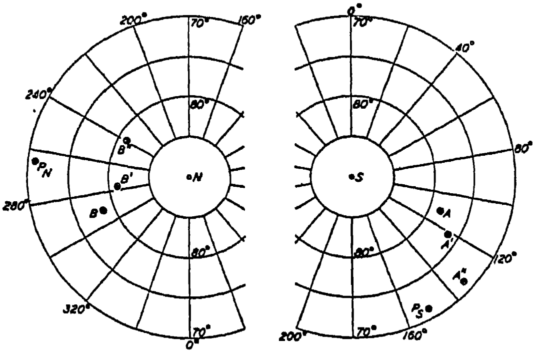


FIG. 12. North polar area FIG. 13. South polar area

N,S = GEOGRAPHIC POLES OF ROTATION
A,B = POLES OF AXIS OF CENTRED DIPOLE, OR UNIFORM MAGNETIZATION
A',B' = INTERSECTION OF AXIS OF ECCENTRIC DIPOLE DRAWN THROUGH MAGNETIC CENTRE C
A'',B'' = POINTS WHERE FIELD-VECTOR OF ECCENTRIC DIPOLE IS VERTICAL
P_N,P_S = MAGNETIC POLES WHERE OBSERVED HORIZONTAL INTENSITY IS ZERO

A,B = POLES OF AXIS OF CENTRED DIPOLE; A',B' = INTERSECTION OF AXIS OF ECCENTRIC DIPOLE SITUATED AT MAGNETIC CENTRE C; A'',B'' = POINTS WHERE FIELD-VECTOR OF ECCENTRIC DIPOLE IS VERTICAL

FIGS. 12 and 13. Indicating the location of various poles, 1922

TABLE 3. *Average numerical values of the three magnetic components, over four zones, for* (1) *the field of the centred dipole*; (2) *the observed field minus the* centred *dipole field*; (3) *the observed field minus the* eccentric *dipole field.*

Unit 0·001 c.g.s. = 100 γ

	North component X			*East component Y*			*Vertical component Z*		
Zone	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
65° N.–25° N.	226	17	22	40	36	36	390	50	48
25° N.–25° S.	307	38	28	40	20	23	69	39	42
25° S.–65° S.	226	34	26	40	32	[illegible]	390	75	39
65° N.–65° S.	264	31	26	40	27	26	240	51	43

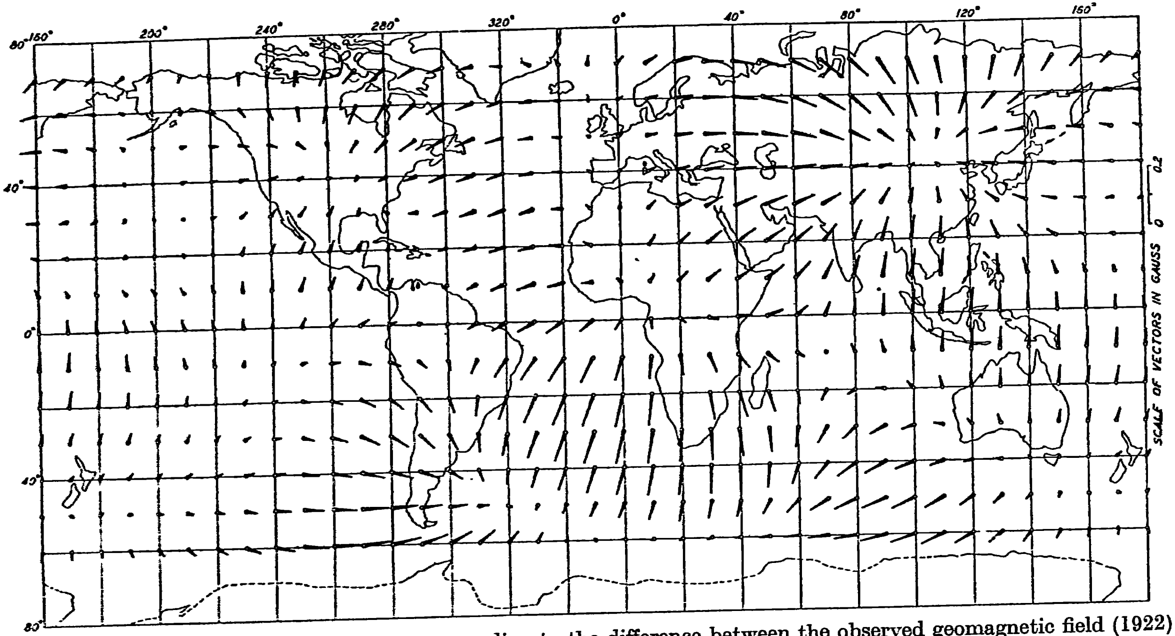


FIG. 14. Horizontal magnetic vectors corresponding to the difference between the observed geomagnetic field (1922) and the field of the centred dipole

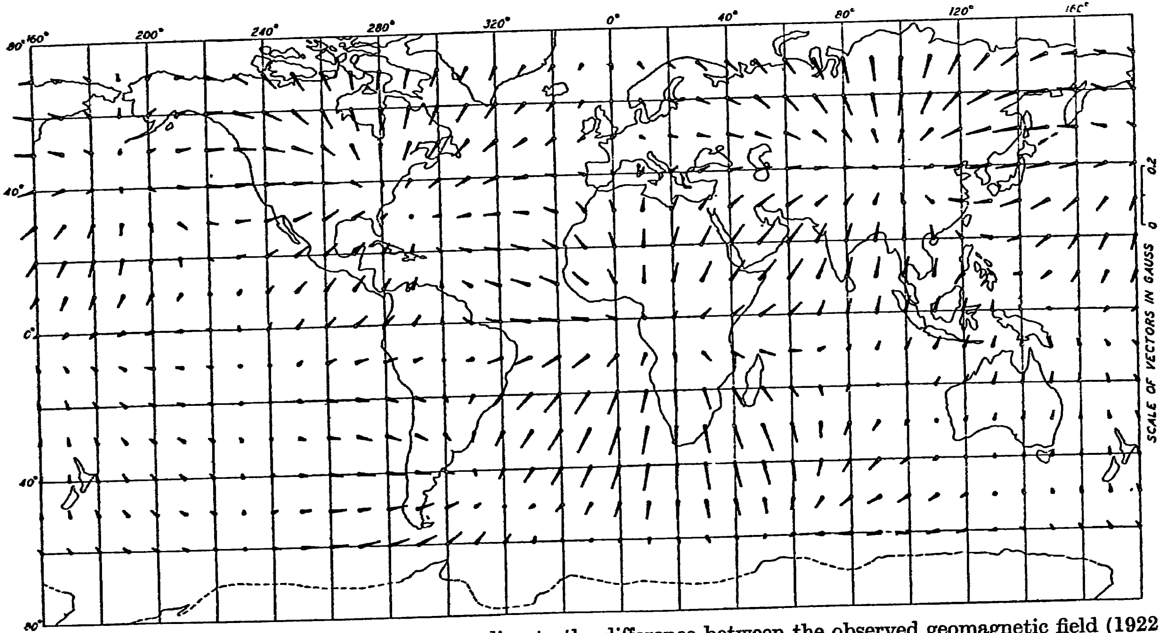


FIG. 15. Horizontal magnetic vectors corresponding to the difference between the observed geomagnetic field (1922) and the field of the eccentric dipole

Inspection of the three sets of charts, for the centred-dipole field (Figs. 2 and 3), the eccentric-dipole field (Figs. 5–11), and the observed field (illustrated by the isomagnetic charts, Figs. 3.2–3.4), leaves no doubt that the eccentric-dipole field gives a truer picture of the observed field than the centred-dipole (or uniform-magnetization) field, especially as regards two characteristic features: (1) the variation of the magnetic intensity with longitude near the equator (with high values between Australia and Asia, and low values over South America), and (2) the form of the zero-isogonics (along which the compass points true north). Figs. 14 and 15, however, show that the average values of the differences from the observed field are only slightly smaller for the eccentric than for the centred dipole field. Table 3 confirms this impression; in fact, in the northern zone, 65° to 25° N., the approximation furnished by the eccentric dipole is even a little inferior to that furnished by the centred dipole. But in the equatorial zone (for X) and in the southern zone, the eccentric dipole gives a decidedly superior approximation, as is already suggested by Figs. 14 and 15, so that for the whole area over which the observations are trustworthy the eccentric dipole gives the better representation. Considering the numerical data given in the table, we find that the differences in X are about one-tenth as large as the observed values; in Z they are higher, being more than one-sixth of the observed values. The differences in Y are, in absolute figures, of the same order as those in X.

The root-mean-square values of the terms in V/a of the first four orders, relative to the earth's centre O, for the epoch 1885, have been computed by Schmidt from (17.22); they are as follows, in the unit $10\,\gamma$:

$$|V_1|/a = 1{,}865, \quad |V_2|/a = 151, \quad |V_3|/a = 85, \quad |V_4|/a = 40. \quad (27)$$

The special importance of $|V_1|/a$ as compared with the subsequent terms is obvious. The main term in V_1 is the axial or P_1^0 component, with mean value 1,828, while the equatorial component, of magnitude 371, is not much greater than V_2. The reduction of $|V_2|$ by the change of origin to the magnetic centre is roughly one-half, and brings V_2 to the same order of magnitude as V_3.

Thus the various stages of approximation to the earth's field are as follows: We start with the field of the dipole alined along the earth's rotation axis; we then introduce the obliquity of the magnetic axis, giving the centred dipole; this is an important first improvement. The next stage, the removal of the dipole to an eccentric position, makes a smaller improvement in the approximation to the observed values.

18.8. The analysis by Dyson and Furner. Dyson and Furner [10] derived the coefficients g and h (forty-eight in all, up to the sixth degree) separately from X, Y, and Z, using values taken from the Admiralty 1922 charts at intervals of 10° in latitude and longitude. Some of the results are given in Table 4. In the case of Z they also made a separate determination—given in column Z (2)—omitting the data for the zones of latitude +70° and +80°. The first four coefficients thus derived were as follows (unit 10γ):

TABLE 4

	X	Y	Z (1)	Z (2)
g_1^0	3,095	..	3,000	3,046
g_1^1	196	232	222	232
h_1^1	−598	−592	−561	−566
g_2^0	133	..	91	45

The differences between the results in the last two columns are the more noteworthy since in the analysis which gave the results Z (1) the zones +70°, +80° had been assigned weights only one-fifth those of the other zones. Hence it was judged unsafe to infer the existence of externally produced parts of the field from the differences between the first two and last two columns.

Suitably weighted means of the coefficients obtained from X and Y were taken, and the calculated values of X, Y, Z derived from the resulting potential were compared with the observed values. Some large differences (rising to 1,500 γ) were found even in the case of X and Y, and Z showed considerable differences in many parts of the world, and very large ones in high latitudes. The latter was to be expected from the way in which Z is determined, by the measurement of H and I.

The ellipticity of the earth was not taken into account in deriving the above figures: if this had been done, g_1^0 as derived from X would be reduced from 3,095 to 3,074, and as derived from Z from 3,046 to 3,042; g_3^0, which is the coefficient most likely to be affected, would be altered from −255 (X) or −186 (Z) to −307 (X) or −206 (Z).

18.9 The external field, and the non-potential field. The most recent spherical harmonic analysis of the earth's field, by Bauer [14], while confirming that most of the field is of internal origin, ascribed about 3 per cent. of the total to external causes (V^e), and about as much to a non-potential part N (due to earth-air currents). The calculations by Dyson and Furner [10], described in § 8 and in 3.7, throw serious doubt on the reality of V^e and N, and it is hard to escape the conclusion that these two results of the spherical harmonic analysis may be mostly due to our incomplete observational knowledge of the field, especially near the poles. However, it seems worth while to consider in some detail the weight of the evidence for an external field.

According to § 3, the first-order harmonic of external origin corresponds to a homogeneous field of intensity H_0^e, in the direction of the line from the earth's centre O to a surface-point having the geographical coordinates: north polar distance $\theta_1^e = (180^\circ - \theta_0^e)$, longitude $\phi_1^e = (180^\circ + \phi_0^e)$. The two available determinations of this part of the external field are as follows:

TABLE 5

Author	*Epoch*	g_1^{0e}	g_1^{1e}	h_1^{1e}	H_0^e	θ_1^e	ϕ_1^e
Schmidt [9]	1885	$-186\,\gamma$	$+241\,\gamma$	$-4\,\gamma$	$304\,\gamma$	52°	171° E.
Bauer [14]	1922	$-523\,\gamma$	$+69\,\gamma$	$+113\,\gamma$	$539\,\gamma$	14°	239° E.

Bauer's calculation, based on far better material than Schmidt's, gave an even higher value for the external field, and the directions of the axis of the first harmonic, in the new determinations, are not very different. Disregarding the obliquity ($\theta_1^e = 14^\circ$) found by Bauer, one may represent the field by a constant vector which *increases* the horizontal field near the equator by about 500 γ, so that the total field near the equator is

$$-(g_1^{0i} + g_1^{0e}) = 30468 + 523 = 30{,}991\ \gamma.$$

The corresponding vertical component of the external field (taking $X^e = 500\ \gamma$ at the equator) is $Z^e = -500 \cos\theta$. Is it possible that the computed external field is merely a result of faulty observations of Z, or of the inclination? In latitude 60° ($\theta = 30^\circ$) this would mean an error of 250 γ in Z. The inclination for the main interior field, given by 3 (1), namely $\tan I = 2 \cot \theta$, is $I = 74^\circ$ at $\theta = 30^\circ$, and the corresponding value of H is 15,000 γ. By differentiating the equation $\tan I = Z/H$, and taking $dH = 0$, we get $dI/\cos^2 I = dZ/H$, and these values of I and H, with $dZ = 250\ \gamma$, give $dI = -4{\cdot}3'$ of arc.

Inclination measurements about 5′ too small in the northern hemisphere, and 5′ too high in the southern hemisphere, would therefore 'produce' the external field. Now the lines of equal dip on the Admiralty 1922 charts, for $I = 70^\circ$ and $I = 75^\circ$, are about 20 mm. apart; the error mentioned would therefore be equivalent to a shift of these isoclines by about 0·3 mm. poleward from their real position. With the necessary smoothing implied in the construction of the charts, the assumption of such an error does not seem at all unreasonable, especially since the cruises of the *Galilee* and *Carnegie* disclosed many regions on the oceans where the errors of the then existing charts were greater than 1°.

We have therefore to look elsewhere for evidence of an exterior field, and we find it in the post-perturbation (9.10) after magnetic storms (the ring-current field of 24.13, 25.9), discussed, in this respect, by Schmidt [21] (see 21.8).

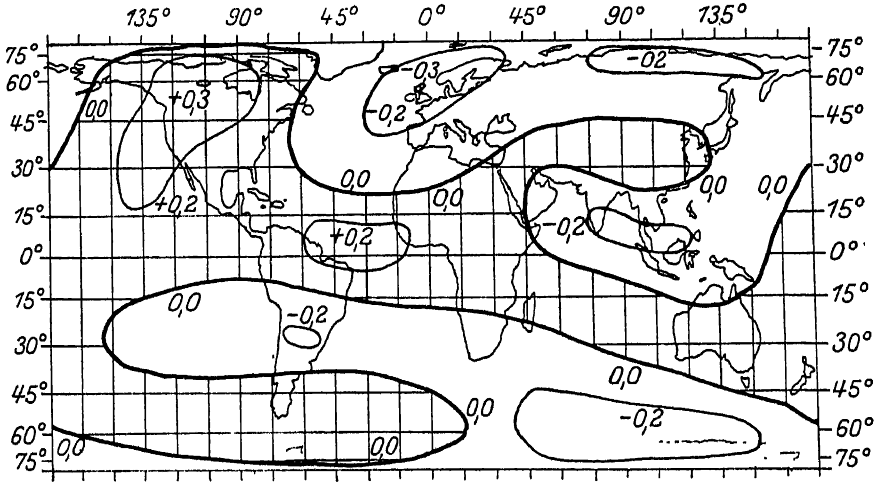


FIG. 16. Indicating the distribution of intensity (in amperes per square kilometre) of vertical electric currents crossing the earth's surface according to A. Schmidt's analysis of the magnetic world charts for 1885. The areas of downward current are those over which the network of meridians and parallels is shown

The discussion in 3.7 of the hypothetical earth-air currents, or the non-potential field, is supplemented here by a map constructed by Schmidt [29] from his analysis for 1885. Current entering the earth is reckoned positive. There is a broad zone of apparent downward current along the equator; at the poles the outward current has a density of about 0·08 ampere per square kilometre. Arguments such as those used with regard to the mean exterior field resulting from spherical harmonic analyses of data taken from world magnetic maps apply also to this non-potential field (see [3.10*b*]).

18.10. The decrease of the field of internal origin, above the earth. The potential of the internal field, expressed in the form (1)

$$V^i = (a^{n+2}/r^{n+1})T^i_n,$$

has coefficients g_n^m, h_n^m which give the field at the surface $r = a$. In order to compute the field at any higher level $a_1 = a+h$, V^i may be conveniently written in the form

$$V^i = (a_1^{n+2}/r^{n+1})T_n^i \, . \, (a/a_1)^{n+2} \tag{28}$$

The ratios $(a/a_1)^{n+2}$ in which the coefficients of order n are diminished at the height $h = a_1 - a$ are given below, for a = 6,370 km. and several values of h; the root-mean-square values of $(V_n)/a_1$ are also given, using the data of (27), unit 10 γ.

TABLE 6

$n =$	*1*	*2*	*3*	*4*	$n =$	*1*	*2*	*3*	*4*
Height h = 0 km.	1·000	1·000	1·000	1·000	$(V_n)/a_1 =$	1865	151	85	40
100 km.	0·954	0·940	0·925	0·911		1779	142	79	36
200 km.	0·911	0·884	0·857	0·831		1699	134	73	33
300 km.	0·871	0·832	0·794	0·759		1625	126	68	30
400 km.	0·833	0·784	0·738	0·694		1553	118	63	28
1,000 km.	0·646	0·558	0·482	0·417		1205	84	41	17
2,000 km.	0·441	0·336	0·255	0·194		823	51	22	8
6,370 km.	0·125	0·062	0·031	0·016		233	9	3	1

This table illustrates how, with increasing height, the higher terms decrease in importance relative to the first-order term (the dipole field). Local disturbances, which correspond to harmonics (or a combination of harmonics) of high degree, are reduced even more effectively. Consider a local anomaly which may be attributed to a dipole at 3 km. depth below the surface; such a dipole, in a spherical harmonic expansion, could only be described by terms of order n higher than 8,000. Let its maximum disturbing force, at the surface, be 2,000 γ. This would be reduced to 54 γ at h = 10 km., 7 γ at h = 20 km., and 2 γ at h = 30 km. This calculation has an obvious bearing on the field of local magnetic masses buried under non-magnetic strata, or at the bottom of the ocean.

For a discussion of the space gradients of the earth's magnetic field, see 2.39 and a paper by Chapman [30].

18.11. The secular variation. The apparent periodicity suggested by the secular variation at some stations with long series of observations (3.9) was studied by Carlheim-Gyllensköld [26 and 27]. He found expressions for the potential at the epochs 1787, 1700, and 1600, in the form

$$V/a = \sum_n \sum_m C_n^m \sin(m\phi + \epsilon_n^m) P_n^m(\theta); \tag{29}$$

his results were partly based on a backward extrapolation (in time) of the secular variations actually observed at a number of stations. He considered that the coefficients C_n^m were approximately constant, and that the ‘phases’ ϵ_n^m are linear functions of the time t (reckoned in years, taking $t = 0$ at the year 1800), so that

$$\epsilon_n^m = \gamma_n^m + p_n^m t. \tag{30}$$

This corresponds to a complete revolution of the field of the harmonic P_n^m, in the time $T_n^m = 2\pi/p_n^m$ years. His numerical values for these periods are

$$T_1^1 = 3{,}147,\ T_2^1 = 1{,}381,\ T_2^2 = 454 \text{ years.} \tag{31}$$

The value of T_2^2 is about equal to the period of the cycle in the secular variation suggested by the declination data for western Europe.

However, this simple conception of the secular variation is not by any means confirmed by Bartels's more recent analysis [28] of it (up to $n = 2$), which was based on the most accurate data available, namely the records of fourteen observatories, for the period 1902–20. We compare his results with those obtained by taking the differences between the results of Dyson and Furner (1922) and of Schmidt, and also with the ‘theoretical’ values computed from Carlheim-Gyllensköld's formula:

TABLE 7. *Spherical harmonic coefficients for the average annual secular variation. Unit* 1 γ.

	Epoch	g_1^0	g_1^1	h_1^1	g_2^0	g_2^1	h_2^1	g_2^2	h_2^2
Dyson-Schmidt	1922–1885	+20	−1	−1	−10	+6	−14	+21	−18
Bartels . .	1920–1902	+42	−9	+12	−7	+8	−25	+13	−8
Carlheim-Gyllensköld	1920–1902	0	+13	+4	0	−4	−12	+13	−17

The following conclusions are suggested:

(*a*) The spherical harmonic series for the secular variation converges much more slowly than that for the main field. The predominance of the first-order term, so conspicuous in the main field, does not appear in the secular variation.

This is well illustrated by the following comparison. The sum of the coefficients of the spherical harmonic terms of degrees 1 and 2 (g_1^0 to h_2^2) in the potential of the main field is eleven times the sum of the coefficients for the terms of degree 3; the corresponding ratio for the potential of the secular variation is only two (in this comparison the main-field potential was computed from the data for the fourteen observatories used in the secular-variation analysis).

(*b*) Gyllensköld's phase-formula (30) is not valid, nor the isomagnetic charts drawn by Fritsche [3.41], for epochs extending back as far as A.D. 1000, based on extrapolations from similar formulae.

(*c*) The apparent decrease in the earth's magnetic moment, by about $\frac{1}{1000}$ of its whole value, per year, which is indicated by a comparison between the results of analyses of the main field at different epochs, appears also in Bartels's secular-variation analysis; he found the value $+42\,\gamma$ for the annual change of g_1^0 (which in 1922 was about $-31{,}500\,\gamma$), or rather more than 0·1 per cent.; the percentage change in other harmonics was much greater (see also p. 131).

(*d*) While the main field may be regarded as a combination of a planetary field (the dipole field) and of weaker regional fields, the secular variation appears to have no outstanding part of planetary character; it consists largely of 6 regional changes which cannot easily be represented by a spherical harmonic series.

18.12. Multipoles. It was mentioned in 17.8 that it is possible, as Maxwell [24] showed, to generate a surface spherical harmonic function S_n of order n by successive differentiations of $(1/r)$ along various axes $h_1, h_2,..., h_n$, so that

$$\frac{d}{dh_1}\frac{d}{dh_2}\cdots\frac{d}{dh_n}\left(\frac{1}{r}\right) = \frac{S_n}{r^{n+1}}. \tag{32}$$

When $n = 1$ the function is the potential of the field of a dipole; when $n = 2$ it corresponds to a quadrupole (a closely adjacent combination of two dipoles with equal and opposite moments); and so on. Each term V_n in the earth's potential V (at $r = a$), involving $2n+1$ spherical harmonic coefficients, can be expressed in this way, by a numerical coefficient (representing a magnetic moment) and the directions (θ, ϕ) of n axes, $h_1, h_2,..., h_n$. Such an expression has been worked out by Umow [25], for the epoch 1885, and for $n = 1$, 2, and 3. Only one other such application of Maxwell's theory of poles (by Bruins [14.92]) has come to our knowledge.

XIX

THE VARIABILITY OF THE HARMONIC COEFFICIENTS FOR THE SOLAR AND LUNAR DAILY VARIATIONS

19.1. Methods for expressing the variability of daily variations. The usual expressions and modes of description for variability, derived in general statistics, are mostly applicable to cases in which each observation is expressed by one quantity (leading to the discussion of frequency-curves, standard deviation, etc.; for example, air-temperature at a station read daily at noon) or by two quantities (leading to the theory of correlation). These methods are not immediately applicable to research on periodic (for instance, daily) variations. The use of ranges (7.8) for this purpose can only be regarded as a simplifying measure adopted in order to obtain a first approximate characterization of the variability of the variation, for a large mass of data; more refined methods of description should afterwards be used.

Consider the solar daily variation S of (say) the declination D at Huancayo (Peru). The D variometer gives a record of S for each day, from midnight to midnight, as a continuous curve. The first simplifying step is to derive the hourly means, and to regard S as given, for any day, by the twenty-five hourly means beginning with the one for 0^h to 1^h of that day, and ending with the one for 0^h to 1^h of the next day; this allows for a possible non-cyclic change. Thus from N days we obtain a series of N *individual* sets of twenty-five values (N of them being repetitions, one per day).

The next step is to derive *average* sets of twenty-five values from these N individual sets, as in 16.29, for all the N days, and also for selected groups of days, chosen, for instance, according to season, sunspot number, or magnetic activity. These average sets represent the solar daily variation S of the element considered, and the differences between S as thus derived from different groups of days indicate how S varies with or is influenced by the season, the sunspottedness, and so on.

This procedure, however, does not disclose the full variability of S. In the case of the noon air-temperature, for example, it would correspond to computing averages for different seasons, for high or low pressure, etc.; but the temperature might also vary considerably from quite different causes, which had perhaps been overlooked. The full variability of the temperature is better represented by the frequency-distribution, which gives the relative number, or percentage, of days on which the noon temperature lies in the successive intervals of 1°. This frequency-distribution is easily determined from the original observations, without regard to any real or supposed cause. Our problem is to find a suitable analogy to it which can represent the full variability of S, as distinguished from the more or less regular changes of S with season, sunspots, or magnetic activity. That part of the variability of S, which can as yet not be accounted for by the influence of season, sunspots, or other well-known causes, may be called irregular or fortuitous. Since it is found to be of the same order of magnitude as the regular changes, it deserves to be studied in detail.

By harmonic analysis, after correcting for the non-cyclic variation, each day is represented by a set of harmonic coefficients a_1, b_1; a_2, b_2,...; a_n, b_n. For each frequency n a harmonic dial is plotted as described in 16.18. The analyses for the N separate days are represented on each dial as a cloud of N points, and the variability of S is now represented by the geometrical properties of these clouds; this can, in many cases, be expressed in the following way, which is more general than the case of circular symmetry considered in 16.24.

19.2. Ellipsoidal clouds of points. Consider a cloud of N points in a plane, with the rectangular coordinates X_ν, Y_ν $(\nu = 1, 2,..., N)$. The coordinates of the centre C are $X_0 = \sum X_\nu/N$, $Y_0 = \sum Y_\nu/N$. The coordinates of the deviations from the centre C may be called

$$x_\nu = X_\nu - X_0, \qquad y_\nu = Y_\nu - Y_0.$$

Let the standard deviations be σ_x, σ_y, and let r be the correlation-coefficient between X_ν and Y_ν (or, equally, between x_ν and y_ν), so that

$$\sigma_x^2 = \sum_\nu x_\nu^2/N, \qquad \sigma_y^2 = \sum_\nu y_\nu^2/N, \qquad r\sigma_x\sigma_y = \sum_\nu x_\nu y_\nu/N. \tag{1}$$

The general Gaussian frequency distribution which fits the cloud best, as judged by the method of least squares, is then expressed by the probability, $w(x,y)\,dxdy$, that a point is situated in the rectangle $(x, x+dx, y, y+dy)$, namely

$$w(x,y) = w_0\, e^{-Q(x,y)/2}, \tag{2}$$

where

$$w_0 = 1/2\pi\sigma_x\sigma_y\sqrt{(1-r^2)}, \tag{3}$$

$$Q(x,y) = \frac{1}{1-r^2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}-\frac{2rxy}{\sigma_x\sigma_y}\right). \tag{4}$$

The lines of equal frequency are ellipses $Q(x,y) =$ constant with the centre C; the major and minor axes all have the same ratio. The major axis makes an angle θ with the x-axis, where

$$\tan 2\theta = 2r\sigma_x\sigma_y/(\sigma_x^2-\sigma_y^2); \tag{5}$$

θ must be chosen between 0° and 90° for $r > 0$, and between 90° and 180° for $r < 0$. For $r = 0$, θ is 0° or 90°, according as σ_x or σ_y is the greater; for $r = 0$ and $\sigma_x = \sigma_y$, the ellipses degenerate into circles.

If new coordinate axes ξ, η are taken, along the major and minor axes of the ellipses (Fig. 1), Q is transformed into

$$Q(\xi, \eta) = \xi^2/\sigma_\xi^2 + \eta^2/\sigma_\eta^2, \tag{6}$$

where

$$\sigma_\xi^2 + \sigma_\eta^2 = \sigma_x^2 + \sigma_y^2 = M^2, \tag{7}$$

say, and

$$\sigma_\xi^2 \sigma_\eta^2 = \sigma_x^2 \sigma_y^2 (1 - r^2). \tag{8}$$

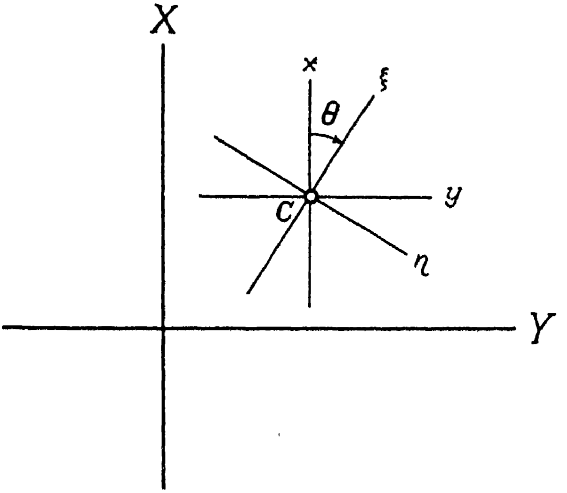


FIG. 1. Illustrating coordinate-systems used in the geometrical description of clouds of points

By means of (7) and (8), the standard deviations σ_ξ, σ_η along the new axes can be calculated from the standard deviations σ_x, σ_y along the original axes, and from the correlation-coefficient r. From (1) and (7) it follows that

$$M^2 = \sum (x_\nu^2 + y_\nu^2)/N, \tag{9}$$

so that M^2 is the average square distance of the points from the centre C; M is useful if it is desired to specify the dispersion of the cloud by a single parameter, disregarding the ellipticity; it may be called the two-dimensional standard deviation. Using the equation 16 (47 a) we find that the average square distance from the origin of the coordinate system is

$$M^2 + x_0^2 + y_0^2. \tag{10}$$

In the ideal case of a perfect Gaussian distribution, the total probability that a point falls outside the ellipse $Q =$ constant is simply

$\exp(-Q/2)$. The *probable-error ellipse* which contains $N/2$ points inside, while $N/2$ lie outside, is therefore given by $\exp(-Q/2) = \frac{1}{2}$, or $Q = \log_e 4$. The major and minor axes, M_1 and M_2, of the probable-error ellipse are therefore, by (6), σ_ξ and σ_η multiplied by $(\log_e 4)^{\frac{1}{2}}$ $(=1{\cdot}17741)$. These may be obtained by solving (7) and (8); both results may be expressed in the following formula (taking the + sign for the major axis M_1, and the − sign for the minor axis M_2):

$$M_1, M_2 = 0{\cdot}83256[(\sigma_x^2+\sigma_y^2)\pm\{(\sigma_x^2-\sigma_y^2)^2+4r^2\sigma_x^2\sigma_y^2\}^{\frac{1}{2}}]^{\frac{1}{2}}. \tag{11}$$

We note further that

$$(M_1^2+M_2^2)^{\frac{1}{2}} = 1{\cdot}1774M. \tag{12}$$

In the case of circular symmetry ($\sigma_x = \sigma_y$, $r = 0$), $M_1 = M_2 = 0{\cdot}833M$.

The actual calculation runs thus: σ_x^2, σ_y^2, and $r\sigma_x\sigma_y$ are computed from (1), M from (7), θ from (5), and M_1 and M_2 from (11). These values can *always* be calculated, because the formulae do not presuppose that the cloud is Gaussian, but give only the constants of a Gaussian distribution which fits the given cloud best. If a diagram should show that the cloud is obviously non-Gaussian—for instance with points clustered around two different centres—the constants M_1 and M_2, etc., will not be *wrong*, though they will have little meaning. In cases referring to the harmonic dials for S and L, however, the cloud is likely to be fairly Gaussian or 'normal' in appearance; an easy, though only partial, test is to draw the probable-error ellipse and count the number of points inside and outside, which should be nearly equal. More severe arithmetical tests of normality are in general not applicable, because they require too much material. If, therefore, we treat a distribution as normal, it is advisable not to press the conclusions too far, and to use the properties of the normal distribution with that 'grain of salt' which is necessary in all applications of the theory of probability.

From what has been said about the definition of the probable-error ellipse, it is easy to show that the probability that a point falls outside an ellipse with the axes κM_1, κM_2 is $(1/2)^{\kappa^2}$; for instance, it is 1/16 for $\kappa = 2$, 1/512 for $\kappa = 3$, 1/66000 for $\kappa = 4$, $1/(33\times10^6)$ for $\kappa = 5$, $1/(7\times10^{10})$ for $\kappa = 6$. For all practical purposes, no point is therefore to be expected outside the ellipse with axes $6M_1$, $6M_2$.

The ratio M_1/M_2 may be called the axis-ratio.

For 'normalized' variables ($\sigma_x = \sigma_y$), $M_1/M_2 = (1+r)^{\frac{1}{2}}:(1-r)^{\frac{1}{2}}$; this applies to cases in which the units of x and y are arbitrary, such as those discussed in 11.10–12, but in harmonic dials, σ_x and σ_y cannot be changed arbitrarily.

19.3. The daily variation of declination at Huancayo in summer. The discussion in § 2 will now be illustrated with reference to S for the magnetic declination at Huancayo on 103 quiet days with international magnetic character-figure 0·0 or 0·1, between the dates November 11 and February 29, from the years 1925 to 1930 (near sunspot-maximum, average sunspot number 71). The four harmonic dials for the 24-, 12-, 8-, and 6-hourly waves in the solar daily variation S are plotted in Fig. 2. The seasonal change of S, slight in that interval, has been eliminated; the amplitude of the lunar daily variation L is also small, only about 8 per cent. of S. The dials therefore exhibit only the 'fortuitous' variability of S. The curious shape of the dials is chosen for economy of space. The probable-error ellipses are drawn and the ends of their axes are indicated by short lines. Numerical data relating to these dials are given in the following table. A change of 1′ in D is equivalent to a magnetic force of 8·64γ at right angles to the direction of H. The angle θ, giving the direction of the major axis of the ellipse, is indicated in Table 1 by the hour which a line through the origin in the direction θ would mark on the dial.

TABLE 1. *Harmonic analysis of the solar daily variation* S *of easterly declination D at Huancayo, Peru: its variability derived from* 103 *quiet days, southern summer, near sunspot-maximum.*

Frequency n =	1	2	3	4
Period, hours	24	12	8	6
The average daily variation				
Amplitude c_n (γ)	11·1	10·0	4·7	1·5
Phase ϵ_n	260·3°	102·6°	283°	180°
Local time of first maximum	12·6^h	11·6^h	3·7^h	4·5^h
The variability				
Standard deviation M (γ)	6·4	5·6	3·5	2·3
Major axis M_1 (γ)	6·43	4·92	3·13	2·10
Minor axis M_2 (γ)	3·95	4·37	2·58	1·69
Direction of major axis	12·3^h	10·2^h	2·5^h	4·8^h
Axis-ratio M_1/M_2	1·63	1·13	1·21	1·24
Ratio c_n/M	1·7	1·8	1·3	0·7

Considering the nearness of Huancayo to the magnetic equator, far from polar disturbances, and, further, the extreme magnetic quietness for which these 103 days have been selected, it is surprising to note the great variability as exhibited in the scattering on Fig. 2. The system of formulae used seems to fit this scattering; the probable-error ellipses fit the distribution fairly well. On counting the number of dots inside

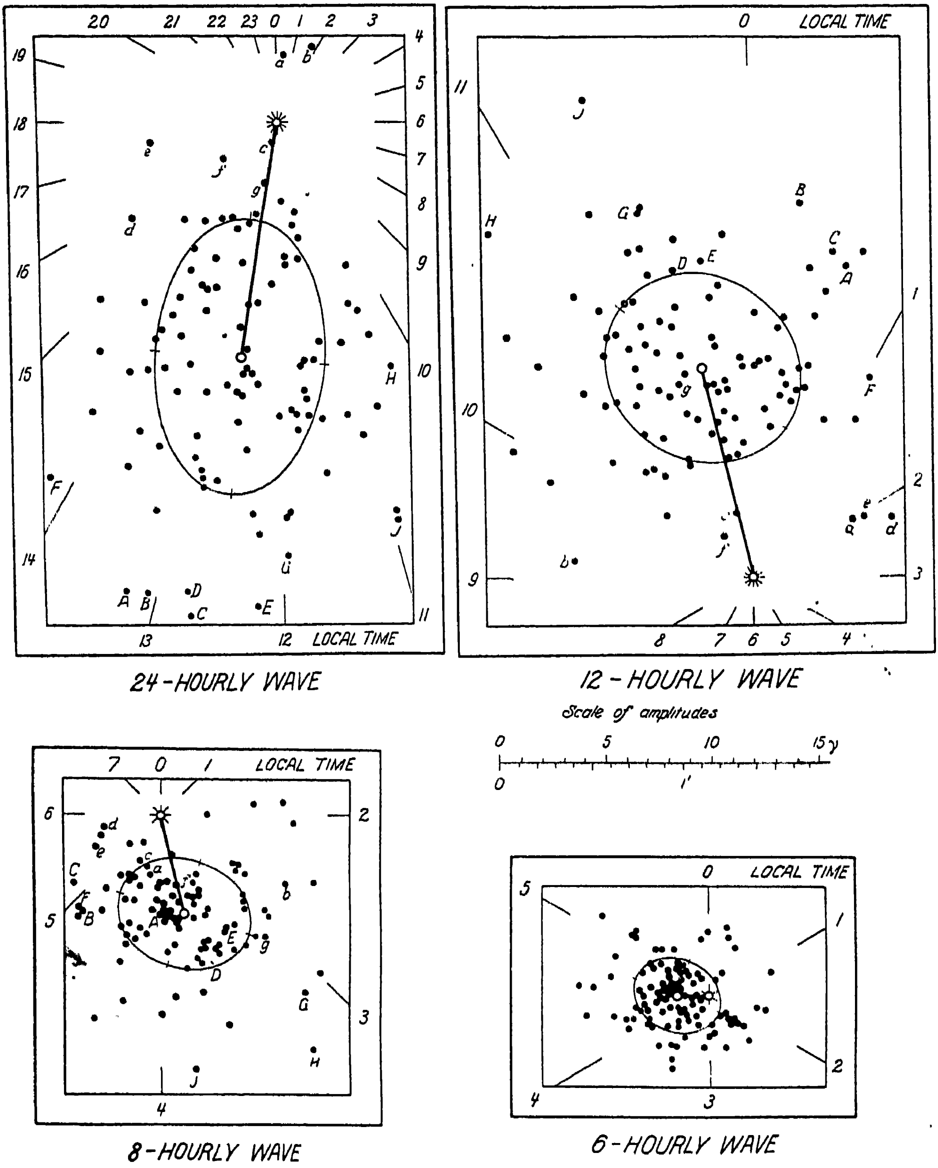


FIG. 2. Harmonic dials for the 24-hourly, 12-hourly, 8-hourly, and 6-hourly sine waves in the daily variation of easterly magnetic declination at Huancayo, Peru. The centre of each dial is 'starred', and the vector from the centre to any marked point on the dial indicates the amplitude, and the local time of the first maximum, of the sine wave. The 103 points on each dial refer to 103 individual very quiet days in southern summer. The letters mark individual days referred to on p. 675. The average vector is drawn, and the probable ellipse

and outside each ellipse, they are found to differ only slightly, so that the assumption of a Gaussian distribution seems to be adequate. Furthermore, we find agreement with the calculations at the end of § 2, according to which, in 103 cases, no dot should be expected outside the ellipse with axes $3M_1$, $3M_2$.

The 24-hourly cloud has the largest axis-ratio. The direction of the major axis indicates that changes in the 24-hourly wave affect the amplitude more than the phase. The range of the amplitudes is at least as great as the mean amplitude. The other clouds have negligible ellipticities. The absolute scatter M diminishes with increasing frequency n, but relative to the average amplitude c_n the fourth wave (6-hourly) has the greatest scatter. This is indicated by the ratios c_n/M in the last line of Table 1.

Each of the 103 days is represented by a dot on each of the four diagrams. The strong interrelations between the diagrams are indicated in the following way. In the 24-hourly dial, the following days were selected as having the smallest amplitudes: (*a*) December 21, 1925; (*b*) January 27, 1930; (*c*, *d*, *e*) February 21–3, 1927; (*f*) January 14, 1928; (*g*) November 21, 1928. The following days were selected as having the largest amplitudes: (*A*) January 27, 1929; (*B*) December 19, 1929; (*C*) February 3, 1929; (*D*) November 28, 1928; (*E*) February 11, 1928; (*F*) November 18, 1929; and (*G*) January 3, 1927. In addition, two days were selected from the 12-hourly dial, namely, (*H*) February 24, 1928, and (*J*) January 9, 1927. These days are marked by the same letters on all the first three dials. Inspection of these dials indicates that days with small or large amplitudes in one wave can be expected to have correspondingly small or large amplitudes in the other waves. A good example of the interdependence of the ranges on successive days, as found by Chapman and Stagg (see 7.8), is furnished by the three successive days (*c*), (*d*), and (*e*) with small amplitudes. That the interrelation between the dials is not restricted to amplitudes is shown by the day (*F*), on which the maxima of all three waves occur more than one hour later than normal, and by the day (*H*), on which they occur earlier than normal. Such outstanding anomalies of the daily variation can of course readily be recognized in the original records, or in the hourly means.

19.4 The daily variation of horizontal intensity at Watheroo. Fig. 3 shows the harmonic dial for the 24-hourly sine wave in H at Watheroo, on 171 very quiet days in southern summer near sunspot-maximum. The average wave has an amplitude of $4{\cdot}5\gamma$ only, with

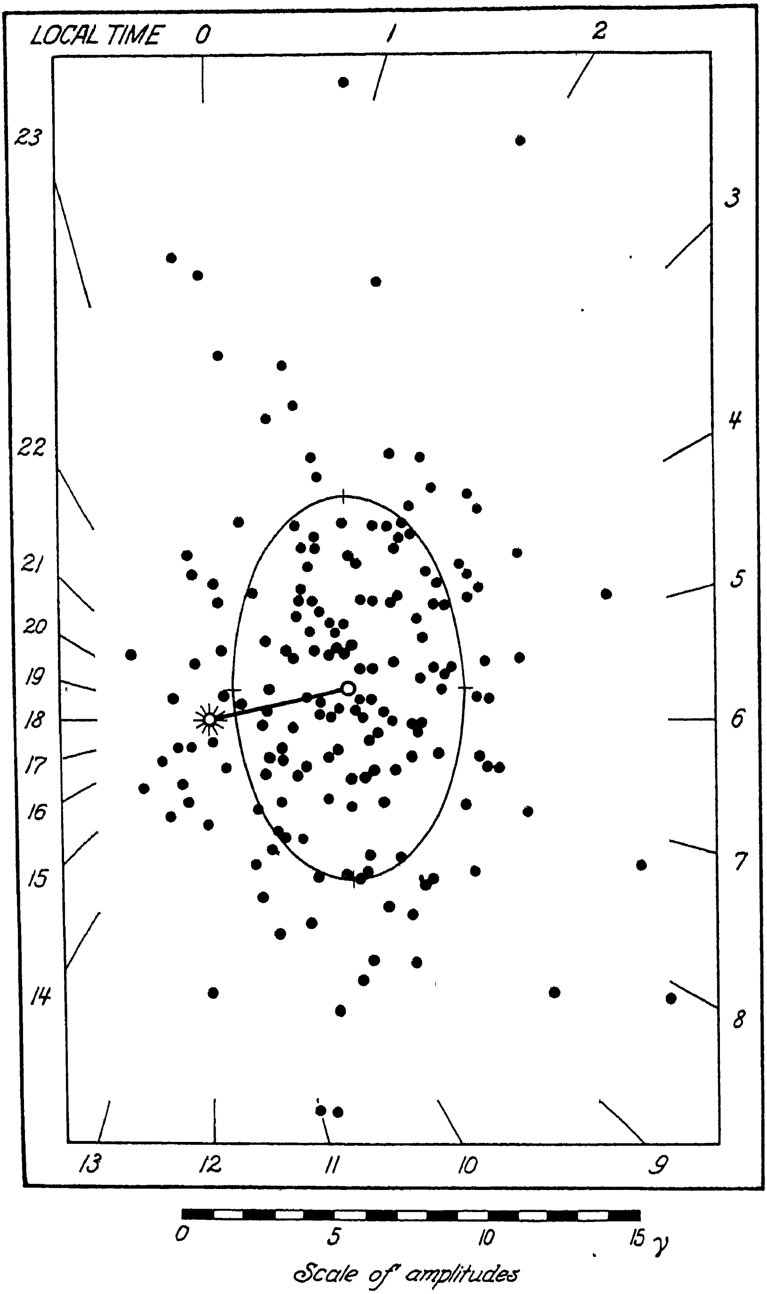


FIG. 3. Harmonic dial for the 24-hourly sine wave in the daily variation of horizontal magnetic intensity at Watheroo, West Australia. The points refer to 171 individual very quiet days in southern summer

the maximum at about 5^h. The axis-ratio M_1/M_2 is very high (1·70). The maximum scattering is along the direction at right angles to the mean direction (ϵ_1), so that the phase-changes are more important than the changes in amplitude.

At equatorial stations the maximum of the 24-hourly wave in H occurs at about noon, whereas at extra-tropical stations in latitudes higher than 45° it occurs at about midnight. Watheroo is near the average latitude at which the phase-shift occurs; that is, the focus of the main current-vortex in the overhead current-systems (Fig. 7.15) passes over Watheroo. On those days which are represented by dots in the upper half of the harmonic dial, Watheroo was an extra-tropical station, that is, the focus must have passed between Watheroo and the equator; whereas on the days whose dots are in the lower half, the focus passed between Watheroo and the south pole. This is the same phenomenon that was discussed later by Hasegawa (7.9) with respect to the northern circuit.

19.5. Simultaneous changes of S in different magnetic elements at the same station. In § 3 it was shown that in S for D at Huancayo the harmonic components of different frequency tend to show similar divergences from their respective mean values, so that if one is larger (or smaller) than the average, so also are the others. In the present section we consider an analogous question, in relation to one particular component of S at Watheroo, namely the 24-hourly component S_1, on 173 very quiet days in the southern summer near sunspot-maximum: namely, whether in different magnetic elements this same component tends to show similar divergences from its respective mean values.

For this purpose we consider the elements Y and Z. Fig. 4*a* shows the harmonic dial for the component S_1 in Z; a similar dial (not reproduced here) was made for Y. Fig. 5 shows the average daily variation, and its 24-hourly wave, for the days considered.

On the dial for Z four regions A, B, C, and D were selected; the regions A and B are indicated in Fig. 4*b* by shading, and likewise for the regions C and D, in Fig. 4*c*; the A region includes the points far from the origin (days of large amplitude) and the B region the points near the origin. The centre of the points in each region is shown by a cross. Similarly the C and D regions include the points (corresponding to not too small amplitudes) for days on which the maximum occurs more than half an hour later (C) or earlier (D) than the average.

The points from the Y dial, for the days thus selected, are plotted in Fig. 4*d* for the A and B days, and Fig. 4*e* for the C and D days; the

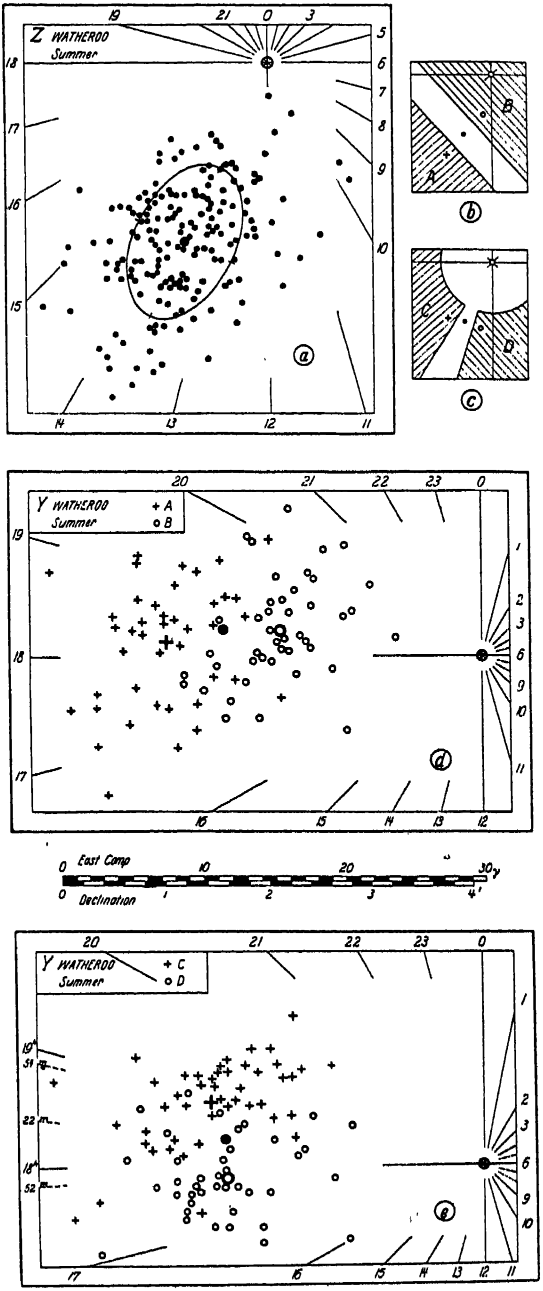


Fig. 4. Harmonic dials for the 24-hourly sine wave in the daily variation of the vertical (Z) and east (Y) components of magnetic force at Watheroo, West Australia, southern summer. These dials illustrate inter-relations of variability on quiet days

points for the A and C days are indicated by crosses, and those for the B and D days by circles. The mean positions are indicated, in like manner, by a heavier cross or circle. These diagrams show clearly that in the Y dial also, the points for A days are far from the origin, and those for B days are near to it. Similarly the mean time of maximum for the 24-hourly wave in Y is more than an hour later for the C than for the D days.

These results suggest considerable changes of intensity, as well as of phase (or position relative to the noon meridian), in the current-systems of Figs. 7.15 and 7.16.

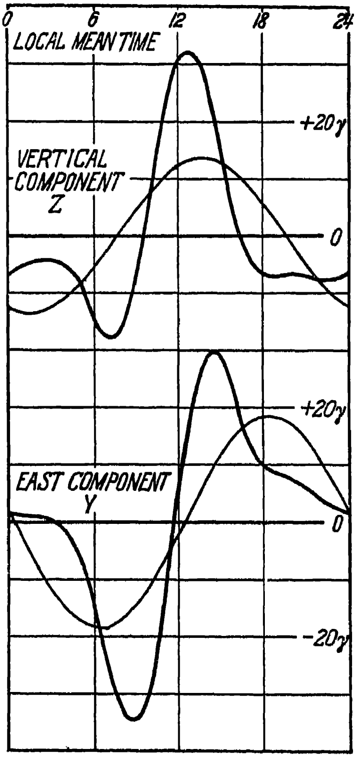


FIG. 5. The average daily variations, and the average 24-hourly waves, for the components and days concerned in Fig. 4

19.6. Quasi-persistence in S on successive days. If the solar daily variation S on an individual quiet day diverges from the average (e.g. an earlier maximum or a larger amplitude), this feature is likely to appear also in the quiet days shortly before and after that day. In other words, if (symbolically) ΔS is the difference between the individual S (corrected for L) and the average S for the particular season and sunspot number, ΔS exhibits quasi-persistence as defined in 16.30. This has been shown by Chapman and Stagg [7.20] as regards the ranges. For Batavia declination, southern summer, Schneider [4] found that the average ΔS, in the 24- and 12-hourly waves, was nearly half as big as the average S, and the parameter $\sigma(6)$, as defined by 16(96), is of the order 2 to 3. This means a serious reduction of the equivalent number of independent days available (16.36).

19.7. S and L in Huancayo H. The results of the analysis described in 8.16 are given in the harmonic dials of Fig. 6. They are dials in solar time for S+L. The six corners of each hexagon are the average dial points for six selected groups of days, namely, groups of days on which the moon culminates (within two hours) at the same solar time. No distinction is made between upper and lower transit of the moon, so that the days of new moon, when the upper transit occurs at (or near) 12 o'clock (solar time) are grouped with those of full moon, when the lower transit occurs at or near this time. The days are conveniently grouped according to the value of Schmidt's parameter μ, which (cf. 8.8 and 5.2) is $24-\nu$, where ν is the age of the moon in lunar hours. The six groups of days are those for which μ has the values (*a*) 0, 1, 12, or 13 (corresponding to lunar transit between 10·5 and 12·5 solar time—that is, days near new or full moon), (*b*) 2, 3, 14, or 15, and so on. New or full moon is marked by a short gap in the hexagon near the dial point for the group (*a*); going round the hexagon in the anti-clockwise direction, we come to the points for the days of groups (*a*), (*b*),....

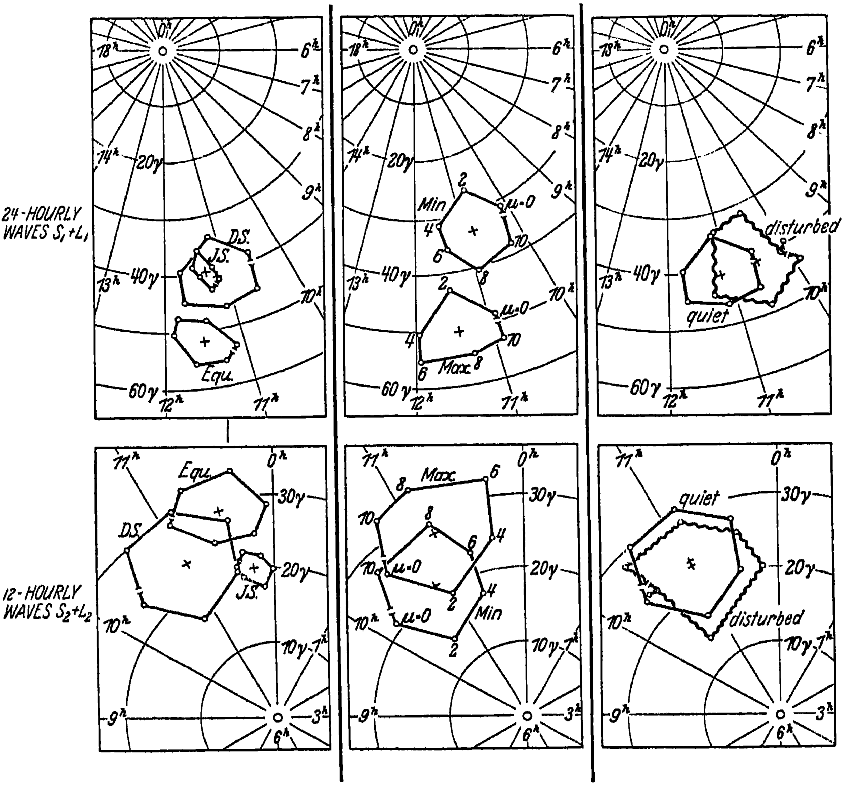
The phase law of change for each component of L, in the form 8 (8), asserts that its phase-angles *regress* by 4π in the course of a month. The centre of the six dial points of each hexagon is the dial point for the S component, and the vector from this centre to any point of the hexagon represents the L component at the corresponding age of the moon. This vector turns clockwise, through two right angles, as expected, in the course of the month. The vector for L at new or full moon is that drawn towards the gap in the hexagon.

The main conclusions to be drawn from Tables 8.8 (p. 264) and 8.9 (p. 266) and Figs. 6–8 are (with reference to H at Huancayo):

Seasons. The phases of both S and L do not change much with season; for S_1 they lie between 11·1 and 11·5^h, and for S_2 between 11·0 and 11·7^h; for L_1 they lie between 5 and 9^h, and for L_2 between 8 and 10^h. But there is a marked difference between S and L with respect to the change with season: whereas the amplitudes s_1 and s_2 are about equal at the two solstices, l_1 and l_2 decrease in the ratio 3:1 from the December to the June solstice.

Sunspots. S is more affected by the increase in sunspot number than L; the ratio of the two values of s_1 found for the groups of days with respectively few and many sunspots (designated min. and max. in Fig. 7) is 1:1·50; for s_2 it is 1:1·29. (Furthermore, the maxima of the 24- and 12-hourly S waves occur definitely later in high sunspot years.) The corresponding ratios for l_1 and l_2 are only 1:1·09 and 1:1·12.

Disturbance. It was to be expected, from former studies, that the influence of magnetic activity on S and L at Huancayo would not be great. Nevertheless it is astonishing how small it is found to be for the 12-hourly wave S_2. In the 24-hourly wave S_1 the influence of disturbance is greater; it adds a solar 24-hourly wave of 7·1 γ amplitude, with maximum about 6^h, as might be expected (9.3).



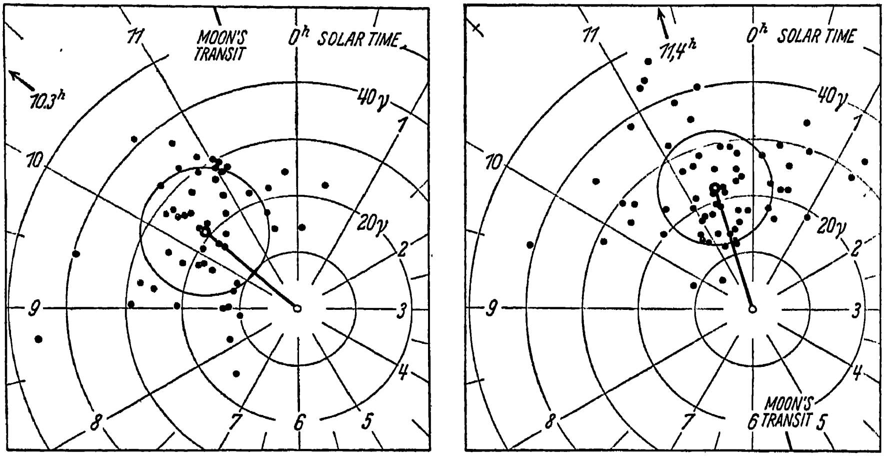
SEASONS
FIG. 6

SUNSPOTS
FIG. 7

ACTIVITY
FIG. 8

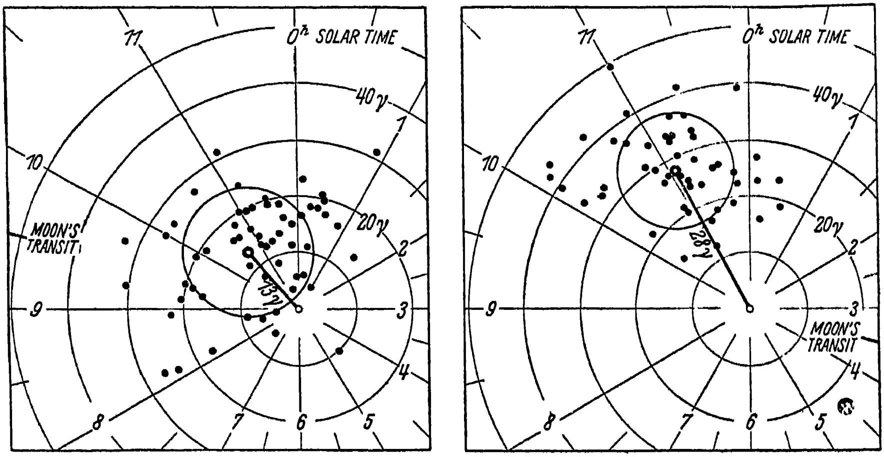
FIGS. 6–8. Harmonic dials, relative to solar time, for the combined solar and lunar waves in the daily variation of the horizontal magnetic intensity at Huancayo; 24-hourly sine waves above, 12-hourly sine waves below. They show, in each case, the dial-points for the two main sine waves, for 6 phases of the moon, with the end-points of the 6 vectors connected to form hexagons. There is a gap in one of the sides corresponding to new or full moon ($\mu = 0$ or 12). From day to day the vector varies so that, within half a month, the hexagon is described, clockwise. FIG. 6. Mean results for the three seasons DS = December solstice, Equinoxes and JS = June solstice. FIG. 7. Mean results (for the December solstice only) for sunspot-minimum and sunspot-maximum (the mean relative sunspot-numbers in the two groups were 8 and 72). FIG. 8. The influence in the December solstice, of the magnetic activity, contrasting the results for quiet and for disturbed days, selected as having international character-figures between 0·0 and 1·1, and between 1·2 and 2·0

19.8. S and L on individual days (for *H* at Huancayo). Figs. 8.8 and 8.9 in 8.16 had already indicated that the large L in the December solstice interferes with S so as to be distinguishable by mere inspection of the records for a few days selected at opposite phases of the moon. The same fact is brought out in Fig. 9 for the 12-hourly harmonic dials,



FULL AND NEW MOON, 51 DAYS

QUARTERS, 62 DAYS



μ = 2 or 14, 58 DAYS

μ = 8 or 20, 48 DAYS

FIG. 9. Harmonic dials for the solar 12-hourly sine waves in the daily variation of the horizontal magnetic intensity at Huancayo, for individual days with spotless sun, at various phases of the moon, showing the influence of L in S+L. The upper dials show the results for days at the two lunar phases $\mu = 0$ or $\mu = 12$, i.e. at full or new moon, and at $\mu = 6$ or $\mu = 18$, i.e. at the moon's quarters; these dials illustrate the greatest difference in the *time of occurrence* of the maximum of the combined solar and lunar wave. The lower dials compare the days at the two phases $\mu = 2$ or $\mu = 14$, and $\mu = 8$ or $\mu = 20$, which show the greatest difference between the *amplitudes* of the combined solar and lunar wave. December solstice

for the December solstice. The result here is perhaps even more striking, because it is based immediately on the observed data. Fig. 9 was constructed as follows.

From Fig. 7 the following pairs of corners of the hexagon were selected as showing the biggest changes: (i) as regards the *occurrence* of the maximum of S_2+L_2, the corners marked 0 and 6, which correspond (roughly) to new and full moon, and to the moon's quarters; (ii) as regards the *amplitude* of S_2+L_2, the corners marked 2 and 8.

For the four sets of days which contributed to these four dial points, four new harmonic dials were constructed. In each case the dial contained the points representing the 12-hourly wave on the individual days, but not for all the days in each set—only for the days on which the daily sunspot number s (p. 176) was zero. The centre of the cloud of points on each dial corresponds (approximately) with the position of the corresponding average dial point in Fig. 7. The probable-error circle for each cloud is indicated. The influence of L is strikingly shown in the phase-shift (by $1{\cdot}1^h$) between the two upper dials, and the increase in amplitude (from 13 to 28γ) between the two lower dials.

It may be mentioned that the influence of the sunspot numbers s on the solar daily variation S has always hitherto been studied by taking, at least, monthly means. Here daily figures of s have been used for the first time, and it remains to be seen whether actually the daily values of s affect S, or whether only the general state of solar activity (expressed, perhaps, by monthly or annual means of s) is significant, as in the case of the relations between s and geomagnetic activity. Good material for such a study is available in the days of the June solstice at Huancayo, because L is then so small that any change in S+L may be ascribed to S alone.

Finally, the radii of the probable-error circles in Fig. 9 for single days may be used to estimate the uncertainty in the positions of the *averaged* dial-points at the corners of the hexagons in the S+L dials. The probable-error radius in Fig. 9 is roughly 10γ. Allowing for quasi-persistence as high as $\sigma = 3$, the probable-error radius for the averaged dial-point from N days, by 16.30, would be $10/\sqrt{(N/3)}\ \gamma$. For each corner in the hexagons in the lower left dial of Fig. 7 N is about 210, so that the radius will be $1{\cdot}2\gamma$. This is enough to explain the slight deformations of the hexagons, as due to the limited number of days, especially in the case of the dials for the 'disturbed days', where the probable-error radius for each corner is estimated as nearly 4γ. For the average S and L, from ($N =$) 1,260 days, the radius of the probable-error circle will be less than $0{\cdot}5\gamma$.

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THE SPHERICAL HARMONIC ANALYSIS OF THE MAGNETIC VARIATIONS

20.1. Introduction. In Chapter XVII a description has been given of the Gaussian method of representing the earth's magnetic field by means of a potential expressed as a series of spherical harmonic functions. The method indicates that nearly the whole field is represented by a single harmonic term, and that this part of the field has its source below the earth's surface. On the other hand Bartels has shown that the spherical harmonic representation is not well adapted for the secular variation field (1902–20), since the series of terms required does not converge at all rapidly.

After the analysis of the main field by Gauss the first important new application of his method to terrestrial magnetic research was made by Schuster [1] (1887), who analysed the field S of the solar daily variation. He was able to represent it approximately by a few principal harmonic terms. The expression indicated that the field is mainly of external origin, though there is also an internal part; the ratio of the corresponding parts of the potential at the earth's surface was estimated by Schuster to be about 4.

Somewhat similar analyses of S have since been made by Fritsche [2], Walker [3], van Bemmelen [4], van Vleuten [5] (criticized by Schmidt [6]), and Chapman [7]. van Bemmelen and Chapman also analysed the field L of the lunar daily magnetic variation. The work of his predecessors was summarized and critically reviewed by Chapman, whose analysis, which will be described here in detail, was more detailed than the earlier analyses, and was also based on a larger amount of modern data, all for the same periods of time. The results will be compared with those of the other investigations.

20.2. General expression for the potential of a daily variation. The general expression for a potential function expanded in a series of spherical harmonic terms has already been given in 18.1 (1) and (2), viz.

$$V = C + \sum_{n=1}^{\infty} \left\{ \frac{r^n}{R^{n-1}} T_n^{(e)} + \frac{R^{n+2}}{r^{n+1}} T_n^{(i)} \right\}, \tag{1}$$

where R denotes the earth's radius, and

$$T_n \equiv \sum_{m=0}^{n} \{g_n^m \cos m\lambda + h_n^m \sin m\lambda\} P_n^m(\cos\theta), \tag{2}$$

with affix e or i added to T, g, and h; λ denotes the longitude measured eastwards from Greenwich.

The coefficients g and h discussed in Chapter XVIII were constant, apart from the slow secular variation; in the case of S and L they are periodic functions of the time; they must therefore be expressible in the form of Fourier series, consisting of terms which are constant multiples of $\cos st'$ or $\sin st'$, where s is any positive integer (the terms $s = 0$ vanish because S and L are expressed as deviations from the daily mean); t' denotes Greenwich time reckoned in angular measure at the rate of 360° per solar day (or per lunar day in the case of L), from the epoch of lower transit at Greenwich. This implies that T_n must be expressible in the form

$$T_n = \sum_{m=0}^{n} \sum_{s=1}^{\infty} \{(g_{n,a}^{m,s} \cos st' + g_{n,b}^{m,s} \sin st')\cos m\lambda + \\ + (h_{n,a}^{m,s} \cos st' + h_{n,b}^{m,s} \sin st')\sin m\lambda\} P_n^m(\cos\theta). \qquad (3)$$

It is convenient to transform this in terms of local (solar or lunar) time by writing

$$t = \lambda + t', \quad \text{or} \quad t' = t - \lambda, \qquad (4)$$

and expressing $\cos st'$ and $\sin st'$ in the forms

$$\cos st \cos s\lambda + \sin st \sin s\lambda$$

and

$$\sin st \cos s\lambda - \cos st \sin s\lambda.$$

Collecting the terms in $\cos st$ and $\sin st$ we have

$$\left.\begin{aligned} T_n = \sum_{s=1}^{\infty} \sum_{m=0}^{n} [\{&p_{n,a}^{m,s} \cos(m+s)\lambda + q_{n,a}^{m,s} \sin(m+s)\lambda + \\ &+ r_{n,a}^{m,s} \cos(m-s)\lambda + s_{n,a}^{m,s} \sin(m-s)\lambda\}\cos st + \\ &+ \{p_{n,b}^{m,s} \cos(m+s)\lambda + q_{n,b}^{m,s} \sin(m+s)\lambda + \\ &+ r_{n,b}^{m,s} \cos(m-s)\lambda + s_{n,b}^{m,s} \sin(m-s)\lambda\}\sin st] P_n^m(\cos\theta). \end{aligned}\right\} \qquad (5)$$

20.3. The assumption of no dependence on longitude. It has been seen in Chapter VII that S varies with the local time ($t = \lambda + t'$), but to a first approximation is not otherwise dependent on λ; there is little positive evidence for the same assumption regarding L, but the only analyses of L yet made have been based on the hypothesis that for L as well as for S, stations at equal latitude have equal daily variations; this is only approximately true. This hypothesis, which we adopt here, implies that λ vanishes from the last expression (except so far as it is involved in t); hence (unless the whole term T_n is zero), all the coefficients p, q, r, and s are zero, except $r_{n,a}^{m,m}$ and $r_{n,b}^{m,m}$, in which

$m = s$; these symbols will be simplified to the form $r^m_{n,a}$, $r^m_{n,b}$. Consequently (slightly changing the notation) we may write

$$T_n = \sum_{m=0}^{n} \{r^m_{n,a}\cos mt + r^m_{n,b}\sin mt\}P_n^m(\cos\theta), \tag{6}$$

or (with a slight further change)

$$\begin{aligned} V &= C + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left\{\left(e^m_{n,a}\frac{r^n}{R^{n-1}} + i^m_{n,a}\frac{R^{n+2}}{r^{n+1}}\right)\cos mt + \right. \\ &\qquad\qquad \left. + \left(e^m_{n,b}\frac{r^n}{R^{n-1}} + i^m_{n,b}\frac{R^{n+2}}{r^{n+1}}\right)\sin mt\right\}P_n^m(\cos\theta) \\ &\equiv C + \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left\{e_n^m\frac{r^n}{R^{n-1}}\cos(mt+\epsilon_n^m) + i_n^m\frac{R^{n+2}}{r^{n+1}}\cos(mt+\iota_n^m)\right\}P_n^m \\ &\equiv C + \sum_{n=1}^{\infty}\sum_{m=0}^{n} V_n^m. \end{aligned} \tag{7}$$

The part depending on r^n is continuous, and satisfies Laplace's equation within the sphere $r = R$; its source is therefore outside the earth. The part depending on $1/r^{n+1}$ similarly arises from within the earth. The letters e and i are chosen to indicate the respectively external and internal origins of the corresponding parts of the potential, as in 18.1.

A term V_n^m in the potential leads to the following terms in X, Y, Z at the surface of the earth ($r = R$):

in X (north):

$$\frac{\partial V_n^m}{R\,\partial\theta} = \{(e^m_{n,a}+i^m_{n,a})\cos mt + (e^m_{n,b}+i^m_{n,b})\sin mt\}\frac{\partial P_n^m}{\partial\theta}; \tag{8}$$

in Y (east):

$$-\frac{1}{R\sin\theta}\frac{\partial V_n^m}{\partial\lambda} = \{(e^m_{n,a}+i^m_{n,a})\sin mt - (e^m_{n,b}+i^m_{n,b})\cos mt\}\frac{m}{\sin\theta}P_n^m \tag{9}$$

(because, at a given instant of Greenwich time t', $\lambda = t - t'$, so that $\partial/\partial\lambda = \partial/\partial t$);

in Z (downwards):

$$\frac{\partial V_n^m}{\partial r} = \{(ne^m_{n,a} - \overline{n+1}\,i^m_{n,a})\cos mt + (ne^m_{n,b} - \overline{n+1}\,i^m_{n,b})\sin mt\}P_n^m. \tag{10}$$

These may be written in the form (using Schmidt's notation 17 (68))

$$(a_n^m\cos mt + b_n^m\sin mt)X_n^m(\theta) \equiv C_n^m X_n^m\cos(mt+\alpha_n^m), \tag{11}$$

$$(-b_n^m\cos mt + a_n^m\sin mt)Y_n^m(\theta) \equiv C_n^m Y_n^m\sin(mt+\alpha_n^m), \tag{12}$$

$$(\mathrm{a}_n^m\cos mt + \mathrm{b}_n^m\sin mt)P_n^m(\theta), \tag{13}$$

where
$$a_n^m = n(e^m_{n,a}+i^m_{n,a}), \qquad b_n^m = n(e^m_{n,b}+i^m_{n,b}), \tag{14}$$
$$\mathrm{a}_n^m = ne^m_{n,a}-(n+1)i^m_{n,a}, \qquad \mathrm{b}_n^m = ne^m_{n,b}-(n+1)i^m_{n,b}, \tag{15}$$
and
$$X_n^m(\theta) = \frac{1}{n}\frac{\partial P_n^m}{\partial\theta}, \qquad Y_n^m(\theta) = \frac{m}{n\sin\theta}P_n^m. \tag{16}$$

The Fourier series for the solar daily variation in X, at a station at north polar distance θ, is consequently

$$\sum_{m=1}^{\infty}\left\{\left(\sum_{n=m}^{\infty} a_n^m X_n^m\right)\cos mt+\left(\sum_{n=m}^{\infty} b_n^m X_n^m\right)\sin mt\right\}$$
$$= \sum_{m=1}^{\infty}(x_{ma}\cos mt+x_{mb}\sin mt), \tag{17}$$

and similarly for Y and Z. The functions P_n^m, X_n^m, Y_n^m are known functions of θ. The process of harmonic analysis simply consists of finding coefficients a_n^m, b_n^m, a_n^m, b_n^m to fit the Fourier coefficients x_{ma}, x_{mb}, y_{ma}, y_{mb}, z_{ma}, z_{mb} determined from observations of the solar (or lunar) daily variation of X, Y, Z at a sufficient number of stations in different latitudes θ. It is clear that a_n^m, b_n^m depend on Z only, and a_n^m, b_n^m depend on X and Y only; a_n^m, b_n^m can be found from the coefficients x_{ma}, x_{mb} alone, or from y_{ma}, y_{mb} alone, and if determined from either set they should lead to the correct values of the other set; if they do not, it is an indication that the field S (or L) is not wholly derived from a potential of the given form. Either the field does not depend only on θ and t, in which case further terms depending on λ must be added, as in (5), or part of the field does not possess a potential, and must therefore be due to electric currents passing from the earth to the air or vice versa.

In the various analyses which have been made, where only a few harmonic terms have been considered, it has been found that the *observed* values of x_{ma}, x_{mb} for X do not agree very closely with the values *calculated* from (11), using the coefficients a_n^m, b_n^m derived from the observed values of y_{ma}, y_{mb}. Different explanations of this have been given: Walker supposed it to be due to the omission of terms in V depending explicitly on λ, i.e. not only on the local time; but Chapman found that this explanation would not account for the greater part of the differences. van Vleuten concluded that the differences indicated the existence of a part of the S field not derived from a potential; Schmidt [6], however, showed that this conclusion was not warranted by the evidence. An alternative and more likely reason for the discrepancies is to be found in the limitation of the harmonic formula to a few terms; even if the field is completely derivable from a quite simple 'smooth' potential function V, this may require a large number of slowly diminishing harmonic terms (perhaps of alternating signs) for its expression, and if only a few terms are taken, these being chosen to correspond best with Y or $(\partial V/\partial\lambda)/R\sin\theta$, the expression may show systematic residuals when $\partial V/R\,\partial\theta$ is compared with X. In addition, the field certainly depends to some extent on λ; and although the corresponding terms in V seemed at first likely to be of relatively small magnitude, compared with the terms in the main part of V which depends only on the local time t, the observations at Huancayo (7.10) have shown that S and L depend on λ to a rather high degree, near the equator.

When a_n^m, b_n^m, a_n^m, b_n^m have been found for Y, or for X and Y, and for Z, the equations (14) and (15) lead to the values of e_{na}^m, e_{nb}^m and i_{na}^m, i_{nb}^m, which indicate the external and internal parts of the field.

20.4. Chapman's analysis of S. In Chapman's analysis of S the Fourier coefficients of the all-day solar daily variations of X, Y, and Z were used as far as the fourth harmonic components ($m = 4$); use was made of data from 21 observatories, 15 northern and 6 southern, lying in widely different longitudes, between latitudes $+60°$ and $-60°$; the observatory at Huancayo, however, had not yet been started then. The data were derived from four quarters of each of two years, 1902 (sunspot-minimum) and 1905 (sunspot-maximum); the four quarters were February–April (March equinox), May–July (N. solstice), August–October (September equinox), and November–January (S. solstice). The data showed a considerable degree of regularity and constancy of type; for example, the 1902 and 1905 values were closely similar, except for the increased amplitude in the later year; they depended mainly on local time, and relatively little on the longitude; they were nearly symmetrical (or in certain cases anti-symmetrical) with respect to the equatorial plane. These features appeared almost as much in the terms of higher frequency ($m = 3, 4$) as of lower ($m = 1, 2$). There were, however, some irregularities, persisting at each station from year to year; on this account the data from the 21 stations were combined into 9 group means, 6 northern and 3 southern.

The data for the two equinoctial quarters differed from one another much less than those for the two solstitial quarters, and the mean of the two former was taken as representing the main part of the phenomenon at the equinoxes. The solstitial data were combined into solstitial mean values and solstitial semi-differences, i.e. the set of means of the summer and winter values of each Fourier coefficient at each station was separately analysed, and likewise the set of semi-differences, for (N solstice$-S$ solstice). The complete analysis of the field at either solstice is, of course, obtainable by adding or subtracting the two sets of harmonic terms thus derived. The mean solstitial data closely resembled the mean equinoctial data, their potential being approximately symmetrical relative to the equatorial plane. The solstitial semi-differences, representing the seasonal variation, were approximately antisymmetrical relative to this plane.

It was found that each of the periodic components in the mean equinoctial and mean solstitial sets of data could be approximately represented by a single harmonic term, of type P^m_{m+1}, while in the solstitial semi-difference set of data each periodic component required two harmonic functions, P^m_m and P^m_{m+2}, for its expression. In some of the other analyses which have been made the higher periodic components ($m = 3, 4$) were not considered, while sometimes more terms were included for the components $m = 1$, 2; but so far as the investigations are comparable, the principal harmonic terms were in all cases the same.

Table 1, in which c and s denote $\cos\theta$ and $\sin\theta$, gives the formulae for P^m_n, X^m_n, and Y^m_n, or rather (cf. 17 (20)) for P_{nm}, X_{nm}, and Y_{nm}; the ratio $P^m_n : P_{nm} = X^m_n : X_{nm} = Y^m_n : Y_{nm}$ is $\{2(n-m)!/(n+m)!\}^{\frac{1}{2}}$; the upper part of the Table gives the numerical value of this ratio for the various values of n and m here considered:

TABLE 1. *Ratios* $P^m_n/P_{n,m}$.

	$n = m$	$n = m+1$	$n = m+2$
$m = 1$	1	$1/\sqrt{3}$	$1/\sqrt{6}$
2	$1/2\sqrt{3}$	$1/2\sqrt{15}$	$1/6\sqrt{5}$
3	$1/6\sqrt{10}$	$1/6\sqrt{70}$	$1/12\sqrt{70}$
4	$1/24\sqrt{35}$	$1/72\sqrt{35}$	$1/360\sqrt{7}$

Formulae for $P_{n,m}$, $X_{n,m}$, $Y_{n,m}$.

$P_{11} = s$ $X_{11} = c$ $Y_{11} = 1$
$P_{21} = 3sc$ $X_{21} = \frac{3}{2}(2c^2-1)$ $Y_{21} = \frac{3}{2}c$
$P_{31} = \frac{3}{2}s(5c^2-1)$ $X_{31} = \frac{1}{2}c(15c^2-11)$ $Y_{31} = \frac{1}{2}(5c^2-1)$
$P_{22} = 3s^2$ $X_{22} = 3sc$ $Y_{22} = 3s$
$P_{32} = 15s^2c$ $X_{32} = 5s(3c^2-1)$ $Y_{32} = 10sc$
$P_{42} = \frac{15}{2}s^2(7c^2-1)$ $X_{42} = \frac{15}{2}sc(7c^2-4)$ $Y_{42} = \frac{15}{4}s(7c^2-1)$
$P_{33} = 15s^3$ $X_{33} = 15s^2c$ $Y_{33} = 15s^2$
$P_{43} = 105s^3c$ $X_{43} = \frac{105}{4}s^2(4c^2-1)$ $Y_{43} = \frac{315}{4}s^2c$
$P_{53} = \frac{105}{2}s^3(9c^2-1)$ $X_{53} = \frac{63}{2}s^2c(15c^2-7)$ $Y_{53} = \frac{63}{2}s^2(9c^2-1)$
$P_{44} = 105s^4$ $X_{44} = 105s^3c$ $Y_{44} = 105s^3$
$P_{54} = 945s^4c$ $X_{54} = 189s^3(5c^2-1)$ $Y_{54} = 756s^3c$
$P_{64} = \frac{945}{2}s^4(11c^2-1)$ $X_{64} = \frac{315}{2}s^3c(33c^2-13)$ $Y_{64} = 315s^3(11c^2-1)$

The values of a_n^m, b_n^m, a_n^m, b_n^m determined by Chapman from Y and Z respectively are as follows:

TABLE 2. *Coefficients of spherical harmonic terms for the* S *field.*
(*Unit of force* 1γ.)

		1905 (sunspot-maximum)				*1902 (sunspot-minimum)*			
		From Y		*From Z*		*From Y*		*From Z*	
n	m	a_n^m	b_n^m	a_n^m	b_n^m	a_n^m	b_n^m	a_n^m	b_n^m
				Mean equinox					
2	1	+18·7	−9·0	+7·1	−2·4	+11·8	−8·7	+5·9	−0·7
3	2	−16·7	+8·6	−6·6	+0·1	−11·8	+8·3	−4·2	+0·2
4	3	+9·0	−8·0	+4·4	−1·5	+6·2	−7·4	+3·6	−1·5
5	4	−2·6	+4·3	−1·7	+0·9	−1·7	+3·3	−1·6	+1·1
				Mean solstice = $\frac{1}{2}(N+S)$ *solstice*					
2	1	+18·0	−7·6	+6·4	−2·1	+10·7	−7·6	+5·4	−1·2
3	2	−14·8	+7·2	−5·6	−0·4	−10·7	+6·7	−4·5	+0·5
4	3	+6·8	−5·8	+3·6	−1·5	+4·8	−4·8	+2·3	−1·2
5	4	−1·8	+1·9	−0·8	+0·4	−0·8	+1·5	−0·6	+0·6
				$\frac{1}{2}(N-S)$ *solstice*					
1	1	+4·2	−2·1	+0·4	+0·1	+4·0	−1·7	+1·0	0·0
3	1	+7·8	+3·3	+3·4	+0·6	+5·9	+1·7	+1·9	+0·9
2	2	−2·7	+5·4	−0·5	+0·9	−1·9	+3·7	−1·2	+0·9
4	2	−3·2	+1·9	−1·9	−0·5	−4·2	+2·0	−1·1	−0·5
3	3	−1·0	−4·0	+0·1	−0·1	−0·1	−3·6	+0·1	−0·1
5	3	−1·8	+0·6	+0·3	+0·7	−0·8	+0·3	+0·2	+0·5
4	4	+2·7	−0·4	+0·7	+0·5	+1·6	−0·6	+0·4	+0·6
6	4	+2·6	−5·7	−1·2	−3·8	+1·1	−5·8	−1·2	−5·0

These coefficients are not the same as those given in the original paper because in this book Schmidt's spherical harmonic functions are used, namely P_n^m instead of P_{nm} (17.4), and also the time-origin used here is midnight instead of noon.

20.5. Comparison with other analyses. These results for a_n^m and b_n^m, derived from horizontal-force data alone, can most conveniently be compared with the results obtained by previous writers by transforming them into the corresponding values of C_n^m, α_n^m (11, 12). In the following table the 'annual terms' correspond to the mean of the solstitial and equinoctial mean values; the seasonal terms in Schuster's and Fritsche's analyses are derived by considering half-years centred at the solstices, and not quarter-years, as here; on this account, other things being equal, Chapman's seasonal values should be the greater. It is to be borne in mind that the year 1870, to which Schuster's data refer, was one of exceptionally high sunspottedness. (The annual mean relative sunspot number was 139 for 1870, and only 64 for 1905.)

TABLE 3. *Comparison of different spherical harmonic analyses of the* S *field.*

Annual terms					*Seasonal terms*				
	Chapman		*Schuster*			*Chapman*		*Schuster*	
	1902	*1905*	*1870*	*Fritsche*		*1902*	*1905*	*1870*	*Fritsche*
C_2^1	13·9	20·1	30·9	20·5	C_1^1	4·3	4·7	5·4	5·9
α_2^1	35°	24°	24°	30°	α_1^1	23°	27°	27°	28°
C_3^2	13·4	17·6	21·3	14·4	C_3^1	6·2	8·4	7·4	4·5
α_3^2	215°	207°	211°	205°	α_3^1	344°	337°	311°	347°
C_4^3	8·2	10·6	12·6	8·4	C_2^2	4·2	6·0	7·8	5·8
α_4^3	47°	40°	67°	35°	α_2^2	243°	243°	255°	242°
C_5^4	2·7	3·8	4·3	..	C_4^2	4·6	3·7	3·5	3·9
α_5^4	242°	235°	258°	..	α_4^2	205°	210°	162°	186°
					C_3^3	3·6	4·1	..	4·3
					α_3^3	91°	104°	..	88°
					C_4^4	1·7	2·8	..	1·9
					α_4^4	20°	8°	..	70°

The analyses by Walker and van Bemmelen contain the following results which may be compared with the above:

Walker: $C_2^1 = 19{\cdot}1 \qquad \alpha_2^1 = 25°$

$C_3^2 = 10{\cdot}0 \qquad \alpha_3^2 = 203°$

van Bemmelen: $C_3^2 = 10{\cdot}0 \qquad \alpha_3^2 = 206°$

Considering the differences between the material, period, and methods of analysis, it appears that there is very fair agreement as to the potential coefficients derived from the horizontal force data. The results obtained from the vertical force agree together less well, and herein lies the main reason for the different conclusions drawn by various writers as to the relation between the inner and outer parts of the field.

20.6. Separation of internal and external terms. The formulae (14) and (15) indicate that

$$e_{na}^m = \frac{(n+1)a_n^m + n\mathrm{a}_n^m}{n(2n+1)}, \qquad i_{na}^m = \frac{a_n^m - \mathrm{a}_n^m}{2n+1}, \tag{18}$$

and similarly for e_{nb}^m, i_{nb}^m. Using the results in Table 2, the values of e_n^m and ϵ_n^m given in Table 4 were obtained, and also (Table 5) the ratios e_n^m/i_n^m of the surface amplitudes of the external and internal terms in the potential, and the corresponding phase differences $\epsilon_n^m - \iota_n^m$.

20.7. Analyses of the L field. The L field has twice been analysed, first by van Bemmelen [4], and secondly by Chapman [7]. The former considered the second (12-hourly) harmonic component alone, as his paper was written prior to Chapman's investigation of the other components (8.5).

TABLE 4. *Spherical harmonic amplitudes and phases for the external part of the S field.* (*Unit* 1 γ *for* e^m_n.)

	Mean equinox				*Mean solstices*				*Solstitial inequality*			
	1905		*1902*		*1905*		*1902*		*Mean of 1902 and 1905*			
m	e^m_{m+1}	ϵ^m_{m+1}	e^m_{m+1}	ϵ^m_{m+1}	e^m_{m+1}	ϵ^m_{m+1}	e^m_{m+1}	ϵ^m_{m+1}	e^m_m	ϵ^m_m	e^m_{m+2}	ϵ^m_{m+2}
1	7·7	24°	5·4	30°	7·1	22°	5·0	30°	3·2	23°	1·8	341°
2	4·4	202°	3·3	210°	3·9	200°	2·9	207°	1·8	241°	0·7	197°
3	2·2	35°	1·8	44°	1·7	35°	1·2	42°	0·9	87°	0·2	220°
4	0·7	232°	0·6	235°	0·4	222°	0·3	234°	0·4	2°	0·9	84°

TABLE 5. *Amplitude ratios* e^m_n/i^m_n *and phase differences* $\epsilon^m_n - \iota^m_n$ *for the external and internal parts of the* S *field.*

	Amplitude ratios					*Phase differences*				
	Mean equinox		*Mean solstice*		*Solstitial inequality. Mean of 1902 and 1905.*	*Mean equinox*		*Mean solstice*		*Solstitial inequality. Mean of 1902 and 1905.*
	1905	*1902*	*1905*	*1902*		*1905*	*1902*	*1905*	*1902*	
m	$n = m+1$		$n = m+1$		$n = m$	$n = m+1$		$n = m+1$		$n = m$
1	2·9	2·7	2·8	3·0	2·5	−5°	−23°	−3°	−20°	−7°
2	2·4	2·0	2·3	2·2	2·3	−18°	−17°	−19°	−18°	−8°
3	2·4	2·5	2·7	2·4	2·0	−21°	−21°	−20°	−21°	−32°
4	2·2	2·9	2·3	3·2	1·7	−23°	−30°	−15°	−24°	−30°
Mean	2·5	2·5	2·5	2·7	2·1	−17°	−23°	−14°	−21°	−19°

van Bemmelen used values of a_2 and b_2 for X, Y, Z from fifteen stations, ranging in latitude from 60° N. to 45° S. The material was somewhat heterogeneous; it related to different epochs, but since L does not vary much in the course of the sunspot cycle this is less important than in the case of S; the data were also derived from periods of very unequal length and in different ways, so that they were not of uniform value. A careful attempt was made, however, to reduce them to a common standard, particularly as regards the inclusion of disturbed days, which, as has been seen, is of importance. New reductions were made for Pavlovsk (1890–1900), Pola (1899–1906), Zikawei (1890–1900), and Manila (1895–1903). The potential coefficients a^2_n, b^2_n, a^2_n, b^2_n were derived for the annual mean variation (alone); in his original paper van Bemmelen concluded from these that the main part of the L field was of internal origin. In a correcting paper (1913) this statement was revised, on account partly of certain numerical errors in the first paper, and also because too much weight had been assigned to early data from Capetown, Melbourne, and Hobarton in the S. hemisphere, extending over a relatively short period of years.

Chapman's analysis was based on data from five stations only, Pavlovsk, Pola, Zikawei, Manila, and Batavia; new reductions were made for all these stations, as previous investigators had not determined the non-semi-diurnal components, which it was desired to include in the work. It was of value, however, to be able to compare the data for the second periodic component with those obtained by van Bemmelen, so far as the different subdivisions of the material rendered this possible; the comparison was on the whole satisfactory. Chapman's reductions for the five stations were made for all three elements D, H, and Z, afterwards transformed to X, Y, and Z; in each case observations for seven quiet years were used, viz. 1897–1903 (except for Batavia, for which the corresponding years 1888–90 of the previous sunspot cycle were taken in place of the years 1899–1901, during which the observatory was being reorganized). All days were used except the most disturbed, numbering about 2 per cent. The results were combined into three seasonal groups, as in 5.1. The analysis was made in the same way as for S, on the assumption that the equinoctial and mean solstitial fields are symmetrical, and that the solstitial-difference field is antisymmetrical with respect to the equatorial plane. Only the principal harmonic terms were considered, viz. those for which $n = m+1$ (in the case of the equinoxes and solstitial means) and $n = m$ (for the solstitial differences). The accidental error of the results is naturally greater than in the case of S, because fewer stations were considered, and because of the smaller intensity of L. The potential expression obtained from Y agreed surprisingly well with the Y data, and fairly well with the X data; the agreement between the Z data and the potential expression derived from them was less satisfactory. On the whole, the equinoctial and mean solstitial results resemble those for S, except as regards absolute magnitude, and a phase difference (L—S) of about 70°; thus they indicate a general similarity of distribution for the L and S fields in the middle belt of the earth. The results are given in the following tables.

TABLE 6. *Spherical harmonic coefficients for the* L *field.*
(*Unit* 1 γ.)

	Mean solstice				*Equinox*				$\frac{1}{2}(N-S)$ *solstice*			
	From Y		*From Z*		*From Y*		*From Z*		*From Y*		*From Z*	
m	a^m_{m+1}	b^m_{m+1}	a^m_{m+1}	b^m_{m+1}	a^m_{m+1}	b^m_{m+1}	a^m_{m+1}	b^m_{m+1}	a^m_m	b^m_m	a^m_m	b^m_m
1	−0·30	−0·99	+0·09	−0·40	−0·01	−1·06	+0·30	−0·16	+0·09	+0·57	+0·21	−0·06
2	+0·09	+1·71	−0·46	+0·36	−0·16	+2·09	−0·63	+0·12	−0·23	+1·54	−0·21	+0·19
3	−0·32	−0·84	0·00	−0·40	+0·06	−1·18	+0·16	−0·45	−0·13	−0·96	+0·08	−0·19
4	+0·23	+0·23	+0·02	+0·20	+0·19	+0·68	−0·04	+0·22	+0·05	+0·13	+0·01	+0·06

TABLE 7. *Spherical harmonic amplitudes and phases for the external part of the* L *field.* (*Unit* 1 γ.)

m	Mean solstice		Equinox		$\frac{1}{2}(N-S)$ solstice	
	e^m_{m+1}	ϵ^m_{m+1}	e^m_{m+1}	ϵ^m_{m+1}	e^m_m	ϵ^m_m
1	0·39	101°	0·36	81°	0·42	72°
2	0·38	263°	0·42	253°	0·51	257°
3	0·17	106°	0·22	83°	0·21	94°
4	0·05	301°	0·09	280°	0·02	290°

It may be noted that van Bemmelen's (corrected) results for e^2_3 and ϵ^2_3 were (in the present notation) 0·34 and 254° for the mean of the year; these are to be compared with 0·38 and 263° for the mean solstice, or 0·42 and 253° for the equinox, in the above table.

TABLE 8. *Amplitude ratios and phase differences for the external and internal parts of the* L *field.*

m	Amplitude ratios, e^m_n/i^m_n			Phase differences, $\epsilon^m_n-\iota^m_n$		
	Mean solstice	Equinox	$\frac{1}{2}(N-S)$ solstice	Mean solstice	Equinox	$\frac{1}{2}(N-S)$ solstice
1	2·8	1·9	2·4	−23°	−29°	−31°
2	1·8	1·5	1·9	−29°	−29°	−6°
3	2·7	2·7	1·9	−19°	−14°	−12°
4	2·7	2·0	2·8	−52°	−17°	+17°

van Bemmelen's results for e^2_3/i^2_3 and $\epsilon^2_3-\iota^2_3$ are 1·25 and −15°; these are to be compared with 1·8 or 1·5, and −29°, in the above table.

20.8. The similarity of S and L. The 'external: internal' amplitude ratios and phase differences obtained from the various harmonic terms in S (Table 5) are independent of one another, and likewise for L (Table 8). Though some of the differences between them may be real, they do not exceed the probable limits of error; in considering the weight to be attached to the various determinations, regard should be had to the amplitudes of the corresponding terms in Tables 2 and 6, or to the values of e^m_n in Tables 4 and 7. It would appear that, for all the harmonics considered, the value of e^m_n/i^m_n is about 2·4 in the case of S, and 2·3 in the case of L, and that the corresponding values of $\epsilon^m_n-\iota^m_n$ are −19° and −21°. There seems to be no reason for doubting the substantial similarity of the S and L fields in respect of the relation between their external and internal parts.

The analysis also shows that the intensity of the S field is about ten times that of the average L field, but that in the former the first periodic component has a larger coefficient than the second, whereas in L the second harmonic has the greater coefficient. The seasonal inequality, relative to the equinoctial average, is much larger in L than in S.

From the data given in Tables 4 and 7 Bartels constructed the diagrams, Figs. 7.15–16, for the equivalent external current-systems for S and L, using the formulae (7) and 17 (83), and assuming a height of 100 km. above the earth's surface for the current layer; the diagrams would be very little altered if the height were assumed to be 200 or 300 instead of 100 km. The equivalent internal current-systems have not been computed, but from 17 (84) and Tables 5 and 8 they may be pictured as carrying about half the current intensities of the external current-systems, with the circulation in each vortex reversed; the centres of the internal current-circuits lie about 15° to the east of those of the external currents.

The S current-system may also be illustrated by Figs. 1, 2, which show the currents of Figs. 15 and 16 of Chapter VII on another projection, namely on an equal-area projection; Figs. 1 and 2 refer to the currents over the day and night hemispheres respectively.

No attempt has yet been made to apply the unabridged formula (5) to the observations of S, allowing for the changes of S with longitude of which Huancayo provides so striking an example. Only approximate determinations have been made (§§ 9 and 10). For an extensive spherical harmonic analysis, Schmidt's method [17.11] seems suitable.

20.9. McNish's analysis. The large solar daily variation of H at Huancayo (7.10) led McNish [8] to make a new spherical harmonic analysis of the S field, in which the data for Huancayo were combined with data from four other American stations (Agincourt, Cheltenham, Vieques, and Pilar, between 64° and 80° W. longitude). Data for the twenty international quiet days of the equinox March, April, September, October 1923 were used. These data, on spherical harmonic analysis, give an asymmetric daily current-distribution, with total current-flows, for the day-circuits, of about 72,000 amperes north, and 165,000 amperes south, of the equator; whereas Chapman's analysis for sunspot-minimum gave two symmetrical circuits of 62,000 amperes each.

The daily variation of the vertical component Z at the equinoxes, for stations lying along any meridian, changes sign at or near the equator. Huancayo was situated slightly south of the magnetic equator (where $Z = 0$) in 1922, but owing to the secular variation the magnetic equator has moved southward, so that in 1935 the magnetic inclination at Huancayo was 2° 11′, corresponding to a station in the northern hemisphere. McNish has pointed out that with this change of situation relative to the magnetic equator, the characteristics of the daily variations of Z in the months of the June solstice have changed slowly but perceptibly from those for a southern station to those for a northern station.

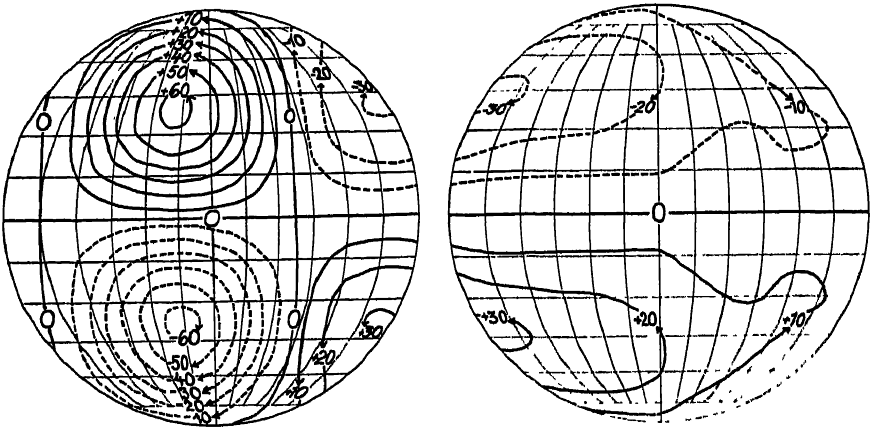


FIG. 1. The overhead electric current-systems corresponding to S_q (the solar daily magnetic variation on quiet days), over the sunlit hemisphere (left) and the night hemisphere (right). These diagrams give an alternative representation of the current-systems already illustrated in Fig. 7.15, referring to sunspot-minimum, at the equinox

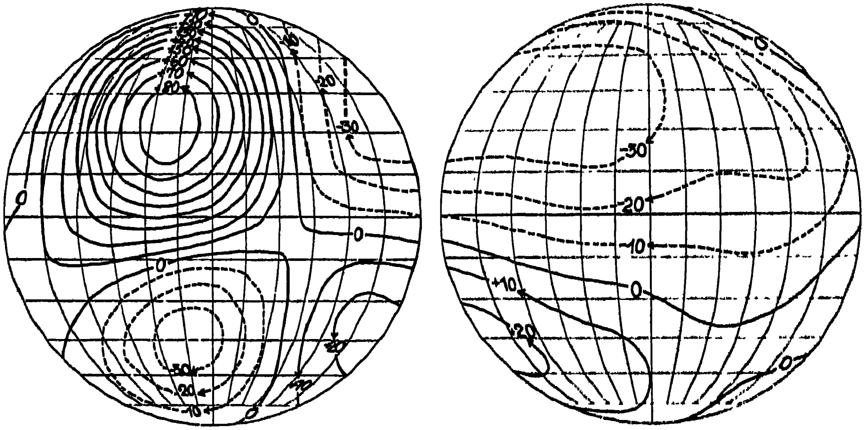


FIG. 2. The current-system of Fig. 7.16, redrawn as in Fig. 1, but referring to sunspot-minimum, in the northern summer

20.10. Hasegawa's analyses. Spherical harmonic analyses for S on a few individual selected quiet days have been made by Hasegawa and Ota, which show striking differences between adjacent days [9, 10]. For equatorial stations the daily variation in X has a noon maximum, which changes to a noon minimum for stations north of the circuit-centre of the current-system. Hasegawa found that the latitude of this centre sometimes altered by as much as 15° between neighbouring days. A few examples are shown in Figs. 7.19–23.

20.11. Analysis of the D_{st} field. As indicated in Chapter IX, the additional magnetic variations present during magnetic storms may be analysed into the storm-time variation, the disturbance daily variations, and an irregular part. The two last have not been subjected to spherical harmonic analysis, but Chapman and Whitehead and Price [22.8, 10] have analysed the storm-time variations. These, being the part of the variation that is symmetrical about the earth's axis, and therefore independent of the longitude, will have a potential involving only zonal harmonics. The observations clearly indicate that the most important harmonic term is the first, $P_1(\cos\theta)$, and that the higher harmonics are relatively small. On the basis of Chapman's analysis of storms (Ch. IX), Chapman and Price deduced from the data for H and Z, at several successive epochs of storm-time, the coefficients e_n and i_n in the expression

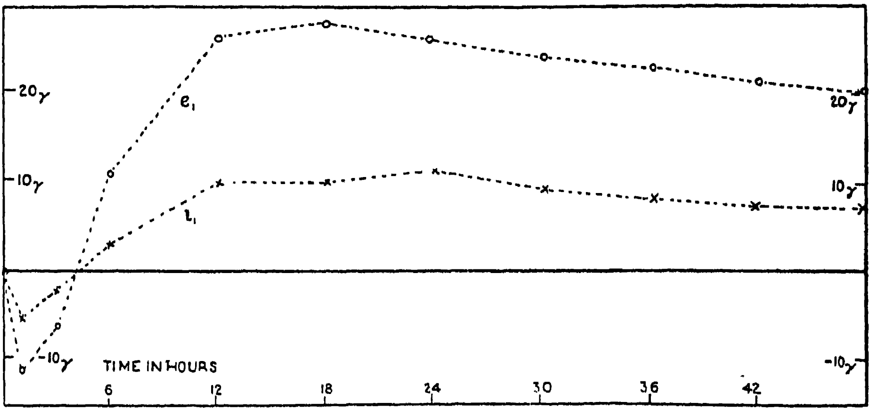


FIG. 3. Illustrating the time-variation, during the first two days of a magnetic storm, of the coefficients e_1 and i_1 of the main spherical harmonic components (P_1) of the disturbance field; e_1 refers to the part whose origin is external, and i_1 to the part whose origin is internal to the earth

$$V = \sum_{n=1,3,5}\left(e_n\frac{r^n}{R^{n-1}}+i_n\frac{R^{n+2}}{r^{n+1}}\right)P_n(\cos\theta) \tag{19}$$

for the potential of the storm-time changes; the coefficients are functions of the time. Their values for $n = 3$ and $n = 5$ are small, and except during the first ten hours of the storm the harmonic P_1 adequately represents the field (in middle and low latitudes). Fig. 3 shows the magnitude of e_1 and i_1 during the first two days of the storm. For the first half hour, when there is a rapid change in the field, the internal part of P_1 is almost exactly half the external part; in the subsequent slower change (from 1^h to 12^h) the ratio is rather less, that is, about 0·42.

The data are based on graphs of the storm-time changes in X and Z referring to the (average) north magnetic latitudes 22°, 40°, 51° (9.2), and the storms are assumed symmetrical with respect to the equator, so that there are no terms P_2, P_4,... .

PART 3

PHYSICAL THEORIES OF GEOMAGNETIC PHENOMENA

In the third part of this book we discuss the physical theories which attempt to explain the various geomagnetic phenomena described in Part 1 and analysed in various ways in Part 2.

It has been shown in Chapters XVIII and XX that by means of spherical harmonic analysis a separation can be made between the internal and external parts of the geomagnetic field (that is, the parts that originate within and above the earth).

In this way it is found that the main field, and the secularly varying part of it, are almost entirely of internal origin. Their causes must therefore be sought within the earth. The very inadequate theories dealing with this part of geomagnetism are described in Chapter XXI.

The transiently varying fields, S, L, and D, are found, on the other hand, to be mainly of external origin, but they have also a part of internal origin, which is a substantial though minor fraction of the whole. The two parts, external (S_e, L_e, and D_e) and internal (S_i, L_i, and D_i), preserve a fairly constant ratio to one another, throughout considerable variations of intensity and form. This suggests that though their immediate causes lie in different regions, the main cause being above the earth and the minor cause within it, the two parts are not really independent. It is therefore natural to inquire whether the inner part can itself be due ultimately to the main, outer cause.

It is certain that since the external fields S_e, L_e, and D_e are variable, they must electromagnetically induce electric currents within the earth, which is to some degree electrically conducting. The induced electric currents will be accompanied by magnetic fields, which, of course, will be of internal origin. Further, if the magnetic permeability of the earth differs appreciably from unity, the external fields will induce *magnetization* within the earth, which also will be accompanied by magnetic fields of internal origin. Let S_e', L_e', and D_e' denote the internal magnetic fields produced by S_e, L_e, and D_e in these two ways. Such induced fields must at least *contribute* to the known internal fields S_i, L_i, D_i, and may be identical with them. If they are identical, we are relieved of the necessity to seek for any separate cause of the internal part of the S, L, and D fields, and have only to consider the cause of the external part.

Chapter XXII examines in detail the possibility that the internal fields S_i, L_i, and D_i are induced, electromagnetically and magnetically, by the external fields S_e, L_e, and D_e. No certain conclusion can be reached, because we do not know the distribution of κ and μ, the electric conductivity and magnetic permeability, throughout the earth. For this reason S'_e, L'_e, and D'_e cannot be definitely calculated from S_e, L_e, and D_e. The most that can be done (and so much *is* done) is to show that no contradiction with existing knowledge is involved in the hypothesis of an inductive origin for the inner transient fields, and that a not unreasonable distribution of κ and μ can be found, which is consistent at the same time with each of the three conditions $\mathsf{S}'_e = \mathsf{S}_i$, $\mathsf{L}'_e = \mathsf{L}_i$, and $\mathsf{D}'_e = \mathsf{D}_i$. This renders it probable that the internal fields are merely induced by the outer ones, and the calculations may also be regarded as giving probable information as to the distribution of κ (and μ) within the earth.

Consequently, physical theories of the S, L, and D fields must be concerned with causes located above the earth, in the earth's atmosphere or beyond it. Such theories are examined in Chapters XXIII (for S and L) and XXIV, XXV (for D). As regards S and L, the conclusion reached is that they must both be due mainly to dynamo action, according to the theory first proposed by Balfour Stewart. Many difficulties remain to be overcome, however, before a clear and comprehensive dynamo theory of S and L can be constructed; these difficulties suggest that the phenomena of motion and ionization in the upper atmosphere are even more complex than has generally been realized.

The account of theories of the D field in Chapter XXV is preceded by a discussion of corpuscular emissions from the sun, since it is considered most likely that such emissions are the cause of aurorae and magnetic storms. The theories of both these phenomena are as yet unfortunately very rudimentary and speculative, and much work in this difficult field remains to be done.

The book concludes with a brief historical account of the development of the subject of geomagnetism.

XXI

THEORIES OF THE MAIN FIELD AND ITS SECULAR VARIATION

21.1. The problem of the field of uniform magnetization. The outstanding feature of the geomagnetic field is its general similarity to the field of a uniformly magnetized sphere or centred dipole—a fact which analytically is represented by the predominance of the first degree terms in the spherical harmonic expression of the potential (cf. 18 (27)). This is usually assumed to imply that the cause of the field must be planetary, affecting the whole earth, and leaving only the superposed irregularities, corresponding to the spherical harmonic components of higher degree, to be explained as regional effects.

Less unanimity exists as to whether or not the obliquity of 11·5° between the earth's axes of rotation and of magnetization is of fundamental significance. Many authors consider that the angle is sufficiently small to warrant the assumption of a physical connexion between the rotation and the magnetization of the earth. Certain geological evidence that suggests a possible complete reversal of the earth's field during past ages is usually disregarded as unreliable—a view which is perhaps too dogmatic.

Unfortunately we are as yet unable to judge whether any other celestial body has a magnetic field or not, with the one exception of the sun (5.13). If more examples were available, and particularly if we could investigate any magnetic fields that may exist on other planets, like Mars or the moon, in physical conditions partly like and partly unlike those of the earth, our opportunities of testing theories of the origin of such fields would be much more favourable than they now are.

Some authors, assuming that the earth's magnetization is a fundamental physical phenomenon, have endeavoured to find an explanation of it that would, at the same time, explain the sun's magnetism. Some theories of the geomagnetic field have attempted to account also for the maintenance of the earth's negative electric charge, and even for gravitation. A review will be given of these various theories [1], although it cannot be said that at present any satisfactory explanation of the earth's main field is available.

21.2. The assumption of permanent magnetization. At first sight the simplest explanation of the earth's field seems to be that the body of the earth is permanently magnetized. Spherical harmonic analysis shows that the main cause of the earth's field is internal, but it gives no clue as to the location and distribution of the magnetization or the electric currents inside the globe (cf. 18.3).

Near the surface the earth-temperature increases with depth at the rate of about 30° C. per kilometre, as far down as observations in deep borings extend. It is rather certain that this increase continues for some distance farther down into the crust. At depths of about 25 km. a temperature of the order of the Curie point for iron, 750° C. (p. 26), can be expected, so that at greater depths all ferromagnetic substances will have lost their magnetic properties, unless the Curie point is higher under the great pressures there met with; but experiments by Adams and Green [2] have shown that for pressure changes up to 3,600 atmospheres (equivalent to 13 km. depth under the continents) the pressure-change in the Curie point is less than 10° C.

We therefore consider the degree of magnetization that must be attributed to the crust, where permanent magnetization might exist, in order to account for the earth's magnetic moment. A sphere of radius a with the uniform specific magnetization I has the moment $\frac{4}{3}\pi a^3 I$ (cf. 1.15); for the earth, if uniformly magnetized, we found $I = 0{\cdot}08$ gauss (18.3). A magnetized *shell* with outer radius a and inner radius b equal to $a-h$ therefore has the moment $\frac{4}{3}\pi(a^3-b^3)I_1$, or, if h is small compared with a, $\frac{4}{3}\pi\, 3a^2hI_1$. The same magnetic moment is obtained as for a uniform sphere of radius a and magnetization I, if $aI = 3hI_1$, or $I_1 = Ia/3h$. If we suppose that the earth's field is due to the uniform magnetization of an outer layer of thickness h, and take $h = 20$ km., we have $a/h = 300$, and $I_1 = 8$ gauss; even if h is as much as 84 km., $I_1 = 2$ gauss.

These values show what average specific magnetization of the surface crust would be required in order to explain the total magnetic field, if the crust is 21 or 84 km. thick. Now the great magnetic anomalies at Kursk and in Lapland are produced by magnetite with a specific magnetization definitely less than 1 gauss. It seems hardly likely, therefore, that such great average intensities of magnetization as 2 to 8 gauss can occur in the outer crust. Other objections to this hypothesis of the permanent magnetization are

(*a*) that, with magnetization confined to this rather thin surface layer, we should expect systematic differences between the magnetic field over oceanic and continental areas—of which no definite evidence has been found;

(*b*) that it would be hard to explain how the magnetization could be everywhere so nearly parallel to the magnetic axis, unless (*c*) some powerful hypothetical process was assumed by which the earth was magnetized at some past epoch, although no trace of this process is now left—because there is now no appreciable external field.

These various objections are unfavourable to the assumption that the geomagnetic field is due to permanent magnetization either in the core or the crust of the earth. Nevertheless the hypothesis of a permanently magnetized ferromagnetic shell or core cannot be entirely rejected in the present state of our knowledge about the earth's internal structure [28; 22.10; 22.16]; and the discussion of the secular variation (§ 6) suggests that a fairly large part of the earth's field—perhaps as much as a quarter—must have its origin in the outer layers of the earth.

Wilde [29] attempted to reproduce the main irregularities of the earth's magnetic field, and its secular variation, in a model of the earth which he called a *magnetarium*. Inside a globe 18 inches in diameter there was a system of wires carrying electric current, which produced a field equivalent to that of a simple dipole at the centre, with its axis inclined so as to cut the globe at a point between the geographical pole and the arctic circle. On the inner surface of the globe Wilde placed thin sheets of iron over areas corresponding to the oceans, and also over certain other areas, apparently not completely specified in his account of his model. He stated that in this way he was able to reproduce the main irregularities of the field. He also made the dipole field revolve about the geographical axis of his model, and claimed that the field then varied in a manner somewhat (though not closely) like the known secular variation of the earth's field in the past.

Rücker [30] discussed Wilde's attempt critically, though not altogether unfavourably; he thought it possible that the magnetization of the crust might be greater under the oceans than under the continents, because the low temperature at the ocean bed would keep the temperature below the value (the Curie point) at which magnetizable materials lose their magnetization, throughout a layer of thickness greater than that under the continents. He considered that the relation between the magnetic permeability of the globe and its more pronounced geographical features is worth further attention. Wilde's experiment was also discussed, more critically, by Bauer [31]. Hitherto, discussions of the world magnetic maps have not revealed any appreciable systematic differences between the geomagnetic field over land and ocean areas (see Fig. 18.15, p. 661).

21.3. Internal currents. An alternative theory is that the earth is an electromagnet, its field being due to internal electric currents, which must have a magnitude of 1,000 million amperes (18.3), and flow from east to west. Since there is no obvious source of electromotive forces within the earth, capable of maintaining such currents, these may be regarded as surviving from some past epoch when, by means unknown, they were originally set up. Since the currents must decay owing to electrical resistance, the original current-system must have been more intense than that which is now postulated, and the corresponding magnetic field must likewise have been greater than the present field.

Lamb (see Chapter XXII) discussed the decay of currents in a uniform spherical conductor, of radius a and electrical resistivity ρ (in c.g.s. units, of dimensions cm./sec.). He found that free currents of particular types would decay exponentially, and that the longest possible time that could elapse before the current had decreased to a fraction $1/e$ or 0·368 of its initial value corresponds to a current-function proportional to P_1^0; in that case the time would be

$$\tau = (4/\pi)a^2/\rho \text{ sec.}$$

(cf. 22 (89), taking $s = 1$, $q = 1$, $\kappa = 1/\rho$). For a sphere of the size of the earth ($a = 6{\cdot}37\times 10^8$ cm.) made of copper ($\rho = 1{,}700$), we obtain $\tau = 10$ million years, and for iron (taking the low value $\rho = 9{,}000$) $\tau = 2$ million years. Slow as this decay may seem, it implies enormous currents 100 million years ago, namely for copper about 2×10^4 times the present value, and for iron 10^{27} times. We know of no processes which could set up such enormous currents.

Larmor [4] suggested the following possibility. Solar surface phenomena seem to indicate an internal circulation, partly in meridian planes. In the presence of a small magnetic field this would induce an electric field, and if a conducting path round the axis were open, a permanent magnetic field might be built up, as in a self-exciting dynamo. The same possibility might apply to the earth's interior; and the secular variation might be regarded merely as an effect of changes in the internal conducting channels. Cowling [5], however, has shown that such self-excitation is not possible.

Consequently Schuster's view [1] still holds, that the 'difficulties which stand in the way of basing terrestrial magnetism on electric currents inside the earth are insuperable'.

21.4. Gyromagnetic effects. Ampère's hypothesis that the elementary magnets in magnetized bodies are permanent molecular whirls of electricity led to the conclusion that each elementary magnet must exhibit the dynamical properties of a spinning body or gyrostat, and that certain gross effects involving both the magnetic and the dynamical properties of the elementary magnets should be observable. Such gyromagnetic effects have been experimentally demonstrated by Barnett [6] and others.

Among these effects, that of magnetization by rotation is of interest here. The magnetization induced in a rod by rotation with an angular velocity of ν revolutions per second was found to be equal to the magnetization that would be induced in the same rod by a uniform magnetic field of intensity $2\pi\nu\lambda$, where λ is a constant which for iron, nickel, cobalt, magnetite, and Heusler's alloy is found by experiment to differ only by a few per cent. from the value m/e, the ratio of mass to charge for an electron, in electromagnetic units. This ratio is $1/1{\cdot}76\times10^7$, so that rotation at the rate of 200 revolutions per second is equivalent to a magnetizing field of only about 7×10^{-5} c.g.s. or 7γ.

The slow rotation of the earth, for which $\nu = 1/8{\cdot}6\times10^4$ sec.$^{-1}$, would therefore be equivalent to a magnetizing field of less than $10^{-6}\,\gamma$; thus, although the sign of the field which the rotation might produce agrees with that of the existing geomagnetic field, the effect is inadequate by a factor exceeding 10^{10}.

In the case of the sun, the magnetizing field would be still smaller, since the angular velocity of the sun is less than that of the earth in the ratio 1/30 approximately. Since the apparent equivalent uniform magnetization of the sun is about eighty times as large as that of the earth, the failure of this rotation-hypothesis of the observed magnetization is even greater for the sun than for the earth.

21.5. Rotating electric charges. Sutherland [3] and Angenheister [8] have considered the magnetic field that would be produced by the rotation, with the earth, of a uniform surface electric charge S, or of a uniform volume charge Q distributed throughout the body of the earth. As Schuster and Schlomka have pointed out, radically different magnetic fields would be observed according as the observer does or does not rotate with the charges and the earth. If charges of one sign only are considered, they must be so great (to explain the observed magnetic field) that they would set up a vertical electric potential gradient 10^8 times as large as that found by atmospheric electrical observations, which is of the order 1 volt/cm. Conversely the negative electric surface-charge of the earth, which is the outstanding feature of atmospheric electricity, would give a magnetic field 10^8 times too small. Angenheister, by assuming that there exists both a surface and a volume charge, equal in total amount and opposite in sign, shows that their external electrostatic effect would be compensated, while retaining a magnetic field. The positive electric charge per unit volume required in order to explain the observed magnetic fields is 4×10^{-15} electromagnetic units for the sun, and 18×10^{-15} for the earth. The charges per unit mass of the sun and earth would therefore be about equal. This numerical similarity is of doubtful significance, however, at least if it is true that (as Hale and his fellow-workers have reported) the magnetic field of the sun varies rapidly with height, because in that case it is not clear what value must be attributed to the sun's magnetic moment (5.13). The great difficulty of the suggested hypothesis is that the *internal* electrostatic field is not compensated; it would in fact be very considerable, and the earth is amply conducting enough to destroy the supposed charge distribution with great rapidity.

An alternative type of geomagnetic hypothesis is to postulate small deviations from the fundamental laws of electrostatic attraction and repulsion between electrons and protons, as Swann, Schlomka, and others have done.

Swann [9] modified the electromagnetic equations slightly, by the addition of small terms depending not only on v, the velocity of the electric charge, as in the classical theory, but also on the acceleration $\dot{v}$ and its rate of change $\ddot{v}$. His additional terms are such that no new effects arise merely through uniform *translation* of matter, that is, the equations remain invariant under the transformation of the restricted theory of relativity; the terms contain a factor the form of which is chosen so that in the case of uniform *rotation* of neutral matter, with angular velocity ω, they imply the existence of a current-density proportional to ω^4r^3, at a distance r from the axis; this law of variation of the hypothetical current-density was shown by Swann to be the only suitable simple law that will give a very small field for rotating bodies in the laboratory, and magnetic fields of the right relative magnitude for the sun and the earth (though owing to the uncertainty as to the true intensity of the magnetic field of the sun, it is unsafe to attach much importance to the observed ratio of the solar and terrestrial intensities). Finally, by appropriate choice of a numerical constant involved in the new terms, the theory is made to fit the actual magnitude of the solar and terrestrial fields. The whole procedure is elaborately *ad hoc*, and therefore somewhat unattractive, despite the skill with which the theory is developed. While the modified equations fulfil their intended purpose without appearing to introduce discrepancies with other observed facts, only one new consequence was inferred from them, namely, that small rapidly rotating non-magnetic bodies should have a minute but just measurable field. Swann and Longacre [12] afterwards tested this prediction, which was not confirmed.

Schlomka [10], following a suggestion by Schuster, started with the simple assumption that, at a given distance r, the magnitudes of the forces between two protons, two electrons, and an electron and proton, are slightly unequal, being in the respective ratios $(1+\alpha):(1+\beta):1$. This would mean that each sphere of matter would be radially electrically polarized, and, by rotation, magnetized; the assumption that $(\beta-\alpha) = 0{\cdot}13\times10^{-18}$ would account for the earth's magnetic field. Electrically neutral bodies would attract one another with a force proportional to $(\beta+\alpha)$, and if this is interpreted as gravitational, we must have $\beta+\alpha = -0{\cdot}80\times10^{-36}$. A copper sphere of 25 cm. radius, rotating 100 times per second, would only give a magnetic field of 10^{-9} c.g.s. $= 10^{-4}\,\gamma$, which is below the limit of observation in laboratory experiments.

However, it seems doubtful whether geomagnetism has any such fundamental significance in physics as in Swann's theory, involving small modifications in the general laws, which reveal themselves only in rotating bodies of great size. Any such hypothesis leads to a field symmetrical about the axis of rotation, whereas neither the sun's nor the earth's field is of this type; the obliquity of the magnetic to the rotational axis is about 4° for the sun, and about 12° for the earth. Thus the transverse component of the field, which a fundamental rotational theory cannot explain, amounts to a few per cent. of the axial component. The secular variation of the earth's field is a further non-axial phenomenon of the same relative order of magnitude.

These asymmetrical features must be ascribed to causes which cannot be fundamental, and as they are not greatly inferior to the axial fields, it seems unnecessary to invoke fundamental hypotheses for the latter. Therefore geomagnetism is probably only a secondary, though possibly widespread, phenomenon, and not a universal fundamental one like gravitation. If this be so, theories of cosmical magnetism must involve factors of an accidental character, not too rigorously determined *a priori*, in order that they may account for the varied secondary features of the solar and terrestrial magnetic fields [13].

21.6. The bearing of the secular variation on theories of the main field. Between the years 1580 and 1820 the declination at London changed from nearly 11° E. to 24° W., that is, by 35°. The corresponding change in the east-component may be estimated as 1,100 γ, or nearly one-fourth of the average total field intensity F.

In conjunction with the fact that the secular variation is a regional, not a planetary phenomenon, its magnitude seems to indicate that the regional parts of the earth's main field are also fairly large. A similar change occurring in the horizontal components of the magnetic field near the poles would mean a shift of the dip poles by as much as 20°, more than the obliquity of the earth's axis.

It therefore seems safe to conclude that a considerable part of the main field must have its origin at depths where the regional differences in the earth's crust are still important. Areas as large as the oceans or the continents seem to be affected in a generally similar way by the processes which produce the regional anomalies of the main field and their secular variations. The observations do not indicate any obvious simple relation of the regional anomalies or the secular variation to the oceanic and continental areas. But from the standpoint of Wegener's hypothesis of continental drift, for instance, the high secular variation on the west coast of South America may seem not to be accidental. The smallness of the regional anomalies found in the Pacific area (Figs. 18.14 and 15) is also a notable fact.

Schmidt [23] has shown that the cyclic features observed in the long series of observations at London, Paris, and Capetown are not inconsistent with the hypothesis of a homogeneously magnetized core, providing about one-eighth of the earth's magnetic moment, which rotates with respect to the crust in the meridian plane with the longitudes 90° and 270°, with a period of 480 years. The total magnetic moment should, on this theory, have passed through a minimum (with the horizontal force H_0 at the equator equal to about 0·25 gauss), at about the year 1600; the moment should have increased thereafter to a maximum ($H_0 = 0{\cdot}32$ gauss) at about 1840, and be increasing again at present. The scarcity of older intensity observations makes the basis for this hypothesis, however, rather uncertain. The main feature of the hypothesis is that it can explain why the loop described by the magnetic vector is nearly circular at stations near the meridian 0° (as at London, Paris, and Capetown), but is elongated at stations near the meridians ±90° (as at Boston and Peiping); but there are few intensity observations more than a century old, so that a quantitative test must await the future course of magnetic history.

21.7. The magnetic effect of earth-movements and ocean currents. The theory of electromagnetic induction, applied to sea water moving with a velocity of 50 cm./sec. (about a knot) across the earth's magnetic field (13.18), predicts electric ocean currents of the order 10^{-8} amp./cm.^{2} In the case of the Gulf Stream, which is about 400 metres deep near Florida, this would mean an equivalent surface-current of 4×10^{-4} amp./cm., and a horizontal magnetic component (p. 19), of about 20γ. However, it is not likely that regional anomalies of this order are actually produced by such ocean currents, because the electric current-circuits will probably not flow in horizontal layers, but will be closed by flow through deep waters, making shallow vertical loops of large horizontal extent, so that the magnetic effect at the surface will be only a fraction of the above value.

According to 13 (10), the equivalent surface current-density induced by a magnetic field of intensity F, in a linear conductor of resistivity ρ and vertical extent h, moving with velocity v across the field, is $i = Fvh/\rho$ in electromagnetic units, and the maximum magnetic effect is $2\pi i$. Consider the vertical movements connected with the tendency towards the isostatic compensation of surface loads on the earth; for Scandinavia, which is rising on account of the gradual decrease of its glacial load (which has been melting since the last ice age), v is of the order of 1 metre in 100 years; similar vertical velocities, of order $v = 10^{-7}$ cm./sec., have been found elsewhere. In order to obtain an upper limit for the electromagnetic effect of such vertical movements, we take $v = 10^{-6}$ cm./sec., $F = 0{\cdot}5$ gauss, $h = 500$ km., and $\rho = 2\times10^{3}$ electromagnetic units $= 2\times10^{-5}$ ohm cm.; this low value for ρ, equivalent to the ρ for copper, is assumed, quite hypothetically, for the lower crust, though it is about 10^9 times the surface value, and also about 10^8 times the maximum value found, from the theory of induction by the S, L, and D fields (Ch. XXII), for any layer down to 500 km.; on these generous assumptions we obtain a current flow which would have a surface field of $8{,}000\,\gamma$. This would be a regional anomaly, and any notable time-change in F or v or h or ρ would yield a secular variation of the right order of magnitude. Gunn [26] suggested this as a possible cause of the secular variation; but the sufficiency of these assumptions rests almost entirely on the low value ascribed to ρ, which is definitely inconsistent with the magnetic evidence provided by the relation between the internal and external parts of the S, L, and D fields (Ch. XXII). If, in this calculation, ρ is given a value of the order 10^{11}, the induction by the rising layer would produce a surface field with an intensity which is only a small fraction of $1\,\gamma$.

Nippoldt [16] suggested, as an explanation of the secular variation, that the level at which the temperature at about 25 km. depth in the earth's crust is equal to the Curie point (§ 2) rises or sinks as the material becomes hotter or colder, and that thus the layers affected can lose or gain in magnetic moment; but Bidlingmaier [27] showed that this suggestion could not survive quantitative examination.

21.8. The external field. Carlheim-Gyllensköld and Schuster [18.22, 26] discussed the hypothesis that the space round the earth has a certain electric conductivity. They supposed that the equatorial part of the earth's moment, representing a rotating magnet, would induce currents in this space, which would cause an appreciable external field. Since, however, the evidence for any appreciable constant external field is still controversial (18.9) and also because of the reduction of κ, in the outside space (cf. 15.10), by the earth's field, we shall not discuss this theory.

Observational evidence for the existence of an external field due to magnetic disturbance, but enduring with appreciable intensity throughout quiet periods, has been described by Schmidt [9.6]. The average magnetic disturbance leaves a diminution in H which is not completely recovered even after a few months, in some cases, and for which the cause is mainly external. He considered that this indicates the existence of an external current system which decays only very slowly, so that its complete disappearance would take some years of unbroken absence of disturbance. He suggested that such complete decay would probably raise the magnetic horizontal intensity near the equator by as much as 250γ above the present average value, or, in other words, that the average external field reduces the internal field, near the equator, by 250γ. He pictured this as due to the effect of a 'ring-current' circulating round the earth in the plane of the magnetic equator; the current is increased during magnetic disturbance, and decreases during the quiet intervals.

How definitely the magnetic axis governs these average disturbance effects is well illustrated by Bartels's stereogram (9.10), which shows the vectorial differences between the average field on quiet and disturbed days at Potsdam; these vectors lie in the vertical magnetic meridian plane (deviating by 19° W. from true north), through the earth's magnetic axis; their horizontal component definitely does not lie in the direction either of the declination (7·2° W. for the years chosen, 1915–28) or of the astronomical meridian.

XXII

ELECTROMAGNETIC INDUCTION WITHIN THE EARTH

22.1. Induction by the external part of the S, L, and D fields. It has been shown in Chapter XX that the transient S, L, and D fields can be separated by spherical harmonic analysis into external and internal parts (indicated in our notation by suffixes *e* and *i*), which originate respectively outside and within the earth. The external parts are, at the earth's surface, between twice and thrice as intense as the internal parts, and there is a phase-difference between corresponding harmonic components of the two parts.

In the introduction to Part 3 it was explained that any varying external field, such as S_e, must produce a varying field S_i' of internal origin, by electromagnetic and magnetic induction within the earth; the field S_i' will at least contribute to the internal part (S_i) of the S field, and may constitute the whole of S_i. It is of importance to know whether $S_i = S_i'$, but a direct comparison of the two is not possible, because the determination of S_i' from S_e requires a knowledge of the electrical conductivity κ and the magnetic permeability μ throughout the earth, whereas our direct knowledge of κ and μ is limited to the outermost layer of the earth.

On this account our discussion of the induced fields must be tentative. The natural procedure is to calculate what they will be, on the basis of various reasonably probable simple assumed distributions of κ and μ, chosen as being amenable to analytic and numerical treatment.

The last consideration requires (at least at the outset) that the earth shall be treated as a sphere in which the distribution of κ and μ is spherically symmetrical (that is, a function of the radius only).

This is quite untrue in the outer layers of the earth so far as concerns κ, though μ is there practically uniform, differing inappreciably from unity except in a few isolated regions containing deposits of ferromagnetic materials. The value of κ, however, varies from 4×10^{-11} or even more in the oceans, to 10^{-13} or 10^{-14} in moist earth, and to still lower values, 10^{-15} to 10^{-16}, in dry earth and rock [1, 2, 3]. The value 4×10^{-11} refers to North Sea water (see p. 424); in the Mediterranean waters κ may be as high as 6×10^{-11}. Owing to these great variations of κ, and the irregular distribution of the oceans over the globe, it might seem that calculations of the induced fields, based on a spherically symmetrical distribution of κ (and μ), would have little practical interest. This question is still under investigation, so far as concern the induction of S_i and L_i; as regards the more slowly varying part c D_i, the outer layers appear to take only a minor share; most of th induction seems to occur at depths of 100 miles or more, and no indica tion has yet been found of any striking departure, at such levels, fror a spherically symmetrical distribution of κ and μ.

The solid part of the earth's outer layer contributes only inappre ciably to S_i and L_i, because of its low conductivity: it seems likely tha below the thin water-bearing layer κ is small compared with 10^{-} (which is about the maximum value found by Gish and Rooney [in their valuable studies of the electrical conductivity of the outermo stratum). The oceans, however, are highly conducting, and later di cussion (§ 3) indicates that appreciable currents must be induced them by S_e and L_e; these current-systems must add some complexiti to the S and L fields, but apparently they do not shield the deeper laye from the S_e and L_e fields; this must be because these fields are in co tinuous westward motion round the earth (following the sun or moor so that the oceanic electric current-systems, of restricted and irregul forms on account of the complicated ocean boundaries, must be co stantly changing, and must presumably be much less intense, on t average, than they would be in even a relatively shallow world-wi ocean.

The outer solid layer of the earth is, however, of importance as regar induction by *rapidly* varying outer fields. Since $\mu = 1$ in these laye the only type of induction there concerned is electromagnetic. Consid electromagnetic induction by a magnetic field which, for simplicity, suppose to vary periodically, proportionately to $\sin pt$. The magne flux f of this field through any circuit drawn horizontally at any le near the surface of the earth will be determined by the form of t circuit and the distribution of the field; it will vary as $\sin pt$, being, sa $f_0 \sin pt$. The induced electromotive force (e.m.f.) round this circuit $-df/dt$, or $-pf_0 \cos pt$, its amplitude thus being proportional to p and f_0. Hence the whole induced current-system in any thin layer, bei proportional to the e.m.f. distribution in the layer and to the value κ there, will be proportional to κp and to the intensity of the induci field. The current-system itself produces a magnetic field within earth which tends to neutralize the external field, or to prevent pe tration to deeper layers. As we go downwards, the layers met w are more and more shielded from the outer field, that is, the result inducing field progressively decreases, and with it the induced curre intensity. The depth at which practically complete shielding from the outer field is attained will be inversely proportional to the general intensity of the induced current-system, that is, to $1/\kappa p$. If either κ or p is large, a thin layer will provide an efficient shield.

The main periodic components of the S and L fields have periods of 24 and 12 hours. For a field of 1-hour period the currents induced in the outer crust of the earth are relatively twelve times as intense as for the 12-hour component of S or L; for such or more rapidly varying fields, induction in the outer layers will be of some importance, because κp for these layers and for such fields is comparable with, or exceeds, the value of κp for S and L in the deeper layers that produce S_i and L_i. For fields of period only a few minutes or less, the outer layers will practically shield the deeper layers from the external field.

There is need for considerable extension of the work that has so far been done in illustrating the influence of electromagnetic induction on the S, L, and D fields. For the large-scale fields of simple world-wide distribution, like S and L and part of the D field, the earth must be treated as spherical; but in discussing induction problems associated with features of these fields (particularly of the D field) that are of more limited geographical extent, it is helpful, and justifiable, to ignore the sphericity of the earth, and to treat its surface as an infinite plane. The most intense part of the D field, for example, is confined to fairly small regions, namely the polar caps within and near the auroral zones: the calculation of the corresponding induced fields must be complicated in any case, even if the part of the field over each polar cap is treated separately, as existing above an infinite plane conductor representing the earth. The theoretical attack on this problem has scarcely been begun; the problem must probably first be subdivided into still simpler elements. An attempt in this direction has been made by considering the induction associated with the concentrated currents flowing along the auroral zones; to illustrate this, the simpler problem of an alternating current flowing along an infinite horizontal linear conductor above a plane conducting earth has been considered. It is clear that the induced currents will not be laterally concentrated like the inducing current: the calculations, made by Price [5], illustrate the current-distribution in the direction transverse to the linear current, and show the distribution of the horizontal and vertical components of the induced field at the surface. The problem differs, of course, in some material respects from those of the polar D current-system, in which the concentrated auroral currents are curved, and flow laterally into and out of the zone.

In the later part of this chapter an outline of the mathematical theory of electromagnetic and magnetic induction in uniform spherical distributions of conductivity and permeability is given. The methods indicated are applicable to any problem in which the conductivity and permeability are distributed uniformly throughout any number of concentric shells, with discontinuous changes from shell to shell. The only case discussed here refers to a uniform sphere.

In all such problems of electromagnetic induction in spheres, it is practically necessary to suppose that the inducing field is first analysed into spherical harmonic components, each of which is considered separately. It has been seen in Chapter XX that the S and L fields can be approximately represented by a few spherical harmonic components: part of the D field can also be so represented, though not the most intense part, namely that in the polar regions. To represent this part adequately, a great many harmonic terms, of varying signs, would be necessary; the calculation of these terms from the observations, and then the calculation and summation of the corresponding harmonic terms in the induced field, with reasonably sufficient accuracy, would be so burdensome that it has not been attempted. Since each harmonic may be treated separately, however, useful results have been reached by consideration of the single outstanding zonal harmonic in D.

Schuster [20.1] was the first to divide one of the transient parts of the earth's field (namely the S field) into its outer and inner parts, and he also first suggested that the latter is merely an induction effect of the former. He tested this hypothesis by using, in conjunction with Lamb, the latter's pioneer studies of electromagnetic induction in a uniformly conducting sphere [6]. This work was extended later by Birkeland [7], Chapman, Whitehead [8, 9], Price [10, 11], Ertel [12], Buchholz [13], and others. The heating of the earth and oceans by the induced currents has been considered by Chapman [17], and found to be unimportant.

22.2. The uniform-core model compatible with the S and L fields. The model distribution of κ and μ that has been most fully considered is that of a uniform sphere, concentric with the earth (of radius a), but possibly of smaller radius qa (where $q < 1$), surrounded by an outer layer $qa < r < a$, in which $\kappa = 0$ and $\mu = 1$. This model includes the case, corresponding to $q = 1$, of uniform κ and μ throughout the whole earth. If $q < 1$, the inner conducting sphere will be called the (conducting) core of the earth.

The relation between the external and induced internal field for any *periodic* spherical harmonic component S_n^m is described in 20.3. The amplitude ratio e_n^m/i_n^m and phase-difference $\epsilon_n^m - \iota_n^m$ are expressed by the formulae (72, 73, 70, 67, 68, 64) in terms of n, m, q, a, κ, and μ.

The formulae involve a quantity β given by

$$\beta = 2\pi qa\sqrt{\frac{\kappa\mu m}{24.60.60}},$$

and if β is sufficiently large it is possible to approximate closely to the phase-difference $\epsilon_n^m - \iota_n^m$ by an expression which involves κ and μ only through a function of β/μ or κ/μ.

The sign of $\epsilon_n^m - \iota_n^m$, as determined in Chapter XX from the observed data for S and L, is negative in all cases, and in this it agrees with the theoretical formula for the 'core' model of the earth. The observed magnitudes of $\epsilon_n^m - \iota_n^m$ for the chief harmonics in the S field are compared in Table 1 with theoretical values calculated by adopting a particular value for β/μ, chosen to fit all the observed values as well as possible.

TABLE 1

Harmonic	P_2^1	P_3^2	P_4^3	P_5^4	P_1^1	P_2^2	P_3^3	P_4^4
(Mean) observed $\epsilon_n^m - \iota_n^m$.	(13°)	18°	21°	23°	7°	8°	32°	30°
Calculated $\epsilon_n^m - \iota_n^m$ . .	18·9	18·7	19·3	20·5	11·6	13·5	15	17
„ „ . .	19·3	18·8	19·5	20·8	11·5	13·4	..	..

Two sets of calculated values are given, the second being derived from the formula (73) for the case $\mu = 1$, and the first from the approximation (75) which depends on κ/μ, without making any assumption as to the value of μ by itself; clearly (75) is quite accurate enough. The best-determined observed values are those for P_3^2, P_4^3, P_5^4, and these agree well with the calculated values. The discrepancies in the other cases are not beyond the accidental range of error in the observed values.

The same particular value of β/μ is in satisfactory agreement with the observed values of $\epsilon_n^m - \iota_n^m$ for the main harmonic terms in the L field, though the observed values are naturally less accurate in this case, on account of the small amplitude of L; the average value of $\epsilon_n^m - \iota_n^m$ for the three principal harmonics P_{m+1}^m in L is $-24°$, as against $-21°$ calculated theoretically.

Having found a single value of β/μ consistent with the phase-differences for the various components of S and L, the corresponding amplitude ratios can be calculated from (72) or (74): they contain the unknown factor q^{-2n-1}, differing from one component to another. By equating the calculated to the observed amplitude ratio, the value of $1/q$

may be inferred separately from each harmonic. The values thus found from the four principal harmonics P_{m+1}^{m} in S are

$$1{\cdot}068,\ 1{\cdot}030,\ 1{\cdot}041,\ 1{\cdot}041,$$

in fair accord with one another. It is satisfactory to find $1/q > 1$, or $q < 1$; a value of $q > 1$ would, of course, have had no physical meaning.

Conversely, adopting 1·04 as the value for $1/q$ (or $q = 0{\cdot}96$) the calculated and observed amplitude ratios for the principal harmonics in the S field can be compared, as follows:

	Observed	*Calculated*
$m = 1$	2·8	2·5
2	2·2	2·4
3	2·5	2·5
4	2·7	2·6
Mean	2·55	2·5

The observed values here given are the means of those in columns 2–5 (for $n = m+1$) of Table 20.5 (p. 692).

For L the corresponding values are

	Observed	*Calculated*
$m = 1$	2·3	2·0
2	1·7	2·5
3	2·7	2·5
4	2·4	2·7
Mean	2·3	2·6

Here the observed values are the means for columns 2 and 4 in Table 20.8. The agreement both for S and for L is as good as could be expected from the nature of the data.

Having determined (or adopted) the value 1·04 for q, we can infer κ/μ from the adopted value of β/μ, which gives the above good fit for the phase-difference; we find

$$\kappa/\mu = 3{\cdot}6 \times 10^{-13} \text{ c.g.s.}$$

Thus with only two disposable constants, q and κ/μ, a good account has been given of the values of at least eight quantities determined directly from observation, i.e. four amplitude ratios and four phase-differences—for the four principal harmonic components of S; the corresponding quantities for the other harmonic components of S, and for the main and subsidiary harmonics in L, although less well determined, are likewise in reasonably good accord with the results calculated from these values of q and κ/μ. Nevertheless, a uniform core surrounded by a non-conducting outer shell must be regarded only as one possible distribution of conductivity compatible with the induction hypothesis of the origin of the fields S_i and L_i; other models may also be compatible with the same data.

So far as concerns this considerable set of data, our discussion favours the hypothesis that the internal part of these transient fields (S and L) is merely an induction effect of the outer part, and has no separate independent cause. This conclusion gains support from the *a priori* probability that each of the fields S, L, and D has one cause only, since each undergoes many variations of intensity from time to time, throughout which the relation between their outer and inner parts seems to remain constant; if separate causes operated, above and below the earth, to produce these respectively outer and inner parts, it would be necessary to suppose that in each case the inner and outer causes varied in unison; this is improbable, in view of the very different physical conditions in the two regions. In particular, a variation of an independent cause, so closely related with solar changes as the observations would require, seems unlikely. Thus we may proceed with some confidence on the hypothesis that the transiently varying inner fields, not only for S and L, but also for D, are merely secondary effects of the outer transient fields.

We shall see later that the above uniform-core model does not well fit the D data, but that an alternative model, or rather a range of models, can be found, which fits the induction hypothesis applied to D as well as to S and L. Before considering this evidence, however, we may note that the amplitude ratios for S and L, calculated from the value of β/μ that fits the observed phase-differences, but taking $q = 1$ (that is, assuming the earth to be uniformly conducting throughout), are definitely inconsistent with the 'observed' values; for the four principal harmonics in S the amplitude ratios thus calculated average 1·8, whereas the corresponding 'observed' value is 2·55.

The adopted value of q (0·96) corresponds to a non-conducting outer layer of depth about 160 miles or 250 km.

The value $3{\cdot}6\times 10^{-13}$ found for κ/μ in the core ($\mu = 1, \kappa/\mu = \kappa = 10^{-13}$ to 10^{-14}) is rather larger than that for moist earth; it is much less than that of sea water ($\kappa/\mu = \kappa = 4\times 10^{-11}$), and much more than that of dry rock ($\kappa/\mu = \kappa = 10^{-15}$ to 10^{-16}).

It may incidentally be noted that the values of q and κ/μ found for the core make β amply large enough to justify the use of the approximate formulae quoted above for $\epsilon_n^m - \iota_n^m$ and e_n^m/i_n^m. Hence the S and L data determine only q and κ/μ, and not q, κ, and μ separately.

If $\mu = 1$ for the core, as it is for almost all of the outermost (solid and liquid) layer of the earth, then $\kappa = \kappa/\mu = 3{\cdot}6 \times 10^{-13}$ is the value to which the present calculations lead. But if the core were highly permeable magnetically, so that μ was large compared with unity, a correspondingly higher value of κ would be inferred. It is only natural that κ should be larger if μ is larger, because in the latter case the inner part of the core is magnetically (as well as electromagnetically) shielded from the inducing field. Hence, for a given ratio of the induced to the inducing field, the induced currents must flow in a thinner layer of the core, which must therefore be of higher conductivity. An additional reason for this is that the magnetically and electromagnetically induced fields are substantially of opposite sign (at the earth's surface), so that if both fields are appreciable, the field of the induced currents must exceed the actual induced field.

22.3. The influence of the surface layers. The above 'core' model of the earth is obviously inconsistent with fact in that it assumes $\kappa = 0$ for $r > qa$, whereas in the oceans κ has a much larger value than that found for the core; and even on the land, Gish and Rooney, in their valuable studies of earth-conductivity in the outermost layer of the earth, have found occasional values of κ as high as 10^{-13}. It is desirable to examine how far our conclusions as to the size and conductivity of the core would be modified by taking account of these surface layers of conducting matter. Any such examination must, of course, fall far short of an exact study of the induction in the surface layers, and especially in the oceans, because of their very irregular and complicated distribution.

As regards the surface layer of moist or water-bearing strata, for which κ is of order 10^{-13} at most, its influence, assuming any reasonable depth of the layer, must be negligible compared with that of all but very shallow seas, on account of the far greater conductivity of the water. A layer of moist earth even 10 miles deep would only possess the same total conductivity as a layer of sea-water 125 feet deep. Thus we shall probably over-estimate the influence of the upper layers of the land regions if we suppose they can be treated for the present purpose as equivalent to shallow seas of depth not exceeding 125 feet.

Having thus, for our present purpose, 'replaced' the continents by shallow seas, we have to consider the influence of the conductivity

$\kappa = 4 \times 10^{-11}$ throughout an outer layer distributed over the whole earth, varying in depth from less than 125 feet to about 5 miles.

Chapman and Whitehead [8] dealt with this problem as follows. They considered a model consisting of an earth containing a uniformly conducting core of conductivity κ', permeability μ, and radius $q'a$, surrounded by a layer in which $\kappa = 0$, $\mu = 1$, itself covered all over by an ocean of uniform depth d and conductivity $\kappa = 4 \times 10^{-11}$. Taking the mean amplitude ratio to be 2·55, the mean phase-difference to be $-19°$ (as found from the first four harmonic components of S), for both the two principal harmonics in S, namely P_2^1 and P_3^2, they calculated the corresponding values of κ'/μ and of the depth d' of the surface of the core (clearly $d' = (1-q')a$) from each of these two harmonics, and for several ocean depths d. Their results were as follows:

Depth of ocean, d	P_2^1		P_3^2	
	$\kappa' \times 10^{13}$	d' (*miles*)	$\kappa' \times 10^{13}$	d' (*miles*)
125 ft.	3·55	168	5·7	191
250 ft.	3·5	178	6·9	210
¼ mile	4·7	198	13·9	336
½ mile	10·4	340	..	..

It thus appears that the estimates of κ' and d' for the core are considerably affected by the presence of even a comparatively shallow world-wide sea, ¼ or ½ mile in depth; the estimates derived from the harmonic component P_3^2 are affected more than those based on P_2^1. A shell of sea-water rather more than ½ mile in depth would cause the phase-difference below the ocean to become positive, which is incompatible with any possible size and conductivity of core.

The question may be otherwise illustrated by calculating the amplitude ratio and phase-difference that would correspond to an earth containing a core of the size and conductivity already deduced on the assumption that $\kappa = 0$ for $r > qa$, but also covered by an ocean of uniform depth d, the primary magnetic field being purely external. The results are as follows [13.70].

Ocean depth d in miles	P_2^1		P_3^2	
	Amplitude ratio	*Phase-difference*	*Amplitude ratio*	*Phase-difference*
0	2·5	−19°	2·4	−19°
½	2·2	−20	2·0	−21
1	2·0	−22	1·7	−20
2	1·8	−18	1·5	−16
3	1·7	−15	1·4	−13
4	1·6	−13	1·4	−10

This table illustrates the extent to which the amplitude ratios and phase-differences are likely to be affected by the existing irregular distribution of land and sea. It suggests also that some difference should be observable between the potentials of S and L if derived wholly from either continental or oceanic observatories. Chapman and Whitehead made a cursory examination of this point, considering six continental stations only, from which they derived a spherical harmonic expression for V, and corresponding amplitude ratios and phase-differences for certain harmonic components; from these data, assuming no outer conducting layer, they deduced for the core a conductivity $\kappa = 1{\cdot}8 \times 10^{-13}$ and a depth (to the surface of the core) of 50 miles. The difference between these and the former values of κ and q may be partly due to the uncertainty of a determination of V from only six stations.

22.4. The distribution of the induced currents in the core. Chapman and Price [10] made an examination of the distribution of the induced currents in the core, corresponding to the principal harmonics of the S and L fields, on the basis of the 'core' model of the earth, $\kappa/\mu = 3{\cdot}6 \times 10^{-13}$ for $r < qa$, $q = 0{\cdot}96$, and $\kappa = 0$ for $r > qa$. As the general theory of induction by alternating fields would suggest, they found that the current-intensity decreases downwards more rapidly for the harmonics of shorter than of longer period; thus for the harmonics P_1^1 and P_2^1 of 24-hourly period, the current-intensity in the core falls to one-tenth its surface value at a depth of about $0{\cdot}33qa$, whereas for the 6-hourly harmonics, P_4^4 and P_5^4, the corresponding depth is only $0{\cdot}14qa$. The phases of the harmonics also advance with the depth, and more rapidly for the shorter than for the longer period harmonics: for example, the increase for P_1^1 and P_2^1 is about 43° for an increase of depth by $0{\cdot}1qa$, whereas for P_4^4 and P_5^4 it is about 85°; down to a depth of $\frac{1}{2}qa$, the rate of increase of phase is nearly but not quite independent of the depth.

The field of the induced currents at the earth's surface $r = a$ is made up of contributions from the currents at all depths in the core, but of course those nearest the surface of the core make the greatest contribution; the deeper layers contribute little, partly because of the downward decrease of the current-intensity, and also on account of the increasing distance from the earth's surface. This may be illustrated by the following table, given by Chapman and Price, showing the approximate *percentage* intensities of the magnetic fields at the earth's surface, due to the induced currents in shells of the core, each of thickness equal to $0{\cdot}1qa$.

Shell radii ÷ qa	P_1^1	P_2^1	P_2^2	P_3^2	P_4^4	P_5^4
1·0 to 0·9	66	70	77	78	86	89
0·9 to 0·8	23	22	18	17·5	13	11
0·8 to 0·7	8	6·5	4	4	1	..
0·7 to 0·6	2	1·5	1	0·5	..	..
0·6 to 0·5	1	..	..	..	..	..

This shows that about 90 per cent. of the surface field comes from the outer part of the core, of thickness $\frac{1}{5}qa$ or less. We may therefore conclude that any variation of κ/μ in the core below this depth would have very slight effect on the induced field at the earth's surface, and therefore that the S and L fields are unlikely to be able to give us much information as to κ/μ below a depth of about 1,000 miles at most.

To obtain information concerning the value of κ/μ at greater depths, more slowly varying fields must be considered, whose induced currents will penetrate farther down: If the varying component of the earth's field which corresponds to the semi-annual variation could be analysed into its inner and outer parts, some knowledge of the value of κ at considerable depths could be obtained. Another possibility is that the 11-year variation in the earth's field may be used for this purpose. This and the 6-monthly variation depend on the greater frequency and intensity of magnetic disturbance near the equinoxes and in sunspot-maximum years than at other times.

Fortunately the aperiodic storm-time changes of the D field are rather gradual in the last phase of a magnetic storm, and they valuably supplement the S and L data in the discussion of the earth's conductivity.

22.5. The magnetic permeability within the earth. It has been seen in § 4 that the relation between the fields S_i and S_e, L_i and L_e, interpreted according to the uniform-core model of the earth, determines q and κ/μ, but that κ and μ cannot separately be determined from such data.

If $\mu \neq 1$, the field of the induced *magnetism* will be proportional to the inducing field, and independent of its rate of change. For the spherical harmonic of order n, the ratio of the coefficients for the magnetically induced and inducing fields is

$$-\frac{nq^{2n+1}(\mu-1)}{n\mu+n+1}.$$

Though the magnetically induced part cannot be separated from the electromagnetically induced part in the case of the daily periodic variations, it may be possible to make such a separation in the case of the storm-time variations.

This is because the external field is varying very slowly at that time, and therefore the electromagnetically induced field is very small, whereas the magnetically induced field is independent of the *rate of change* of the external field. Chapman and Price, in their quantitative study of induction by the D field, found that possibly after two or three days from the beginning of a storm the relation between D_i and D_e might indicate whether $\mu \neq 1$. When the induced currents have practically died away, the internal magnetically induced field will oppose the horizontal components and reinforce the vertical component of the inducing field, in ratios which would be quite considerable if μ differed materially from 1. For example, if $\mu = 10$, the combined horizontal component would only be $\frac{1}{4}$ that of the external field alone, and the corresponding ratio for the vertical component would be $\frac{5}{2}$.

Chapman and Price searched for such an effect, by examining the X and Z data at several observatories for nine or ten days after magnetic storms; they found no reliable indication that $\mu \neq 1$. The search is difficult, because it depends on the X and Z components of the storm-time part of the D field when this has sunk to a low value; since the undisturbed state of the earth's field is not definitely known, these components are then difficult to determine with certainty.

Swann has suggested that the permeability deep within the earth might be tested by experiment. His proposal was that long-distance telegraph wires might for a brief period be linked together to form two large circuits, not too small even compared with the earth's radius. A current generated in one circuit would produce an artificial change of the earth's field and induce a current in the other circuit: the magnitude of this induced current would depend in a calculable way upon the value of μ within the earth. Swann's proposal [14] was originally set out very briefly, and it is of interest to quote it, as follows:

> 'Suppose we have two cables parallel to each other around the earth's equator, and start a current in one of them. We shall get an induced current, and a definite time integral of current in the other. The magnitude of the latter will depend upon the dimensions of the circuits, the value of the primary current, the resistance of the secondary circuit, and the average permeability of the earth, and will vary by a factor of the order of magnitude of 3 according as the average permeability is comparable with unity, or large compared therewith. Now a simple calculation will show that by starting a current of one ampere in the primary cable, the order of magnitude of the induced flow in a secondary cable, of resistance say 10,000 ohms, will be 0·017 coulomb, a very large quantity when considered in relation to what is measurable with a sensitive galvanometer. It will be obvious that the ideal condition stipu-

lated above is not necessary for the purpose of obtaining measurements from which conclusions could be drawn. It is not necessary for the circuits to be immediately adjacent, or to be drawn around the equator, or to be parallel to each other. In fact, a little consideration will show that all that is necessary is that the dimensions of the circuits shall be comparable with the depths to which we wish to extend our information as to the permeability, and their distance apart must not be greater than is comparable with their dimensions. The practical question involved is of course how far it would be possible to utilize existing cables for such a project as this.'

In a later paper [15] Swann solved and discussed various mathematical problems bearing on this proposal. Unfortunately the suggested experiment has not yet been made.

It seems likely, on general grounds, that μ is approximately unity everywhere throughout the earth (except in a few local deposits of special minerals). A large value of μ in the core would presumably imply the existence of ferromagnetic material there. The central part of the earth is indeed generally supposed to consist largely of iron, but at ordinary pressures iron loses its permeability at about 750° C., and this temperature is attained at about 25 km. depth—much less than the thickness of the relatively non-conducting outer shell, corresponding to the above uniform-core model of the earth. Increased pressure appears only to lower the temperature at which iron loses its permeability.

Hence it seems probable that we may take $\mu = 1$, in which case κ for the uniform core would be $3{\cdot}6 \times 10^{-13}$. If, however, μ were to exceed unity, the estimate of κ would have to be increased μ-fold.

22.6. Induction by aperiodic fields. The theory of induction by aperiodic fields, even in a uniformly conducting sphere, is more difficult than that of periodic induction. This is because in the aperiodic case it is necessary to consider the satisfaction of certain initial conditions, whereas in the periodic case we consider only the permanent purely periodic régime which, whatever the initial conditions, will ultimately be attained.

In the theory of aperiodic induction in a uniform sphere, the inducing field is supposed analysed at each instant into spherical harmonic components. Each of these components gives rise to a part of the induced field which involves only the corresponding internal harmonic. If the time variations of these corresponding parts of the inducing and induced fields are represented by $f_e(t)$ and $f_i(t)$ respectively, the theory enables $f_i(t)$ to be calculated when $f_e(t)$ is known. When the time-factor of the inducing field is of the simple form $(1-e^{-\alpha t})$, that of the induced field is given by (87) or (87 a). Hence one obvious method of making the calculations is to represent $f_e(t)$ by a sum of terms of this type, and superpose the results obtained for the separate terms. This method was adopted by Chapman and Price in their investigation (§ 7) of the internal part of the D_{st} field. They found that the numerical work involved was greatly reduced by choosing the α's from among the series of particular numbers α_{ns}, defined by (59) and (60), though by so doing only a very rough representation of the actual $f_e(t)$ could be obtained.

An alternative method, which avoids this difficulty, consists in first finding the time-factor for the induced field, say $\phi(t)$, which corresponds to a sudden increase by unity in the time-factor of the inducing field; the result for any other aperiodic inducing field can then be obtained by means of a general formula (viz. (93)) which is applicable to all aperiodic induction problems. This method, which enables the *exact* time-function $f_e(t)$ to be used, has been applied by Price and Lahiri (§§ 8, 9).

22.7. Induction by the storm-time disturbance field D_{st}. In the further discussion we shall suppose that $\mu = 1$. We shall also consider only the main zonal harmonic P_1 in the storm-time field D_{st} (as observed in the middle belt of the earth, between latitudes $\pm 50°$). In this case ($\mu = 1, P_n = P_1$), the special numbers α_{ns}, or α_s as they may be written (dropping the suffix $n = 1$), are very simple, namely

$$\alpha_s = s^2 A,$$

where

$$A = \pi/4\kappa q^2 a^2.$$

In order to examine the induction by the outer storm-time field in a model earth of the simple type compatible with the S and L fields, that is, $\kappa = 3{\cdot}6\times 10^{-13}$ for $r < qa$, where $q = 0{\cdot}96$, and $\kappa = 0$ for $r > qa$, A must correspond to these values of q and κ for the core: they give $A = 5{\cdot}74\times 10^{-6}$.

Adopting this value of A, Chapman and Price found that the time-factor $f_e(t)$ or e_1, for the harmonic P_1, determined as described in 20.11, can be represented by a sum of four terms of the above form $a_s(1-e^{-\alpha_s t})$; these terms correspond to $s = 1, 3, 6, 8$, and to $a_1 = 40 = -a_3$, $a_6 = -96 = -a_8$. Thus

$$e_1 = -96(e^{-36At}-e^{-64At})+40(e^{-At}-e^{-9At}),$$

the unit being 1γ. The values of e_1, calculated from this formula, for the first four days of a storm, are illustrated by a full line in Fig. 1;

the 'observed' values (for the first two days), as derived in 20.11, are shown by a dotted line (cf. Fig. 20.3): the agreement is moderately good. It could be improved only by taking values of α not included in the series of special numbers (α_{ns}) already mentioned. This, however, was not done by Chapman and Price, because it would have greatly increased the numerical work involved.

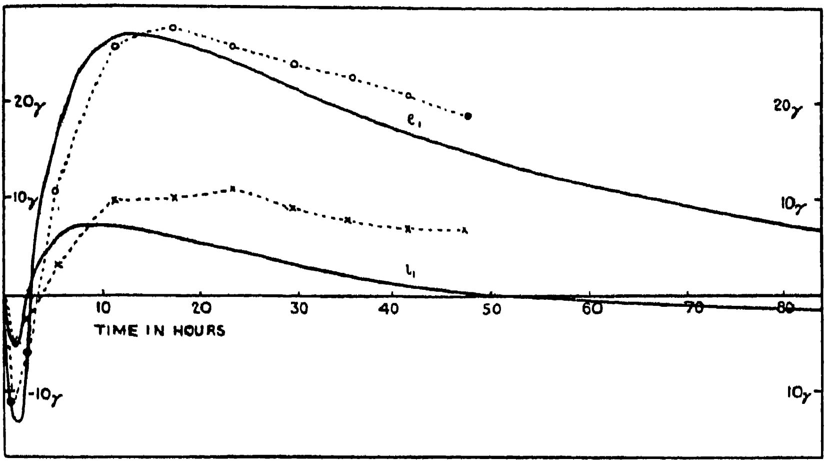


FIG. 1. The dotted lines are the curves given in Fig. 20.3 (p. 697). The full lines illustrate an attempt to represent these curves by theoretical expressions; here i_1 corresponds to the field of internal electric currents such as would be induced, in a certain 'model earth', by the field corresponding to the 'external' curve e_1

The corresponding value of $f_i(t)$ or i_1, calculated from the formulae given by Price, was found to be

$$i_1 = -96(\phi_8 - \phi_6) + 40(\phi_3 - \phi_1),$$

where ϕ_p is the function given in (87 c). The values of i_1 calculated from this formula are shown in Fig. 1 by a full line, for comparison with the 'observed' values (dotted line).

The 'theoretical' e curve decreases somewhat too rapidly after about 15 hours from the storm commencement; the calculated i curve is also then too low: but the proportional discrepancy between the calculated and observed values is much greater for i than for e. This indicates that the simple core model of the earth, with the values of κ and q found from S and L, does not accord well with the relation between the outer and inner parts of the more slowly varying D_{st} field in the later phase of a magnetic storm.

The D_{st} data alone can, however, be reconciled with the hypothesis that the inner part is wholly due to induction by the outer part, even if the earth is represented by a simple core model, provided that values of q and κ different from those found for the S and L data are adopted. It is not so easy to deduce q and κ separately in the case of aperiodic fields as in that of periodic fields, but if we suppose $q \leqslant 0{\cdot}96$, then κ must be at least 44×10^{-13} to fit the D_{st} data; the smaller the value of q, the larger must be the value of κ. The values $q = 0{\cdot}94$, $\kappa = 44\times 10^{-13}$ lead to a fair agreement, after the first twelve hours of a storm, between the calculated induced P_1 field and the internal part of this harmonic as deduced by analysis of the D_{st} field.

This discrepancy between the conclusions as to the core derived from S and L, on the one hand, and D_{st} on the other, should not be regarded as necessarily conflicting with the hypothesis that the inner parts of these fields are induced by the outer parts: Chapman and Price regarded it as indicating merely that the simple core model of the earth is inadequate, and that the value of κ probably increases downwards rather rapidly in the region around $q = 0{\cdot}96$.

This will imply a more rapid downward decrease of induced current-intensity in the core than would exist if the core had the same value of κ throughout, as at the core surface. Chapman and Price showed, by considering the simple core model found from S and L, that in such a core the D_{st} induced currents decay distinctly less rapidly than do those for even the slowest (24-hourly) component of the S field; thus, whereas in the latter case the current-intensity falls to one-tenth of its surface value at a depth $0{\cdot}33qa$ below the surface of the core, the D_{st} currents (due to P_1) would not be diminished in this ratio until a depth of about $0{\cdot}6qa$ was reached. If, however, κ increases downwards in the core, all the induced currents, for S and L as well as for D, will penetrate less far down.

The above discussion shows that, to determine whether a single model earth can be found which will give a good fit between observation and the induction hypothesis, for all three of the S, L, and D fields, it is necessary to consider the problem of induction in a sphere of non-uniform conductivity. This extension of the discussion has been successfully made by Price and Lahiri, whose work will now be described.

22.8. Non-uniform core models compatible with both the S and D_{st} fields. Taking the distribution of conductivity within the earth to be given by $\kappa = k\rho^{-s}$ for $\rho < 1$, $\kappa = 0$ for $\rho > 1$, where $\rho = r/qa$, Price and Lahiri [16] showed that the discrepancies in the induction theory of the internal parts of S and D_{st} are much reduced —though not entirely removed—by taking a value of s considerably greater than the zero value that corresponds to a constant κ. Their method was first to investigate the range of values of s which permit the induction hypothesis to account for the internal part of the harmonic P_3^2, this being one of the best determined harmonics in the S field. When a particular value of s is taken, the values of e/i and $\epsilon-\iota$ for any one harmonic, as determined from the observations, fix the corresponding values of k and q. It has already been seen in § 2 that when $s = 0$, i.e. when the core is uniformly conducting, the values of k and q are approximately $3{\cdot}6\times 10^{-13}$ e.m.u. and 0·96. As s is increased, it is found that k must be decreased and q increased; but since q cannot increase beyond unity, this gives an upper limit to the possible values of s. This upper limit is slightly less than 30, and the corresponding value of k is about 4×10^{-14} e.m.u. (q, of course, being unity).

The sets of values of s, k, and q which are found suitable for P_3^2 were then used to calculate the time-factor $i(t)$ in the potential of the induced field (P_1) which corresponds to the observed $e(t)$ of the *storm-time* variations. These calculations showed that, as s is increased, $i(t)$ is increased, and consequently the fit between the calculated and observed $i(t)$ is improved. But even when $s = 30$ (which is actually slightly larger than the P_3^2 observations allow) $i(t)$ is still somewhat too small. This can be seen from the curves a', b', and c' of Fig. 2, which show the values of $i(t)$ calculated for the distributions of κ obtained by taking $s = 2$, 8, and 30 respectively, with (in each case) the corresponding values of k and q, already found to be suitable for the P_3^2 harmonic of the S field. These distributions of κ are shown graphically in Fig. 3, curves a, b, and c. Each gives the correct values of e/i and $\epsilon-\iota$ for P_3^2, except that in the case of c the phase-difference is slightly too small. These results suggest that the curve c is a considerably better approximation to the actual distribution of κ within the earth than the curve a (which is almost the same as the uniform core distribution considered in §§ 2–7 above). At the same time the D_{st} observations appear to require a higher value of κ for the lower layers than even the s, k, q model can allow, if it is to remain consistent with the results for P_3^2.

The conductivity of the lower layers could be further increased if they were shielded more effectively, in the case of the S variations (which are more rapid than the D_{st} variations), by upper conducting layers. This could be done by having more conductivity near the surface of the earth, either concentrated in a thin shell at the surface—in which case it might represent the influence of the oceans (§ 3)—or spread throughout the thick shell surrounding the more highly conducting core. To examine the first of these possibilities, Price and Lahiri repeated the above calculations for a non-uniform core (s, k, q), surrounded by a thin uniform conducting shell at the earth's surface.

If the integrated conductivity of this shell is denoted by $\kappa_0 \partial$ and its thickness by ∂, it is found that, corresponding to any value of s, there is a definite range of values of $\kappa_0 \partial$ consistent with the observed results for P_3^2. The upper limit of this range is independent of s, and is equal to $5{\cdot}1 \times 10^{-6}$ e.m.u.$\times$cm. The lower limit increases as s increases, and it coincides with the upper limit when s becomes infinite. The same limiting distribution is obtained if either s or k tends to infinity. It consists of a thin shell of integrated conductivity $5{\cdot}1 \times 10^{-6}$ e.m.u.$\times$cm. —which is roughly equivalent to a uniform ocean of depth 1 km.—surrounding an infinitely conducting core of radius $0{\cdot}903a$. This distribution is represented by the curve e of Fig. 3.

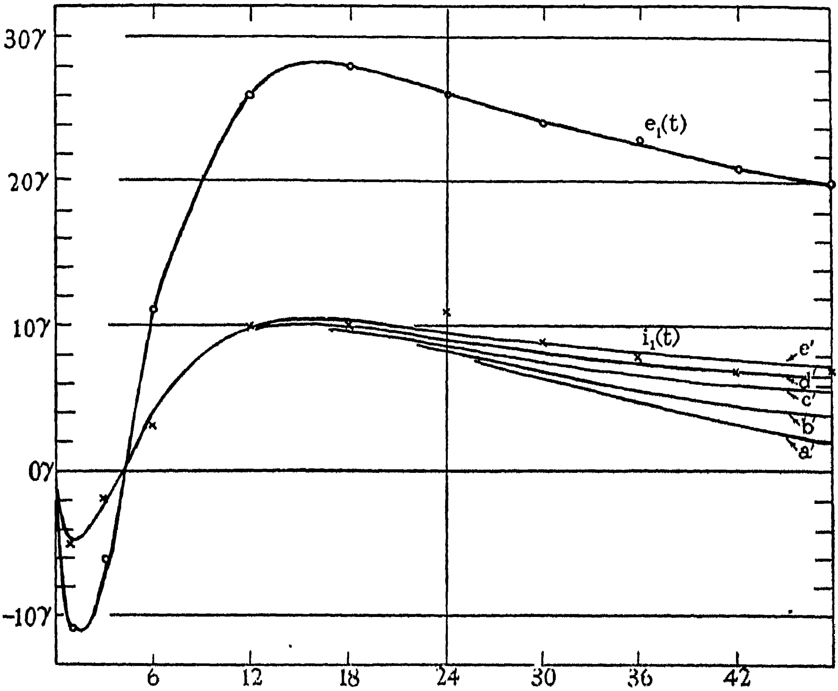


FIG. 2. Showing the variation of i_1 calculated theoretically, for different distributions of electric conductivity within the earth (as in Fig. 3)

When the storm-time variations are considered, it is found that the *greatest possible* 'induced' value of $i(t)$, consistent with the P_3^2 observations, corresponds to the extreme distribution e. This theoretical maximum for $i(t)$ is shown as curve e' in Fig. 2. It will be seen that the observed values of $i(t)$ are very close to this maximum, but do not exceed it except in one case, namely at $t = 24$ hours. (This exceptionally high value is possibly due to some irregular fluctuation, which has not been completely eliminated in the analysis of the observations.) This is a good confirmation of the induction hypothesis; at the same time it indicates that the observed results for P_3^2 and D_{st} will not, by themselves, indicate an *upper* limit to the value of κ below a depth of about $0{\cdot}1a$. It should also be noted that instead of taking κ infinite below $r = 0{\cdot}903a$, as in e, it may have any value greater than about 10^{-11} e.m.u., because all such values lead to results differing inappreciably from each other.

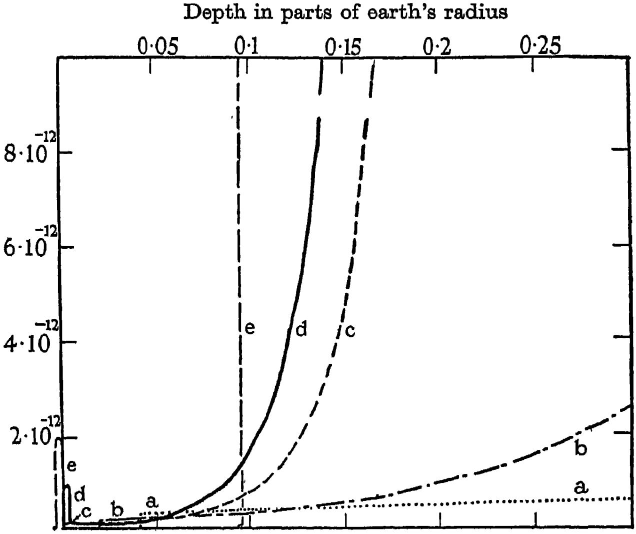


FIG. 3. Illustrating distributions of electrical conductivity within the earth, compatible with the relation found in 1919 by Chapman, between the external and internal parts of the harmonic P_3^2 in the field of the daily magnetic variation S; of these curves, only d and e are compatible with the storm-time variation of the magnetic disturbance field (cf. Fig. 1). In the case of d and e there is a thin shell of relatively high conductivity near the surface; the area under the curve d (or e) near the surface gives an indication of the total conductivity $k_0 \partial$ of this shell, but other values of the thickness ∂ are equally compatible with the data, provided that $k_0 \partial$ has the value indicated by the curve d

The least permissible value for $\kappa_0 \partial$ *in the above model* is about 2×10^{-6} e.m.u. × cm., because the observed results for P_3^2 then allow s to be as great as 37, and this value of s is just large enough to give a suitable internal field for D_{st}. With this value of $\kappa_0 \partial$, the observed amplitude ratio and phase-difference for P_3^2 can be obtained by taking $s = 37$, $k = 4 \times 10^{-14}$ e.m.u., and $q = 1$; this distribution is shown graphically as curve d of Fig. 3; the corresponding $i(t)$ for the storm-time variations is shown as curve d' of Fig. 2. It will be noted that the curves d' and e' do not differ by more than 1γ at any time, and with one exception (at $t = 24$ hr.) the observed values of $i(t)$ lie on or between these curves. It appears, therefore, that any distribution of κ between those represented by d and e of Fig. 3 is compatible with the observed results for both P_3^2 and D_{st}.

In the case of e, the shielding of the core would be due entirely to the oceans; in the case of d it would be due partly to the oceans and partly to the layer of moderate conductivity between the surface and the core. The value 2×10^{-6} e.m.u. $\times$ cm. for $\kappa_0 \partial$ in d does not, however, *necessarily* represent a lower limit to the possible influence of the oceans, because there is still the possibility, noted above, that the observations are compatible with a distribution in which κ, throughout the upper layers, is greater than the value necessarily assigned to it when the formula $\kappa = k\rho^{-s}$ is used.

It was shown, however, by Price and Lahiri, that the observations are *not* compatible with any distribution in which κ does not somewhere decrease (either suddenly or gradually) with increasing depth, before the region is reached where it rapidly increases. The distribution of this type which comes *nearest* to fitting the observations is that in which κ remains constant and equal to $2{\cdot}3 \times 10^{-13}$ e.m.u. down to a depth $0{\cdot}11a$, and then suddenly increases to infinity (or any value greater than about 10^{-11} e.m.u.). This distribution gives the correct relation between the external and internal parts of P_3^2; also it gives a time factor $i(t)$ for D_{st} which lies between the curves c' and d' of Fig. 2, and is slightly too small to give a satisfactory agreement with the observed $i(t)$. This shows that *if the influence of the oceans on P_3^2 and D_{st} is negligible* there must be a region, not far below the earth's surface, where κ is greater than $2{\cdot}3 \times 10^{-13}$ e.m.u., and that between this region and the much more highly conducting core there is a region where κ is somewhat less than $2{\cdot}3 \times 10^{-13}$ e.m.u. This, however, seems rather improbable, especially in view of the fact that the measured conductivity of the surface rocks is seldom greater than 10^{-15} e.m.u.

22.9. The distribution of conductivity within the earth. One deduction which it seems can be drawn from the above results is that a very rapid increase in the conductivity of the earth occurs at a depth of about 600 or 700 km. (about one-tenth of the earth's radius), because this feature is common to *all* the distributions which can give results in satisfactory agreement with the observations. Below this depth there is a region in which κ is *at least* 10^{-11} e.m.u. The observations at present being considered cannot, however, afford much further information about this region because the corresponding induced currents do not penetrate it very deeply. In the case of the extreme theoretical distribution *e*, the currents do not, in fact, penetrate below the surface of the core; in the case of *d*, their intensity is greatest at $r = 0{\cdot}88a$, and decreases quickly to a negligibly small value at $r = 0{\cdot}81a$.

In the region between the surface and the highly conducting core, the average conductivity is possibly of the same order as that of the surface rocks (i.e. about 10^{-15} or 10^{-16} e.m.u.). If this is so, then this discussion indicates that the oceans must have roughly the same influence on S and D_{st} as a uniform ocean of depth between $\frac{1}{2}$ km. and 1 km. This does not seem unlikely since the actual oceans, if uniformly distributed, would cover the earth to a depth of 3 or 4 km. The observations so far considered do not, however, preclude the possibility of the mean conductivity of the upper layers being as high as $2{\cdot}3\times10^{-13}$ e.m.u., and if this is the case the influence of the oceans must be appreciably less than the above. An independent estimate of the influence of the oceans, based on an investigation of the induced currents in a suitable non-uniform thin shell, is desirable to distinguish between these alternative hypotheses.

The above information as to the distribution of conductivity within the earth might be rendered more precise by considering the other harmonics in the daily variations. An indication of this is afforded by the phase-differences of the harmonics P_2^1, P_3^2, P_4^3, P_5^4, the observed values of which are respectively 13°, 18°, 21°, 23° (p. 715). Now the values calculated for these phase-differences, using the original uniform-core model, were 19°, 19°, 20°, 21° (Table 1). These increase less rapidly, with the order of the harmonic, than do the 'observed' values. On the other hand, the values obtained when the extreme distribution *e* of Fig. 3 is used are 8°, 18°, 25°, 30°, which increase too rapidly. This shows that the distribution *e* is too extreme. It would evidently be possible to choose a value of $\kappa_0\,\partial$, somewhat less than the value $5{\cdot}1\times10^{-6}$ e.m.u. of the distribution *e*, together with a suitable 'core', which would give phase-differences for the above harmonics in reasonably good agreement with the observed values. It is, however, doubtful whether it would be possible, with the observations at present under review, to distinguish between a gradual downward increase of κ in the core, as, for example, in *d*, and a very abrupt increase in κ (as in *e*, but to a finite value instead of infinity). In any case, the conclusions based on the present series of observations should not be pushed too far; it is desirable to check their reliability by an independent analysis of a more recent and extensive set of magnetic data, before more elaborate calculations are made. It may also be noted that if the non-cyclic variation were analysed into external and internal parts, in the same way as the D_{st} variations, the relation between these parts would be a further check on the results, and would also probably extend our knowledge of κ to slightly greater depths.

THE MATHEMATICAL THEORY OF CURRENT-INDUCTION BY VARYING MAGNETIC FIELDS IN REGIONS BOUNDED BY CONCENTRIC SPHERES

22.10. The general equations. The notation here used is as follows:

$\boldsymbol{E}$ = electric intensity.
$\boldsymbol{H}$ = magnetic force.
$\boldsymbol{B}$ = magnetic induction.
$\boldsymbol{A}$ = magnetic vector potential.
$\boldsymbol{c}$ = electric current-density.
μ = magnetic permeability.
κ = electric conductivity.

These quantities will be supposed expressed in c.g.s. electromagnetic units.

We assume that in each conducting region, where

$$\boldsymbol{c} = \kappa \boldsymbol{E}, \tag{1}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}, \tag{2}$$

μ and κ are constant, and that in the non-conducting regions (where $\kappa = 0$) $\mu = 1$, so that there $\boldsymbol{B} = \boldsymbol{H}$. We consider only fields which are 'quasi-stationary', meaning by this that their rate of variation is sufficiently small for it to be possible to ignore displacement currents.

The magnetic vector potential $\boldsymbol{A}$ satisfies the equations

$$\boldsymbol{B} = \operatorname{curl} \boldsymbol{A}, \tag{3}$$

$$\operatorname{div} \boldsymbol{A} = 0. \tag{4}$$

This implies that, to our degree of approximation,

$$\operatorname{div} \boldsymbol{c} = 0,$$

and also that just within a conductor the components of $\boldsymbol{c}$ and $\boldsymbol{E}$ normal to the surface are zero.

It is also assumed that there are no free charges in any dielectric region (such as the atmosphere or any supposedly non-conducting layer of the earth). As regards volume charges in any uniformly conducting region, the general theory of the electromagnetic field (whether or not account is taken of displacement currents) shows that any such charge originally present will be dissipated with great rapidity; hence such volume charge need not be considered here.

Since

$$\operatorname{curl} \boldsymbol{B} = \operatorname{curl}\operatorname{curl} \boldsymbol{A}$$

$$= -\nabla^2 \boldsymbol{A} + \operatorname{grad}\operatorname{div} \boldsymbol{A}, \tag{5}$$

it follows from (4) and (2) that

$$\nabla^2 \boldsymbol{A} = -\operatorname{curl} \boldsymbol{B} = -\mu \operatorname{curl} \boldsymbol{H}. \tag{6}$$

The magnetic flux N through any surface S bounded by a circuit s is given (in vector notation) by

$$N = \int_S \boldsymbol{B}\cdot d\boldsymbol{S} = \int_S (\operatorname{curl} \boldsymbol{A})\cdot d\boldsymbol{S}$$

$$= \oint_s \boldsymbol{A}\cdot d\boldsymbol{s},$$

by Stokes's theorem.

By Faraday's law the electromotive force round the circuit s is given by

$$\oint_s \boldsymbol{E}\cdot d\boldsymbol{s} = -\frac{\partial N}{\partial t} = -\oint_s \frac{\partial \boldsymbol{A}}{\partial t}\cdot d\boldsymbol{s}.$$

Hence

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \operatorname{grad}\phi, \tag{7}$$

where ϕ is (at present) an arbitrary scalar function of position and time. If ϕ is identified with the electrostatic potential of an instantaneous distribution of charge, this distribution (in the assumed absence of volume charges) must be situated on interfaces between conductors and dielectrics. A complete theoretical treatment, not restricted by the assumptions already mentioned, shows that such surface distributions are very minute; their order of magnitude in the case of quasi-stationary fields is $E_n/4\pi c^2$ e.m.u., where $c = 3\times 10^{10}$ cm./sec., and E_n is the normal component of E just outside the conductor. Nevertheless their electrostatic field is important, because it is responsible for the reduction of E_n just within the conductor to zero, and for the deflexion of any currents approaching the surface so that they are made to flow parallel to it.

It is, however, not necessary to identify ϕ with the electrostatic potential of the charges present; if this is not done, A will of course include a part depending on these charges. It is in fact possible, in the case of certain distributions of κ, including uniform distributions, to take $\phi = 0$, consistently with the equations (1)–(4). When κ is uniform, our assumed equation $\operatorname{div} \boldsymbol{c} = 0$ implies also $\operatorname{div} \boldsymbol{E} = 0$, which is consistent with (4) and with the form of (7) corresponding to $\phi = 0$, namely

$$\boldsymbol{E} = -\frac{\partial A}{\partial t}. \tag{7 a}$$

When $\boldsymbol{B}$ and $\boldsymbol{E}$ are known, this equation, together with (3) and (4), defines A uniquely (apart from an unvarying gradient vector which would not affect $\boldsymbol{B}$ and $\boldsymbol{E}$).

When A is thus defined, the normal component of any variable part of A must be zero just inside the surface of any conductor.

From (1) and (7 a) we deduce that

$$\boldsymbol{c} = -\kappa\frac{\partial A}{\partial t}. \tag{8}$$

By Ampère's law, for any circuit s bounding a surface S,

$$\oint_s \boldsymbol{H}\cdot d\boldsymbol{s} = 4\pi \int_S \boldsymbol{c}\cdot d\boldsymbol{S},$$

and since the left-hand side is equal, by Stokes's theorem, to

$$\int (\operatorname{curl} \boldsymbol{H})\cdot d\boldsymbol{S},$$

it follows that

$$\operatorname{curl} \boldsymbol{H} = 4\pi \boldsymbol{c}.$$

Hence, by (8),

$$\operatorname{curl} \boldsymbol{H} = -4\pi\kappa\frac{\partial A}{\partial t},$$

and further, by (6),

$$\nabla^2 A = 4\pi\kappa\mu\frac{\partial A}{\partial t}. \tag{9}$$

In non-conducting material this reduces to

$$\nabla^2 A = 0, \tag{10}$$

implying that $\operatorname{curl} \boldsymbol{B} = 0$, by (6), so that $\boldsymbol{B}$ is derivable from a magnetic potential V, i.e.

$$\boldsymbol{B} = \boldsymbol{H} = -\operatorname{grad} V, \tag{11}$$

and since $\operatorname{div} \boldsymbol{B} = 0$,

$$\nabla^2 V = 0. \tag{12}$$

22.11. Solutions for regions bounded by spheres. We seek solutions of equations (9) and (12) that are appropriate to regions whose boundaries are concentric spheres. It is therefore convenient to use spherical polar coordinates of position, r, θ, ϕ, as in 17.1, the origin being at the centre of the spheres. The corresponding components of vectors will be distinguished by the suffixes r, θ, ϕ.

In terms of such coordinates and components, the equation $\operatorname{div} A = 0$ is

$$\operatorname{div} A = \frac{\partial(r^2 A_r)}{r^2\,\partial r}+\frac{\partial(A_\theta \sin\theta)}{r\sin\theta\,\partial\theta}+\frac{\partial A_\phi}{r\sin\theta\,\partial\phi} = 0; \tag{13}$$

the r, θ, ϕ components of $\operatorname{curl} A$ are respectively

$$\frac{\partial(A_\phi \sin\theta)}{r\sin\theta\,\partial\theta}-\frac{\partial A_\theta}{r\sin\theta\,\partial\phi},\quad \frac{\partial A_r}{r\sin\theta\,\partial\phi}-\frac{\partial(rA_\phi)}{r\,\partial r},\quad \frac{\partial(rA_\theta)}{r\,\partial r}-\frac{\partial A_r}{r\,\partial\theta}. \tag{14}$$

22.12. Non-conducting regions. In non-conducting material $B = -\operatorname{grad} V$ (cf. (11)). Hence

$$B_r = -\frac{\partial V}{\partial r},\quad B_\theta = -\frac{\partial V}{r\,\partial\theta},\quad B_\phi = -\frac{\partial V}{r\sin\theta\,\partial\phi}.$$

Since $\nabla^2 V = 0$ (cf. (12)) V can be expressed as a series of spherical harmonic terms of the form

$$a(\rho^n E_n^m+\rho^{-n-1}I_n^m)S_n^m(\theta,\phi), \tag{15}$$

where a is a constant (a length), S_n^m is a spherical surface harmonic of degree n and order m, and

$$\rho = r/a; \tag{16}$$

E_n^m and I_n^m are functions of the time only, and we suppose that they are expressible as sums of *exponential* functions of the time, i.e.

$$E_n^m = \sum_\alpha E_n^{m\alpha}e^{-\alpha t},\qquad I_n^m = \sum_\alpha I_n^{m\alpha}e^{-\alpha t}, \tag{17}$$

for a series of known values of α, which may be real, complex, or imaginary. For each value of α there is a term in the expansion of V, and likewise of $-\operatorname{grad} V$ or B and its components. All these terms may be distinguished by the appropriate affixes n, m, α, but these will be omitted for simplicity in printing. We shall consider a typical term in V, namely

$$a(E\rho^n+I\rho^{-n-1})e^{-\alpha t}S_n^m. \tag{18}$$

At a spherical boundary ($r = a_1$ or $\rho = \rho_1$, say) between conducting and non-conducting regions, the normal magnetic induction (B_r) and the tangential magnetic force (H_θ, H_ϕ) must be continuous, and this applies to each of such n, m, α terms separately. The values of these quantities, at $r = a$, are as follows:

$$B_r = -\{nE\rho_1^{n-1}-(n+1)I\rho_1^{-n-2}\}e^{-\alpha t}S_n^m, \tag{19}$$

$$H_\theta = -\{E\rho_1^{n-1}+I\rho_1^{-n-2}\}e^{-\alpha t}\partial S_n^m/\partial\theta, \tag{20}$$

$$H_\phi = -\{E\rho_1^{n-1}+I\rho_1^{-n-2}\}e^{-\alpha t}\partial S_n^m/\sin\theta\,\partial\phi. \tag{21}$$

The surface harmonic S_n^m is of the form $P_n^m(\cos\theta)\cos(m\phi+\epsilon)$. It is convenient to regard E and I as complex constants, with the convention that in all our formulae we are dealing with the real part of each whole complex expression. The complex constants E and I can thus represent the phase as well as the amplitude of the external and internal harmonics, i.e.

$$E = E_0 e^{i\epsilon}, \qquad I = I_0 e^{i\iota}, \tag{22}$$

where E, I, E_0, I_0, ϵ, ι all bear the appropriate affixes n, m, α.

22.13. Conducting regions. The induction $\boldsymbol{B}$ and the vector potential A in the conducting region are likewise supposed expressed as sums of parts $\boldsymbol{B}^\alpha$, A^α, each depending on the time only through a time-factor $e^{-\alpha t}$ (the series of α's being the same as for the non-conducting region). (In $\boldsymbol{B}^\alpha$, A^α, $\boldsymbol{c}^\alpha$, the α is an affix, not an index.) Each such part must separately satisfy (9), which thus becomes

$$\nabla^2 A^\alpha + 4\pi\kappa\mu\alpha A^\alpha = 0,$$

or, if

$$k^2 = -4\pi\kappa\mu\alpha, \tag{23}$$

then

$$(\nabla^2 - k^2)A^\alpha = 0. \tag{24}$$

The corresponding part of $\boldsymbol{c}$, denoted by $\boldsymbol{c}^\alpha e^{-\alpha t}$, is given by

$$\boldsymbol{c}^\alpha = \kappa\alpha A^\alpha \tag{25}$$

by (8).

Lamb showed that the solutions of (24) and (3) which are appropriate to spherical boundaries are of two types. The one that is of importance here is a sum of terms of the form

$$A_n^{m\alpha} = ae^{-\alpha t}f_n(r)(\boldsymbol{r} \wedge \operatorname{grad} S_n^m), \tag{26}$$

where $\wedge$ is the sign for a vector product, S_n^m is, as before, a spherical surface harmonic of degree n and order m, $\boldsymbol{r}$ denotes the *vector* from the origin to the point (r, θ, ϕ), and $f_n(r)$ is a function of r only, which satisfies the equation

$$r^2\frac{d^2f_n}{dr^2}+2r\frac{df_n}{dr}-\{n(n+1)+k^2r^2\}f_n = 0. \tag{27}$$

(For remarks on the other type of solution see § 18.)

The r, θ, ϕ coordinates of $\boldsymbol{r}\wedge\operatorname{grad}S_n^m$ are

$$0,\quad -\frac{\partial S_n^m}{\sin\theta\,\partial\phi},\quad \frac{\partial S_n^m}{\partial\theta},$$

so that those of $A_n^{m\alpha}$ are

$$0,\quad -ae^{-\alpha t}f(r)\frac{\partial S_n^m}{\sin\theta\,\partial\phi},\quad ae^{-\alpha t}f(r)\frac{\partial S_n^m}{\partial\theta},\tag{28}$$

which also give, when multiplied by $\kappa\alpha$, the components of the corresponding part of $\boldsymbol{c}$, by (25).

On inserting the components of A from (28) in (13) we verify that (26) satisfies $\operatorname{div}A = 0$.

The components of $\operatorname{curl}A$ or $\boldsymbol{B}$ are found by substitution from (28) in (14); using the equation satisfied by S_n (cf. 17 (34–7), p. 618) we get (omitting affixes)

$$B_r = -\frac{1}{\rho}n(n+1)f(r)e^{-\alpha t}S_n^m,\tag{29}$$

$$B_\theta = -\frac{1}{\rho}\frac{d}{dr}\{rf(r)\}e^{-\alpha t}\frac{\partial S_n^m}{\partial\theta},\tag{30}$$

$$B_\phi = -\frac{1}{\rho}\frac{d}{dr}\{rf(r)\}e^{-\alpha t}\frac{\partial S_n^m}{\sin\theta\,\partial\phi}.\tag{31}$$

Inserting these again into equations similar to (14), but with $\boldsymbol{B}$ substituted for A, we find that $-\operatorname{curl}\boldsymbol{B}$ or, by (4, 5), $\nabla^2 A$, has no r component, while the θ and ϕ components are respectively $-\partial S_n^m/\sin\theta\,\partial\phi$ and $\partial S_n^m/\partial\theta$ multiplied by

$$\frac{a}{r}\left[\frac{d^2}{dr^2}\{rf(r)\}-n(n+1)\frac{f(r)}{r}\right]e^{-\alpha t}.$$

Substituting these components into (24), it is readily verified that this is satisfied subject to (27).

22.14. The conditions satisfied at a spherical boundary. At a spherical boundary $r = a_1$ or $\rho = \rho_1$, between conducting and non-conducting regions, B_r, H_θ, and H_ϕ must be continuous. This condition must be fulfilled for each n, m, α term separately. Hence, from (19–21), (29–31) we have

$$n\rho_1^n E-(n+1)\rho_1^{-n-1}I = n(n+1)f(a_1),\tag{32}$$

$$\mu\{\rho_1^n E+\rho_1^{-n-1}I\} = \left[\frac{d}{dr}\{rf(r)\}\right]_{r=a_1}.\tag{33}$$

22.15. The functions $f_n(r)$. The equation (27) can be expressed as a Bessel equation by changing the dependent and independent variables f_n and r to w and z, where

$$f_n = r^{-\frac{1}{2}}w,\qquad z = kr.$$

The equation then becomes

$$z^2\frac{d^2w}{dz^2}+z\frac{dw}{dz}-\{(n+\tfrac{1}{2})^2+z^2\}w = 0,$$

of which the general solution is

$$w = AI_{n+\frac{1}{2}}(z)+BK_{n+\frac{1}{2}}(z),$$

in the standard notation of Bessel functions. Consequently

$$f_n(r) = r^{-\frac{1}{2}}\{AI_{n+\frac{1}{2}}(kr)+BK_{n+\frac{1}{2}}(kr)\}.$$

It is convenient, however, to give some details of the direct solution of (27). This equation is analytic at all points except $r = 0$, which however is a regular point. The indicial equation has the roots n, $-n-1$, so that

$$f_n(r) = Cr^n\phi_n(r)+Dr^{-n-1}\phi_{-n-1}(r), \tag{34}$$

where C and D are arbitrary constants, and ϕ_n and ϕ_{-n-1} are integral functions, which satisfy the respective equations

$$r^2\frac{d^2\phi_s}{dr^2}+2(s+1)r\frac{d\phi_s}{dr}-k^2r^2\phi_s(r) = 0 \tag{35}$$

for $s = n$ and $s = -n-1$ respectively; these equations are obtained by substituting $r^n\phi_n$ or $r^{-n-1}\phi_{-n-1}$ for f in (27). It is clear that ϕ_{-n-1} must be arbitrary to the extent of any multiple of $r^{2n+1}\phi_n$, since such an addition to ϕ_{-n-1} only alters the arbitrary constant C in (34). Hence (35) may be expected to have two integral solutions when $s = -n-1$. It may easily be verified that when $s = n$ (> 0), only one solution is integral. This is of the form (valid whether s is positive or negative)

$$F_s(k^2r^2) = 1+\frac{(kr)^2}{2(2s+3)}+\frac{(kr)^4}{2.4(2s+3)(2s+5)}+\dots = \frac{\Gamma(s+\frac{3}{2})}{(\frac{1}{2}ikr)^{s+\frac{1}{2}}}J_{s+\frac{1}{2}}(ikr). \tag{36}$$

When $s = -n-1$, the solution of (35) has the form $CF_{-n-1}+Dr^{2n+1}F_n$, where C and D are arbitrary constants. Note that $F_n(k^2r^2)$ corresponds to the $R_n(k_r)$ of Chapman and Whitehead [9], and to the $R_n(-k^2r^2)$ of Price [11]; $G_n(kr)$, introduced in (40), corresponds to the $R_{-n-1}(kr)$ of Chapman and Whitehead.

A solution of (27) may also be obtained in the simple form

$$f = Ce^{kr}\rho_n(kr),$$

where C is an arbitrary constant; $\rho_n(x)$, which satisfies the equation

$$x^2\frac{d^2\rho}{dx^2}+2(x+x^2)\frac{d\rho}{dx}-[n(n+1)-2x]\rho = 0, \tag{37}$$

(obtained from (27) by substitution of the present expression for f) proves to be a finite polynomial in $1/x$, of degree $n+1$, i.e.

$$\rho_n(x) = \frac{1}{2x} - \frac{n(n+1)}{(2x)^2} + \frac{(n-1)n(n+1)(n+2)}{2!\,(2x)^3} - \dots. \tag{38}$$

Since (27) is unaltered by changing the sign of k, another and independent solution of (27) is $De^{-kr}\rho_n(-kr)$. Both these solutions are of the form $r^{-n-1}\phi_{-n-1}(r)$. The solution $r^nF_n(k^2r^2)$ already found from (27) must clearly be a combination of these two new forms, and it is not difficult to verify that

$$(kr)^nF_n(k^2r^2) = \frac{(2n+1)!}{2^n.n!}\{e^{kr}\rho_n(kr)+(-1)^ne^{-kr}\rho_n(-kr)\}. \tag{39}$$

For the second part of f in (34), which, as shown above, is ambiguous to the extent of a multiple of $r^{2n+1}F_n$, we adopt the form

$$(kr)^{-n-1}G_n(kr) = -\frac{2^n(n-1)!}{(2n-1)!}e^{-kr}\rho_n(-kr). \tag{40}$$

Reversing the order of the terms in the polynomial (38) for ρ_n, we have

$$G_n = e^{-kr}\left\{1+kr+\frac{2(n-1)}{2!\,(2n-1)}(kr)^2+\frac{2^2(n-2)(n-1)}{3!\,(2n-2)(2n-1)}(kr)^3+\dots\right\}. \tag{41}$$

It is clear from (38, 39) that F_n must be expressible as a sum of hyperbolic functions each divided by some power of kr; for example,

$$F_0 = \frac{\sinh kr}{kr}, \qquad F_1 = \frac{3}{k^2r^2}\left(\cosh kr - \frac{\sinh kr}{kr}\right). \tag{42}$$

Moreover, from (36) it is readily shown that

$$F_{n+1} = \frac{2n+3}{k^2r}\frac{dF_n}{dr}, \tag{43}$$

$$F_{n-1} = F_n + \frac{r}{2n+1}\frac{dF_n}{dr}. \tag{44}$$

From (41) we find

$$G_0 = e^{-kr}, \qquad G_1 = e^{-kr}(1+kr), \tag{45}$$

and also we can readily prove the following relations, analogous to those for F_n:

$$G_{n-1} = -\frac{2n-1}{k^2r}\frac{dG_n}{dr}, \tag{46}$$

$$G_{n+1} = G_n - \frac{r}{2n+1}\frac{dG_n}{dr}. \tag{47}$$

Again, from (46), we find

$$\frac{G_{n-1}}{G_n} = -\frac{2n-1}{k^2r}\frac{d\log G_n}{dr}, \tag{48}$$

and by expressing $\log G_n$ as a power series we deduce that

$$\frac{G_{n-1}}{G_n} = \frac{2n-1}{kr}\left\{1-\frac{n}{kr}+\frac{n(n+1)}{2(kr)^2}-\frac{n(n+1)}{2(kr)^3}-\frac{(n-2)n(n+1)(n+3)}{8(kr)^4}\ldots\right\}. \tag{49}$$

Since, by (40),
$$\frac{G_{n-1}}{G_n} = \frac{(2n-1)}{kr}\frac{\rho_{n-1}(-kr)}{\rho_n(-kr)}, \tag{50}$$

the second factors on the right of (49, 50) are equal. Hence by reversing the sign of kr in the bracket of (49), we obtain $\rho_{n-1}(kr)/\rho_n(kr)$.

When the real part of kr is large and positive, $(kr)^nF_n$ is approximately equal to

$$\frac{(2n+1)!}{2^n.n!}e^{kr}\rho_n(kr), \tag{51}$$

and then also we have the approximate relation

$$\frac{F_{n-1}(k^2r^2)}{F_n(k^2r^2)} \doteqdot \frac{kr}{2n+1}\frac{\rho_{n-1}(kr)}{\rho_n(kr)} = \frac{kr}{2n+1}\left\{1+\frac{n}{kr}+\right.$$
$$\left.+\frac{n(n+1)}{2(kr)^2}+\frac{n(n+1)}{2(kr)^3}-\frac{(n-2)n(n+1)(n+3)}{8(kr)^4}\ldots\right\}. \tag{52}$$

The proportionate error in (51) is, by comparison with (39), of magnitude $e^{-2kr}\rho_n(-kr)/\rho_n(kr)$: if the real part of kr exceeds 7·5, $|e^{-2kr}| < e^{-15} = 10^{-6\cdot 51}$, which is very small. Hence, unless by some chance the ratio $\rho_n(-kr)/\rho_n(kr)$ happens to be large, (51) will be valid for such values of kr.

22.16. Induction in a uniform solid sphere. The preceding analysis will first be applied to the case of a uniform solid sphere of radius qa, surrounded by non-conducting matter or free space. Since the field must be finite at $r = 0$, $f(r)$ will consist only of the part Cr^nF_n, of positive degree, or (changing C)

$$f_n(r) = C\rho^nF_n. \tag{53}$$

Hence, by (28, 25), the components of the n, m, α term in $\mathbf{c}$ are

$$0,\quad -\kappa\alpha aCe^{-\alpha t}\rho^nF_n\frac{\partial S}{\sin\theta\,\partial\phi},\quad \kappa\alpha aCe^{-\alpha t}\rho^nF_n\frac{\partial S}{\partial\theta}. \tag{54}$$

The boundary conditions (32, 33) at $r = qa$, $\rho_1 = q$, are therefore

$$nE-(n+1)q^{-2n-1}I = n(n+1)CF_n(k^2q^2a^2) \tag{55}$$

and $$\mu\{E+q^{-2n-1}I\} = C\left\{(n+1)F_n(k^2q^2a^2)+r\frac{dF_n}{dr}\right\}$$

$$= C\{(2n+1)F_{n-1}(k^2q^2a^2)-nF_n(k^2q^2a^2)\}, \tag{56}$$

after using (44). Consequently, provided that E is not zero,

$$\frac{I}{E} = \frac{n}{n+1}q^{2n+1}\left[1-\mu\left\{\frac{F_{n-1}(k^2q^2a^2)}{F_n(k^2q^2a^2)}+\frac{n(\mu-1)}{2n+1}\right\}^{-1}\right], \tag{57}$$

$$\frac{C}{E} = \frac{\mu}{n+1}\left\{F_{n-1}(k^2q^2a^2)+\frac{n(\mu-1)}{2n+1}F_n(k^2q^2a^2)\right\}^{-1}. \tag{58}$$

22.17. The 'free' current-systems. If E is known, corresponding to a given external inducing field, these two equations determine I and C, which may be regarded as giving, for the spaces outside and within the sphere, the particular integral of the differential equations of induction. But these equations have solutions, infinite in number, even when $E = 0$; they correspond to the complementary function in the solution of an ordinary linear differential equation, or to the free vibrations of an elastic system. When $E = 0$, the number α in the exponential time-factor $e^{-\alpha t}$ associated with E (and hence also with I and C) ceases to be assigned; thus k also, which depends on α (cf. 23), is unknown. But when $E = 0$, the equations (55, 56) are equivalent to two equations, one giving the ratio I/C for any assigned k, while the other is independent of I/C, and represents a condition to be satisfied by k. This equation is

$$(2n+1)F_{n-1}(k_{ns}^2q^2a^2)+n(\mu-1)F_n(k_{ns}^2q^2a^2) = 0, \tag{59}$$

where the suffix to k indicates that the equation is satisfied only by certain definite values of $k^2q^2a^2$. These are real, negative, and infinite in number; we suppose that the suffix s $(= 1, 2, 3,\ldots)$ is assigned to them in ascending order of magnitude.

Writing $$x_{ns}^2 = -k_{ns}^2q^2a^2 = 4\pi\kappa\mu q^2a^2\alpha_{ns}, \tag{60}$$

(59) is equivalent (cf. 36, 44) to

$$x_{ns}J'_{n+\frac{1}{2}}(x_{ns})+(n\mu+\tfrac{1}{2})J_{n+\frac{1}{2}}(x_{ns}) = 0. \tag{61}$$

Since $k_{ns}^2q^2a^2$ is negative, α_{ns} is real and positive, so that the associated current-systems decay exponentially without change of type. (When $\mu = 1$, (59) is equivalent to $F_{n-1}(-x_{ns}^2) = 0$ or $J_{n-\frac{1}{2}}(x_{ns}) = 0$.)

To each value of α_{ns} (or k_{ns}^2) there corresponds a value of the ratio I/C, given by

$$I_{ns} = -nq^{2n+1}C_{ns}F_n(k_{ns}^2q^2a^2) = -nq^{2n+1}C_{ns}F_n(-x_{ns}^2). \tag{62}$$

The absolute values of I_{ns} and C_{ns} are arbitrary to the extent of a numerical factor, the same for both. The infinite series of freely decaying current-systems associated with any given integers n and m enable the initial conditions of any problem of induction in the sphere to be satisfied.

22.18. Lamb's second solution for A. Lamb's second solution of (24) corresponds to a current-system in which the flow is not in general perpendicular to the radius vector, and which possesses no magnetic field outside. Such solutions exist only for certain special (positive) values of α; they are the solutions of the equation

$$F_n(-4\pi\kappa\mu q^2a^2\alpha) = 0.$$

Changes in the magnetic field outside the sphere cannot generate or affect such currents. (This is not quite true if we take account of the finite velocity (c) of propagation of electromagnetic waves. But the necessary modification is unimportant, if α is small compared with c/a, which is approximately 47 for a sphere of the size of the earth. Now e^{-47t} decreases to one-tenth of its value in 0·05 second, so that only those free current-systems would be affected for which the rates of decay are extremely rapid.) Consequently, and also because such currents, if they existed, would not be discovered by surface observations of the field, they need not be discussed here.

22.19. Periodic induction in a uniform solid sphere. We now consider induction by a purely periodic external field, which is proportional to e^{ipt} (where p is real) and depends only on local time. In the case of the solar daily variations, in so far as these depend solely on local time, $p = m$, the order of the harmonic S_n^m, that is, the time-factor is e^{imt}, if time is reckoned in angular measure (2π per day); if, however, as we shall suppose, t is measured in seconds, the value of α in $e^{-\alpha t}$ will be given by

$$\alpha = -\frac{2\pi im}{24\times60\times60}. \tag{63}$$

(In the case of the lunar daily variations, the factor 24 in (63) has to be replaced by 25·74, 24·84, 24·56, 24·41, according as $m = 1, 2, 3, 4$, i.e. by $24/(1-2/mM)$, where $M = 29{\cdot}53$, the length of the mean synodic month in mean solar days; the second of these numbers, 24·84, is the mean length of the lunar day in solar hours.)

We write $$k^2q^2a^2 = 2i\beta^2, \qquad \beta^2 = \frac{4\pi^2\kappa\mu mq^2a^2}{24.60.60} \tag{64}$$

(cf. 23), so that β^2 is real and positive. Then

$$kqa = \beta(1+i), \qquad 1/kqa = (1-i)/2\beta. \tag{65}$$

It will appear later that, in connexion with the daily magnetic variations, the lowest value of β (the real part of kqa) exceeds 7·5, so that the approximations of (51) and (52) may be accepted as valid. Writing (52) in the form

$$\frac{F_{n-1}(k^2q^2a^2)}{F_n(k^2q^2a^2)} = A'+iB, \tag{66}$$

where A' and B, which are real, are given by

$$A' = \frac{\beta}{2n+1}\left\{1+\frac{n}{\beta}+\frac{n(n+1)}{4\beta^2}+\frac{0}{4\beta^3}\cdots\right\}, \tag{67}$$

$$B = \frac{\beta}{2n+1}\left\{1-\frac{n(n+1)}{4\beta^2}-\frac{n(n+1)}{4\beta^3}\cdots\right\}, \tag{68}$$

we deduce from (57) that

$$\frac{E}{I} = \frac{n+1}{n}q^{-2n-1}\left\{\frac{A+iB}{A-\mu+iB}\right\}, \tag{69}$$

where $$A = A'+\frac{n(\mu-1)}{2n+1} = \frac{\beta}{2n+1}\left\{1+\frac{n\mu}{\beta}+\frac{n(n+1)}{4\beta^2}+\cdots\right\}. \tag{70}$$

The ratio E/I is a complex quantity, expressible as $(E_0/I_0)\exp i(\epsilon-\iota)$, in the notation of (22) or, in the notation of Chapter XX, with appropriate affixes,

$$\frac{E}{I} = \frac{e_n^m}{i_n^m}\exp i(\epsilon_n^m-\iota_n^m). \tag{71}$$

From (69) we infer that

$$\frac{e_n^m}{i_n^m} = \frac{n+1}{n}q^{-2n-1}\left\{\frac{A^2+B^2}{(A-\mu)^2+B^2}\right\}^{\frac{1}{2}}, \tag{72}$$

$$\epsilon_n^m-\iota_n^m = \tan^{-1}\frac{B}{A}-\tan^{-1}\frac{B}{A-\mu}. \tag{73}$$

For the lowest value of β that occurs in our applications, which is about 7·9, n is 1 or 2, so that $n(n+1)/4\beta^2$ does not exceed 1/41; the highest value of n that we need to consider is 5, and since the associated value of β is then twice as great as before, $n(n+1)/4\beta^2$ in this case does not exceed 1/50. These values are small enough to justify the neglect of the term $n(n+1)/4\beta^2$ and later terms in (68) and (70), in comparison with the preceding terms (1 or $1+n\mu/\beta$); (72), (73) then become approximately equivalent to

$$\frac{e_n^m}{i_n^m} = \frac{n+1}{n}q^{-2n-1}\left\{1+\frac{(2n+1)(\frac{1}{2}-\beta/\mu)}{\beta^2/\mu^2+n\beta/\mu+\frac{1}{2}n^2}\right\}^{-\frac{1}{2}}, \tag{74}$$

$$\epsilon_n^m-\iota_n^m = -\tan^{-1}\frac{(2n+1)\beta/\mu}{2\beta^2/\mu^2-\beta/\mu-n(n+1)}. \tag{75}$$

These expressions depend only on n, q, and β/μ, which is

$$\frac{2\pi q a\sqrt{m}}{\sqrt{(24.60.60)}}\sqrt{\frac{\kappa}{\mu}}; \tag{76}$$

hence they show that, to this degree of approximation, the field induced by a given external periodic field depends only on the ratio κ/μ, and not on κ and μ separately. Not until the approximation is pushed farther do (72) and (73) come to involve β separately from β/μ, and consequently $\kappa\mu$ separately from κ/μ.

The current-density $\mathbf{c}$ is given (cf. 25, 26, 53, 58) by

$$\frac{2\pi i\kappa m a}{24.60.60} C\rho^n F_n(k^2r^2)e^{-\alpha t}(\mathbf{r}\wedge\operatorname{grad} S_n^m); \tag{77}$$

when $\mu = 1$, C is specially simple, its value being $1/(n+1)F_{n-1}(k^2q^2a^2)$.

In the present periodic case we are usually not concerned with any initial conditions; we are interested only in the induced currents and their field, after any transient 'free' current-systems, produced in first setting up the periodic external field, have died away. Thus the solutions of § 17 need not be considered here.

22.20. Aperiodic induction in a uniform solid sphere. The general analysis of § 16 will next be applied to induction in a uniform solid sphere, during the exponential rise of an external field to an ultimately steady value. Thus we suppose that initially there is no current flowing within the sphere, and that a particular harmonic S_n^m in the external field varies with the time as $(1-e^{-\alpha t})$ for $t \geqslant 0$, and is zero for $t \leqslant 0$: α is here to be real and positive. To satisfy this condition it is necessary to take the free current-systems of § 17 into account, as well as the induced system with the 'external' time-factor $e^{-\alpha t}$.

By means of a combination of time-factors $\sum a_s(1-e^{-\alpha_s t})$ it is possible to approximate to any ordinary function of the time, whose value is initially zero: in particular, to a function which finally tends to zero, like e in 20.11. This function e is in fact the application in view in the present section.

Thus we consider a term in the potential of the external field, given by

$$aE\rho^n(1-e^{-\alpha t})S_n^m. \tag{78}$$

The first part of this expression corresponds to a constant field; this induces a constant magnetization in the core (if $\mu \neq 1$), giving a constant magnetically induced field having a potential

$$-aEq^{2n+1}\rho^{-n-1}\frac{n(\mu-1)}{n\mu+n+1}S_n^m; \tag{79}$$

no *electromagnetically* induced field is associated with the first term of (78).

The second term of (78), $-aE\rho^n e^{-\alpha t} S_n^m$, is of the form considered in §§ 12–16. In the region $r > qa$ the potential of the associated field, induced both magnetically and electromagnetically, consists of a 'particular integral' term (§ 17) whose value is

$$-aI\rho^{-n-1}e^{-\alpha t}S_n^m, \tag{80}$$

where I is given by (57) together with a series of 'complementary function' terms (§ 17) of the form

$$-a\rho^{-n-1}S_n^m \sum_{s=1}^{\infty} I_{ns}\, e^{-\alpha_{ns} t}; \tag{81}$$

here α_{ns} is one of an infinite series of real positive numbers, given by (60) in terms of $k_{ns}^2 q^2 a^2$, which is a solution of (59); I_{ns} may have any value so far as concerns the equations of induction, but is determined by the initial conditions of the problem.

In the present case the initial conditions are that the induced current-intensity $\mathbf{c}$, and the induced field, are everywhere zero. As regards $\mathbf{c}$, the radial component c_r is identically zero, while (cf. (54) and (62))

$$c_\phi = \kappa a\rho^n \frac{\partial S_n^m}{\partial\theta}\Big[-\alpha C F_n(-4\pi\kappa\mu\alpha r^2)e^{-\alpha t} - \\ -\frac{1}{nq^{2n+1}} \sum_{s=1}^{\infty} \alpha_{ns}\, I_{ns} \frac{F_n(-4\pi\kappa\mu\alpha_{ns} r^2)}{F_n(-4\pi\kappa\mu\alpha_{ns} q^2 a^2)}\, e^{-\alpha_{ns} t}\Big], \tag{82}$$

wherein C is given by (58); c_θ is obtained from c_ϕ by substituting $-\partial S/\sin\theta\,\partial\phi$ for $\partial S/\partial\theta$. Thus $\mathbf{c}$ will vanish everywhere, when $t = 0$, if for all values of r from 0 to qa

$$\frac{1}{nq^{2n+1}} \sum_{s=1}^{\infty} \alpha_{ns} I_{ns} \frac{F_n(k_{ns}^2 r^2)}{F_n(k_{ns}^2 q^2 a^2)} + \alpha C F_n(k^2 r^2) = 0. \tag{83}$$

(In (82) the negative signs before C and I_{ns} come from the negative signs in (78) and (62).)

The potential of the induced field in the region $r > qa$ is

$$a\rho^{-n-1}\phi_n^m(t) S_n^m, \tag{84}$$

where $$\phi_n^m(t) = -\frac{n(\mu-1)}{n\mu+n+1} E q^{2n+1} - I e^{-\alpha t} + \sum_{s=1}^{\infty} I_{ns} e^{-\alpha_{ns} t}, \tag{85}$$

I being given by (57). The initial condition

$$\phi_n^m(0) = 0, \tag{86}$$

together with (83), suffices to determine all the coefficients I_{ns} as multiples of E.

The determination of I_{ns} may be effected in more than one way. The functions F_n in (83) may be expanded in powers of r^2; since the total coefficient of each power of r in (83) must vanish, this gives an infinite set of equations, from which the I's may be derived as ratios of infinite determinants. This method, for $\mu = 1$, has been worked out by Price. Later he gave a second method, without restriction to $\mu = 1$, using the Heaviside operational calculus. Only the results will be quoted here.

The explicit form of (85) is found to be

$$\phi_n^m(t) = \frac{nq^{2n+1}}{n+1}\Bigg[-\frac{(n+1)(\mu-1)}{n\mu+n+1}(1-e^{-\alpha t})+ \\ +2(2n+1)\mu k^2\sum_{s=1}^{\infty}\frac{(e^{-\alpha_{ns}t}-e^{-\alpha t})}{\{n(\mu-1)(n\mu+n+1)-k_{ns}^2q^2a^2\}(k^2-k_{ns}^2)}\Bigg]. \tag{87}$$

An alternative form of (87) is

$$\phi_n^m(t) = \frac{nq^{2n+1}}{n+1}\Bigg[-e^{-\alpha t}+ \\ +(2n+1)\mu\frac{e^{-\alpha t}}{(2n+1)\{F_{n-1}(k^2q^2a^2)/F_n(k^2q^2a^2)\}+n(\mu-1)}+ \\ +2(2n+1)\mu k^2\sum_{s=1}^{\infty}\frac{e^{-\alpha_{ns}t}}{\{n(\mu-1)(n\mu+n+1)-k_{ns}^2q^2a^2\}(k^2-k_{ns}^2)}- \\ -\frac{(n+1)(\mu-1)}{n\mu+n+1}\Bigg]. \tag{87a}$$

Likewise the expression found for c_ϕ is

$$c_\phi = \frac{\kappa Ea}{n+1}\rho^n\frac{\partial S_n^m}{\partial\theta}\Bigg\{-\alpha\frac{F_n(k^2r^2)}{F_{n-1}(k^2q^2a^2)}e^{-\alpha t}- \\ -2(2n+1)\sum_{s=1}^{\infty}\frac{\mu\alpha_{ns}k^2F_n(k_{ns}^2r^2)e^{-\alpha_{ns}t}}{\{n(\mu-1)(n\mu+n+1)-k_{ns}^2q^2a^2\}(k^2-k_{ns}^2)F_n(k_{ns}^2q^2a^2)}\Bigg\},$$

which reduces, when $\mu = 1$, to

$$c_\phi = \frac{\kappa Ea\alpha}{n+1}\rho^n\frac{\partial S_n^m}{\partial\theta}\Bigg\{-\frac{F_n(k^2r^2)}{F_{n-1}(k^2q^2a^2)}e^{-\alpha t}+ \\ +\frac{2(2n+1)}{q^2a^2}\sum_{s=1}^{\infty}\frac{F_n(k_{ns}^2r^2)e^{-\alpha_{ns}t}}{(k^2-k_{ns}^2)F_n(k_{ns}^2q^2a^2)}\Bigg\}. \tag{88}$$

Should k coincide with one of the numbers k_{ns}, say with k_{np}, the term $s = p$ must be omitted in (88), and the first term within the bracket must be replaced by

$$[F_n(k^2r^2)\alpha t - \tfrac{1}{4}(2n+1)\{2F_{n-1}(k^2r^2) - F_n(k^2r^2)\}]\frac{2(2n+1)e^{-\alpha t}}{k^2q^2a^2F_n(k^2q^2a^2)}.$$

Likewise in (87) the term $s = p$ must be omitted from the sum, and

$$\frac{-2(2n+1)\mu k^2te^{-\alpha_{np}t}}{\{n(\mu-1)(n\mu+n+1)-k^2q^2a^2\}4\pi\kappa\mu} \tag{87 b}$$

added to the first term. In a sense this case corresponds mathematically to resonance in a vibrating system, but in the present case there is of course no physical phenomenon analogous to resonance, because ϕ_n^m, regarded as a function of k, has in general no exceptional character when k coincides with k_{ns} for any s.

When $\mu = 1$, $n = 1$, $m = 0$, as in the case of the main harmonic in the D_{st} field, the equation (61) for the roots x_{ns} reduces to $J_{\frac{3}{2}}(x) = 0$ or $\sin x = 0$, so that

$$x_{ns} = s\pi, \qquad k_{ns}^2 = -s^2\pi^2/q^2a^2,$$
$$\alpha_{ns} = s^2\pi^2/4\pi\kappa q^2a^2 = s^2A, \tag{89}$$

where, as in § 7, $A = \pi/4\kappa q^2a^2.$

If in (87) $\alpha = \alpha_{np} = p^2A$, the expression (87, 87 b) for the function $\phi_n^m(t)$, which now ($n = 1$, $m = 0$, $\mu = 1$, $\alpha = p^2A$) will be denoted by $\phi_p(t)$, may readily be transformed to

$$\phi_p(t) = \frac{3q^3}{\pi^2}\left[\left\{At + \frac{21-2p^2\pi^2}{12p^2}\right\}e^{-p^2At} - p^2\sum_{s=1}^{\infty}{}' \frac{e^{-s^2At}}{s^2(s^2-p^2)}\right], \tag{87 c}$$

the term $s = p$ being omitted from the summation.

Price has shown that the function $\phi_n^m(t)$ has the following properties:

(*a*) $\phi(0) = 0$ and $\phi'(0) = nq^{2n+1}\alpha/(n+1)$, which is essentially positive; hence (84) has initially the same sign as (78).

(*b*) $\phi(t) \to -nq^{2n+1}(\mu-1)/(n\mu+n+1)$ as $t \to \infty$; this corresponds to the field of the permanent magnetization induced by the ultimately steady external field; it vanishes if $\mu = 1$.

(*c*) $\phi(t)$ has only one maximum value, which occurs at the time t such that $\phi(t) = \Phi(t)$, where

$$\Phi(t) \equiv \frac{nq^{2n+1}}{n+1}\left\{2(2n+1)\mu\sum_{s=1}^{\infty}\frac{e^{-\alpha_{ns}t}}{n(\mu-1)(n\mu+n+1)-k_{ns}^2q^2a^2} - \frac{(n+1)(\mu-1)}{n\mu+n+1}\right\}. \tag{90}$$

The function $\Phi(t)$ is the limit to which $\phi(t)$ tends as $\alpha \to \infty$, i.e. as the change in the external field becomes instantaneous; $\Phi(t)$ decreases steadily from $nq^{2n+1}/(n+1)$ to $\phi(\infty)$ as t increases from 0 to ∞. It follows that the graphs of $\phi(t)$ and $\Phi(t)$ are as shown in Fig. 4; the maximum of $\phi(t)$ will be less, the slower the rate of change of the external field.

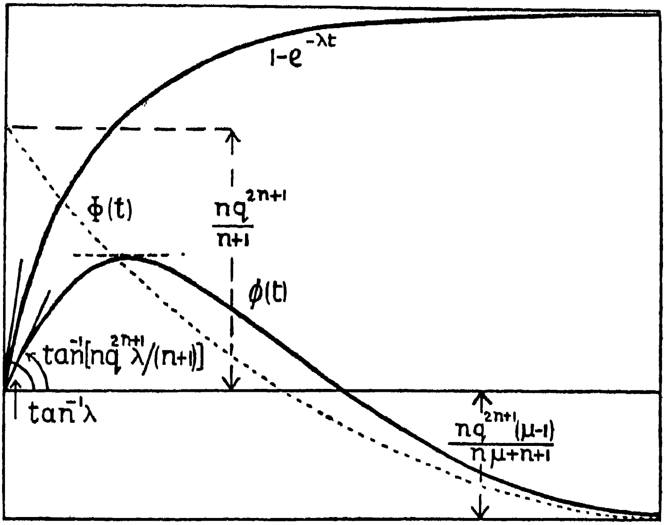


FIG. 4

The series in the expressions (87), (87 a) for ϕ, and (90) for Φ, converge rather slowly when t is small. Price has shown how to obtain alternative expressions which converge rapidly for small values of t. These expressions are very simple when $n = \mu = 1$, namely

$$\phi_1(t) = \tfrac{1}{2}(1-e^{-\alpha t})-\frac{3}{2k^2}(1-\alpha t-e^{-\alpha t})+$$

$$+\frac{2}{k}\sqrt{\frac{(\alpha^3 t^3)}{\pi}}\left\{1-\frac{2}{5}\alpha t+\frac{2.2}{5.7}(\alpha t)^2-\frac{2.2.2}{5.7.9}(\alpha t)^3...\right\}, \tag{91}$$

$$\Phi_1(t) = \frac{1}{2}-\frac{3}{2\pi qa}\sqrt{\left(\frac{t}{\kappa}\right)}+\frac{3t}{8\pi\kappa q^2 a^2}. \tag{92}$$

When the function Φ has been determined, the induced field corresponding to *any* inducing field $e(t)$ can be found without expressing $e(t)$ in terms of exponentials, by using the definite integral formula

$$i(t) = \frac{d}{dt}\int\limits_0^t e(t-u)\Phi(u)\,du = e(t)\Phi(0)+\int\limits_0^t e(t-u)\Phi'(u)\,du, \tag{93}$$

given by Price [11, p. 244].

22.21. Plane-earth induction problems. The preceding discussion, of 'spherical' induction problems is necessary for an accurate treatment of the induction of *world-wide* systems of earth-currents, by world-wide varying geomagnetic fields (like S, L, and parts of the D field). The treatment becomes increasingly difficult, however, the greater the local variations in the field-distribution. Hence it seems impracticable, without immense labour and the inclusion of very many spherical harmonic terms, to discuss accurately the earth-currents associated with the D field near the auroral zones. In such cases it is often possible to make some progress by considering a 'model' in which the local features of the field are represented, but relative to a plane earth. Several simplified induction problems intended to illustrate earth-current phenomena in polar latitudes have been considered by Price, Vestine, and El Wakil, but the results (some of which are not yet published) will not be included here.

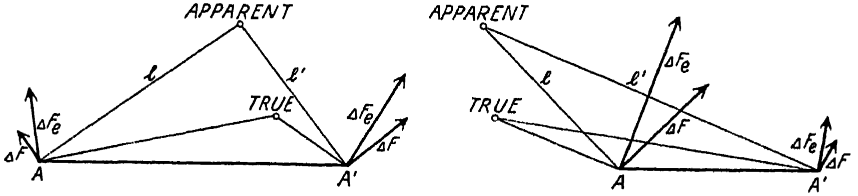


FIG. 5. Illustrating the apparent and true heights of a linear electric current in the upper atmosphere, flowing parallel to the ground (perpendicularly into the paper), as inferred from the magnetic disturbing vector observed at two points A, A′ on the ground, in a plane normal to the current. The arrows marked ΔF_e represent the true magnetic vectors due to the external current; lines drawn at right angles to them give the true location of the current. The vectors ΔF are the observed magnetic disturbing vectors, which are the vector sums of ΔF_e and of ΔF_i, the vector due to induced currents in the ground; ΔF_i has a horizontal component in the same sense as ΔF_e, and a vertical component in the opposite sense. The apparent location of the overhead current, inferred from ΔF, is too high

In conclusion, we may refer to the currents induced in the earth (treated as having an infinite plane surface) by a varying current i along an infinite rectilinear path parallel to the earth's surface; if i is periodic, the field of the induced currents, at the earth's surface, will be periodic with the same period; apart from a moderate difference of phase, they will increase the horizontal and decrease the vertical component of the external periodic field. Chapman [9.19] has pointed out that this will cause the height of the currents along the auroral zone, when calculated by the method of Birkeland (p. 318), to be overestimated. This is illustrated by Fig. 5.

XXIII

THEORIES OF THE SOLAR AND LUNAR DAILY MAGNETIC VARIATIONS (S_q AND L)

A. THE DYNAMO THEORY OF S AND L

23.1. The external part of the S_q and L fields. In this chapter we discuss the physical theories that have been proposed to account for the daily magnetic variations, solar and lunar. The part of their fields which is of internal origin is attributed to electric currents induced in the earth (Ch. XXII) by the external part, so that only the latter has to be considered here.

The solar daily variation may be divided into a part S_q, observable in its pure form on quiet days, though present also on other days, and a part S_D associated with magnetic disturbance; S_D will be discussed in Chapter XXV, as part of the disturbance field. In this chapter, therefore, the discussion relates to the external part of the S_q and L fields.

It is convenient to take as our starting-point the very able and lucid discussion by Stewart [1] in an article on 'Terrestrial Magnetism' in the *Encyclopaedia Britannica*, 9th ed., 1882. He did not distinguish between S and S_q, but this does not affect his arguments.

23.2. Untenable hypotheses. Stewart dismissed the hypothesis [16] that the daily magnetic variations are due to direct magnetic action by the sun, because the observed variations are not of the type that such action would produce. From Hale's researches on the sun's magnetic field (5.13) it would appear that the sun may possess a magnetic field, but that this is altogether too weak to produce an observable daily variation at the earth's surface; its intensity at the sun's surface is of the order 50 Γ, so that, if it decreases as the inverse cube of the distance, its intensity at the earth cannot exceed $0{\cdot}4\gamma$; actually the Mount Wilson researches indicated that the sun's magnetic field decreases outwards still faster than this (presumably because of electric currents in its atmosphere), though there seems to be some doubt as to these results.

Faraday [14] found that oxygen is paramagnetic, and becomes less so when heated; he suggested that the change in the paramagnetism of the oxygen of the atmosphere, due to the daily heating and cooling, may be responsible for the observed daily variation of the earth's field. Stewart pointed out that this associates S with the main mass of the atmosphere, and that the sun's heating effect on the atmosphere as a whole cannot vary to the extent required (on Faraday's hypothesis) to explain the variations of S in the course of the sunspot cycle. Again, the hypothesis would involve a well-marked annual variation of the earth's field, of opposite phase in the northern and southern hemispheres: actually the annual and semi-annual variations in the field are of a different nature, and very small. Schuster subsequently added a further argument, which is conclusive against Faraday's hypothesis; the paramagnetism of the air is so small that the change in the surface field would be insensible even if the whole of the atmosphere were removed.

Christie, Moser, and de la Rive supposed [15] that the sun's heat produces thermo-electric currents in the atmosphere and the earth's crust, and that their magnetic field is responsible for S. Stewart remarked that 'it is not easy to perceive how we could have thermo-electric currents in the upper regions of the atmosphere', and rejected the idea that such currents in the earth's crust can be the cause of S, because it seems impossible to allow that any solar heating action on the earth's crust can be as much greater at sunspot-maximum than at sunspot-minimum as S is found to be.

Schuster's subsequent proof [2] of the external origin of S precludes the view, which has often been proposed, that S is primarily due to earth-currents. In his second paper [3] on S, Schuster disposed of another suggested cause, showing that neither the regular nor irregular magnetic variations can be due to the motion of masses of electrified (charged) air, on any scale likely to occur in the atmosphere.

✓ **23.3. Stewart's dynamo theory.** Stewart concluded 'by the method of exhaustions' (§ 128) that the *upper* atmosphere is the most probable seat of the solar action that produces S; his chief reason was that neither the solid earth nor the lower atmosphere is affected by the sun in any way that changes greatly in amount, like S, from sunspot-minimum to maximum. He further remarked that 'the only conceivable magnetic cause capable of operating in such regions must be an electric current', and 'we know from our study of the aurora that there are such currents in these regions—continuous near the poles and occasional in lower latitudes'. Hence he proposed, as the primary cause of S, 'that convective currents established by the sun's heating influence in the upper regions of the atmosphere are to be regarded as conductors moving across lines of magnetic force, and are thus the vehicle of electric currents which act upon the magnet' (§ 124). It might be objected that these regions were insufficiently conducting, but Stewart thought there was ground for imagining that 'their conductivity may be much greater than has hitherto been supposed' (§ 128), and quoted (§ 141) some experiments by Hittorf on the subject of conductivity in rarefied gases.

It is convenient to term Stewart's hypothesis the (atmospheric) *dynamo* theory, because the supposed action is similar to that of a dynamo, in which the earth itself is the magnet, and the moving air is the armature, of which the windings are represented by the convected conducting ionosphere.

Stewart did not work out this theory in any detailed quantitative way, but drew some analogies between the S field and the distribution of temperature and convection in the lower atmosphere (§ 129), though recognizing that in the upper regions these phenomena may have a rather different character. He seemed to refer the sunspot cycle of change in S to a variation in the rate of convection in the upper atmosphere, corresponding to changes in the temperature-variation; he accounted for the absence of such a sunspot cycle of change in the lower atmosphere by supposing that at sunspot-maximum the water content of the atmosphere is increased, and that this prevents an increase in the amount of that part of the sun's radiation (supposed to be more intense at such times) which reaches the lower atmosphere. He suggested (§ 148) that the S and L fields change more rapidly by day than by night because the conductivity of the air is greater over the sunlit hemisphere than over the night hemisphere. He recognized (§ 133) that parts of S may be due to secondary electric currents induced in the earth.

As regards L, he stated (§ 146) that since it varies as nearly as possible inversely as the cube of the moon's distance from the earth, 'it is impossible to refrain from associating it either directly or indirectly with something having the type of tidal action, but in what way this influence operates we cannot tell'. In a later passage (§ 148) he implied that L is due to electric currents in the upper atmosphere.

23.4. Schuster's development of the dynamo theory.

Schuster [3] put the dynamo theory into a quantitative form. He first showed [2] by mathematical analysis of the observations that the greater part of S is of external origin, and that the inner part can reasonably be attributed to earth-currents induced by the varying outer field. He also showed later that the external electric currents are of the type that would arise if convective air-motions, such as are observed at ground-level, exist in the conducting layer, and on this assumption he estimated the total conductivity of this layer [3]. He also contributed, along with other physicists of his time, to our present knowledge of ionization and electrical conductivity in rare gases. He ascribed the conductivity of the upper atmosphere to the ionizing power of solar radiation, which he supposed to be increased at times of many sunspots; 'the most plausible cause', he suggested, was the sun's ultra-violet radiation.

He remarked that the electric currents producing the magnetic variations circulate only in the upper layers of the atmosphere, where the pressure is too small to affect the barometer. This has been confirmed in recent years by radio measurements, which show that the main ionized layers, in which the currents must flow, do not extend downwards much below a height of 100 km. (Ch. XV), although there may be ionized layers below this, of a kind less easily detectable by radio observations, because the electron density in them is very small compared with the ionic density. Schuster recognized that the observed barometric variations and the observed magnetic variations originate mainly in different layers, at low and high levels respectively, and that these layers may to some extent oscillate independently; while he found it possible to give a good account of the observed S field by supposing that the whole atmosphere has one united oscillation, he remarked that 'there are certain difficulties which are most easily explained by assuming possible differences in phase and amplitude between the upper and lower layers. If the two oscillations are quite independent, the conducting power, depending on the now unknown amplitude of the periodic motion, cannot be calculated, but must still be large, unless the amplitude reaches a higher order of magnitude than we have any reason to assume' ([3], p. 164).

In the mathematical development of his theory, Schuster assumed that the ionization and conductivity of the atmosphere vary with the sun's zenith angle, being greater by day than by night, and in summer than in winter. He pointed out an apparent difficulty in supposing that the observed atmospheric oscillations can produce the observed magnetic variations: the main term in the daily barometric variations is semi-diurnal (i.e. of 12-hour period), and though there is also a 24-hour term, this is usually small, especially over the great sea area of the globe (cf. 15.12), whereas in S the 24-hour term is in general much greater than the 12-hour term. He stated that the daily variation of the conductivity helps to overcome this difficulty, because the 12-hourly convection, combined with the varying conductivity, can produce an appreciable 24-hour periodicity in the magnetic change, whereas the 24-hourly convection does not contribute appreciably to the 12-hourly magnetic change. He was, however, driven to assume either that the atmospheric oscillation of the upper layers is greater in summer than in winter, and is to that extent independent of the oscillation of the lower layers, or that the ionizing power of solar radiation is in some degree accumulative, and that atmospheric conductivity is therefore 'not completely determined by the position of the sun at the time'.

In his mathematical analysis Schuster took account of both the diurnal and semi-diurnal oscillations, basing his numerical applications on Hann's formula for the daily barometric change at the equator, namely

$$0\cdot3\sin(\phi+t_0)+0\cdot92\sin\{2(\phi+t_0)+156^\circ\}, \tag{1}$$

measured in millimetres of mercury; here $t_0 =$ Greenwich mean solar time, increasing from 0 to 360° from midnight to midnight, $\phi =$ eastern longitude, and $t = \phi+t_0 =$ local mean solar time. Schuster took these two terms to correspond to terms P_1^1 and P_2^2 in the velocity potential of the oscillations. First treating the conductivity as the same all over the earth, he deduced for the magnetic potential a value which, expressed in terms of Schmidt's harmonics (17.4), is

$$Ka\{13\cdot7P_2^1\cos(\phi+t_0-180^\circ)+24\cdot8P_3^2\cos2(\phi+t_0-102^\circ)\}, \tag{2}$$

where K denotes the conductivity integrated through the thickness of the layer, and a denotes the earth's radius. Comparing this with his own determination of the principal terms in the potential of the observed S field, namely

$$10^{-6}a\{154P_2^1\cos(\phi+t_0-156^\circ)+71P_3^2\cos2(\phi+t_0-74\cdot5^\circ)\}, \tag{3}$$

he deduced $K = 11\times10^{-6}$ from the diurnal term, and $K = 3\times10^{-6}$ from the 12-hour term. The calculated phases for the 24- and 12-hour terms lag behind the observed phases by 1·6 hours and 1·8 hours.

He proceeded to recalculate the magnetic potential on the assumption that K varies as $1+\cos\chi$, where χ is the sun's zenith angle; he was thus able to account better, though not completely, for the observed preponderance of the 24- over the 12-hour term in the magnetic variations. His final value for K at the equator at noon was 3×10^{-6} electromagnetic units.

He examined the influence of self-induction, and (taking $K = 3\times10^{-6}$) found that it would reduce the calculated value of the magnetic potential by about 14 per cent.; it would also retard the phase, by about 1 hour for the diurnal and 1½ hours for the semi-diurnal term, increasing by these amounts the phase-discrepancies already noted.

He also examined the influence of the obliquity of the axis of the main magnetic field (the field of the atmospheric dynamo) on the S field, which should on this account include a part not dependent solely on local time. He did this 'somewhat in detail as it will give us a good test of the proposed theory when suitable observations will be available' (p. 170). He concluded that 'the new terms come well within the range of our observational powers. It would be interesting to trace them, but it should be remarked that only observations made near the equinox are suitable for the purpose, as the seasonal terms . . . would otherwise interfere' (p. 172).

From his inferred value of K, he deduced the value 10^{-13} e.m.u. for κ, the specific conductivity, by assuming that the thickness of the conducting layer is 300 km.; he regarded 10^{-13} as a not impossible value for the specific conductivity of the air at high altitudes.

He suggested that the heating effects of the S system of atmospheric electric currents should be appreciable, and might help to explain the isothermal character observed in the (lower) stratosphere; these heating effects have since been discussed by Chapman [5, 13], and found to be unimportant.

Schuster [3] concluded by urging a further and more accurate test of the theory: 'the most promising line of attack seems to me to be the investigation of the diurnal variation near the equator, where . . . it should not only vary with local time, but possess a term depending on the time of the meridian which passes through the magnetic axis' (p. 185). He also urged the desirability of further investigation of L, remarking that 'it is quite possible that the effects may depend on a tidal disturbance of the upper regions of the atmosphere. If so, we may expect to get a valuable test of our theory by their investigation' (p. 181).

23.5. Later investigations of the dynamo theory of S and L. Schuster's spherical harmonic analysis of the S field was later repeated by Fritsche, Walker, van Vleuten, van Bemmelen [20.2–6], and Chapman; the two latter also analysed the L field. A critical account of the work of the other writers here cited is given in Chapman's two papers [4, 5] on the analysis, and on the dynamo theory, of the S and L fields. In these papers Schuster's mathematical theory of S was extended by using a more general expression for the dependence of K on the sun's zenith distance, namely

$$K = K_0+K_1\cos\chi+K_2\cos^2\chi. \qquad (4)$$

The last term, which was absent from Schuster's formula, enables a greater difference in K between day and night to be represented, and led to a better theoretical agreement with the S data, particularly for the third and fourth periodic components. The theoretical discussion of atmospheric ionization in Chapter XV, however, shows that though the rate of ion-*production* doubtless mainly depends on $\cos\chi$, the distribution of ion-*content* (and conductivity), which depends also on the ionic and electronic recombinations, will not be symmetrical with respect to the meridian plane through the sun. Consequently K must involve also terms in $\sin\chi$; indeed it must be a function of the separate variables θ (the co-latitude) and ϕ (the longitude or local time), and not simply a function of χ.

23.6. The phase-law in L. Chapman also discussed the dynamo theory of the L field, and showed [8.5] that it readily accounts for the peculiar monthly cycle of change of phase of the non-semi-diurnal components of L (Ch. VIII); these components mathematically express the intensification of the L field over the sunlit hemisphere, due to a purely semi-diurnal lunar tidal convection in an atmosphere whose conductivity depends on solar time. The essence of this phase-change can be understood by considering the product of a factor $a\sin(2\tau+\alpha)$ representing the tide (τ denoting here lunar time, increasing by 2π per lunar day) with a factor

$$k_0+k_1\sin(t+\beta_1)+k_2\sin(2t+\beta_2) \tag{5}$$

representing the conductivity K expressed as a function of solar time t (increasing by 2π per solar day) instead of as a function of the zenith angle χ. The product, which will indicate the periodicity of the combined effect (such as the induced electric currents and their magnetic field), may be evaluated by formulae similar to 16 (16); it is of the form

$$\begin{aligned}&k_0 a\sin(2\tau+\alpha)+\\&+c_1\sin(2\tau-t+\epsilon_1)+c_3\sin(2\tau+t+\epsilon_3)+\\&+c_0\sin(2\tau-2t+\epsilon_0)+c_4\sin(2\tau+2t+\epsilon_4).\end{aligned} \tag{6}$$

The lunar day is about 50 minutes longer than the solar day, and therefore the solar time t increases slightly faster than the lunar time τ. Let (p. 163)

$$t=\tau+\nu; \tag{7}$$

then the variable ν increases uniformly with the time, by 2π in the course of a lunar (synodic) month, because in this period the moon completes one circuit of the earth, and the number of solar days is one more than that of the lunar day. Making this substitution for *t*, the above expression may be written:

$$k_0 a \sin(2\tau+\alpha)-c_0\sin(2\nu-\epsilon_0)+c_1\sin(\tau-\nu+\epsilon_1)+$$
$$+c_3\sin(3\tau+\nu+\epsilon_3)+c_4\sin(4\tau+2\nu+\epsilon_4). \quad (6\,\mathrm{a})$$

This shows that the luni-solar action produces a variation which includes a semi-diurnal component of constant phase (relative to the lunar time τ), and non-semi-diurnal components which change in phase by $\pm 2\pi$ or by $+4\pi$ throughout the month, as described in Chapter VIII; in addition there should be a change of fortnightly period.

It is convenient to take $\nu = 0$ at new moon, when the sun and moon are on the same meridian.

23.7. The dynamo theory of the L field. The dynamo theory of the L field [7.1] can be discussed more definitely than that of the S field, because of the greater certainty as to the period of the convective motion in the lunar case; all the available evidence indicates that the lunar tidal convection must be semi-diurnal, whereas the solar daily convection may include other frequencies as well. Hence it is possible to show conclusively that the dynamo theory of L is not in conformity with observation, if the tidal convective motion in the ionosphere has the phase indicated by the lunar semi-diurnal barometric variation observed at the ground (cf. 15.12). The latter is expressible in the form

$$a\sin(2t+\alpha),$$

where α, in the mean of a year, is rather less than 90°; it varies from about 55° to 110° at different stations, but the mean may be taken as about 80°. If the tidal pressure variation has the same phase in the ionosphere also, and if the conductivity is a simple function of the sun's zenith angle χ, with its maximum immediately beneath the sun ($\chi = 0$), then at new moon the time-factors of the various periodic terms in the magnetic potential of L should be $\sin(n\tau+170^\circ)$, except for a not very large correction due to self-induction. Actually the time-factors for these terms are all approximately the same, namely $\sin(n\tau+348^\circ)$. The discrepancy of phase is nearly 180°, indicating either that the lunar tidal convection is reversed at high levels, or that the simple dynamo theory of L is not valid. Recent studies of atmospheric oscillations make the former conclusion the more probable of the two, particularly since no alternative non-dynamo theory of L is available.

If, however, the lunar atmospheric tide is reversed at high levels, ere is no reason to suppose that the amplitude or speed of the tidal rculation in the ionosphere is the same as near the ground.

Chapman [7.1] revised Schuster's estimate of K, the integrated conuctivity of the ionosphere above the place which has the sun at its nith; he based the estimate on L instead of S, because of the definitess of the period (at least) of the atmospheric oscillation concerned; suming that the speed of the tidal circulation is the *same* as at the ound, he found that K must at sunspot-maximum approach the value 5×10^{-5} e.m.u., nearly ten times that found by Schuster. If, however, e circulation in the ionosphere is f times as rapid as at the ground, is estimate of K must be divided by f.

23.8. Comparison with radio data. The height-integral K of the ctrical conductivity κ for the E and F layers of the atmosphere has eady been discussed in Chapter XV. It was pointed out by Pedersen 5.21] that for current-flow along the direction of the earth's magnetic ld the value of κ is independent of this field, but that for current-flow any other direction the conductivity is reduced by the field. The luction is greatest for the directions transverse to the field; the conctivity in such directions will be denoted by $\kappa_\perp$ (specific) or $K_\perp$ tegrated with respect to height). In the ionosphere the conductivity arallel to the field (denoted by $\kappa_\parallel$) may be much greater than $\kappa_\perp$. was first pointed out by Pedersen, a correct mathematical development of the dynamo theory should take account of the difference tween $\kappa_\parallel$ and $\kappa_\perp$, by considering separately the effect of the electrotive force components along and perpendicular to the field; this has t yet been done. The distributions of the electric current-flow and gnetic intensity which have been deduced on the assumption that κ independent of the direction will need some modification. The rent-flow in the ionosphere must be mainly horizontal, and theree except at the magnetic equator it *must* have a component transse to the magnetic field; and even on the magnetic equator the flow l be largely transverse to the field. Hence, since the total fall of tential round a current-circuit, due to resistance, is mainly deterned by the more resisting parts of the circuit, the current-intensity the ionosphere is likely to depend more on the reduced transverse ductivity, $\kappa_\perp$, than on $\kappa_\parallel$. Consequently, the value of κ inferred from hould be compared with the estimate of $\kappa_\perp$ in Chapter XV, rather n with $\kappa_\parallel$.

The maximum estimate of $K_\perp$ based on radio data (15.11) was $\times 10^{-5}$ e.m.u.; this is the ionic contribution to $K_\perp$, and is almost entirely due to the E layer, and the lower part of the F_1 layer; the ionic contribution of the F_2 layer, and the electronic contributions o both the E and F layers, were found to make negligible additions to $K_\perp$. The above estimate of $K_\perp$ assumed that in the E layer the ratio of negative ions to electrons, denoted in 15.8 by λ, is of the order 100. Thus the maximum estimate of $K_\perp$ from the radio data, $1{\cdot}2\times10^{-5}$ e.m.u., is not much less than the value of $K_\perp$ required by the dynamo theory of L, assuming that f, the ratio of the horizontal lunar tidal velocity of the air in the ionosphere, to that at the ground, is equal to 1.

According to Pekeris [7] the value of f, the amplification of the tidal circulation in the ionosphere, may be as large as 100, in which case λ in the E layer would not need to be estimated as more than unity; the reversal of the phase of the tidal circulation in the ionosphere, indicated by L, is also in accord with his calculation of the tidal oscillation. But Appleton and Weekes [8] find from radio measurements that the height of the E layer is greatest at the lunar upper and lower transits, suggesting that the phase of the lunar semi-diurnal variation of *pressure* is similar to that observed at the ground. The lunar semi-diurnal variation of height which they find is ±1 km., which is surprisingly large, and at present very difficult to explain. It suggests that the factor f may be even greater than 1000.

At present, therefore, the explanation of the observed phase of L is uncertain, and there seems to be almost enough transverse conductivity in the ionosphere to explain the amplitude of L without assuming any magnification of the lunar tidal air motion.

The dynamo theory of L is the only available theory that seems likely to be true; but it cannot be considered complete until the theory of the atmospheric oscillations is properly understood. Some uncertainty must attach to the latter theory, and to the estimate of f, until the large annual change in the phase of the lunar atmospheric tide (15.12) has been explained.

If the dynamo theory of L is adopted, the magnetic data may be said to indicate the value of $K_\perp v$, where v is the average magnitude of the lunar tidal convective velocity in the ionosphere. If $K_\perp$ can be inferred from the radio data, then the combined radio and magnetic data provide the value of v as a datum for the theory of the atmospheric oscillations. If, on the other hand, the latter can provide a satisfactory value of v, then together with the magnetic data it affords an estimate of $K_\perp$; this in its turn indicates the value of the factor λ, which is still uncertain so far as the radio evidence goes.

23.9. Daily and seasonal changes of the ionospheric conductivity. The velocity potential of the lunar tidal circulation was taken by Chapman to be proportional to the harmonic function P_2^2; this choice was made partly for mathematical convenience, but is unlikely to be far from the truth. This velocity distribution, in conjunction with the assumed distribution of K, determines the relative magnitudes of the various periodic harmonic terms in L; the theory indicates, in good agreement with observation, that the main terms will be P_2^1, P_3^2, P_4^3, P_5^4. If K had the form considered by Schuster, namely

$$K \propto 1+\cos\chi, \tag{8}$$

the calculated coefficient of the fourth harmonic in L, P_5^4, would be much smaller than is observed. Chapman took

$$K \propto 1+3\cos\chi+\tfrac{9}{4}\cos^2\chi = (1+\tfrac{3}{2}\cos\chi)^2, \tag{9}$$

which is never negative, and gives a much greater ratio of day K to night K than does Schuster's expression. Though Chapman's expression certainly does not closely agree with the variation of K, as suggested by radio data or by the theory of 15.7, it leads to a very fair representation of the relative importance of the first four harmonics in L; the calculated relative coefficients are proportional to 44, 50, 33, and 9, and their ratios to the observed coefficients are 1·0, 1·0, 1·3, and 0·8; not much change would be required in the formula for K to make these four numbers equal.

The formula (9) implies a considerable seasonal change in the daily variation of K at stations in middle and high latitudes, and the theoretical predictions as to L, based on this formula, include a seasonal change in L; the theory indicates that this should be represented mainly by harmonics of the form P_1^1, P_2^2, P_3^3, P_4^4, which is in agreement with observation (Ch. XX); but the 'observed' coefficients of these harmonics are much larger than the calculated values, assuming that the atmospheric tide has no seasonal variation.

As stated in 15.12, the lunar atmospheric tide at the ground has an *annual* variation, consisting largely of a notable change of phase from the June to the December solstice; but it has no important *seasonal* variation, meaning an increase of amplitude in one hemisphere and a decrease in the other, following the motion of the sun to and fro across the equator. Thus the unexpectedly large seasonal variation of L seems to require that there is a decidedly larger seasonal variation of K in the ionosphere than the formula (9) would imply.

This is somewhat remarkable, since the formula (9) already corresponds to a greater seasonal variation of K, except in high latitudes, than the radio data would suggest. The latter seem to agree quite well, as regards the seasonal variation, with what is indicated by the simplest form of theory of the ionization of the ionosphere (15.7), according to which the noon ionization should vary approximately as $\sqrt{\cos\chi}$ in the E and F layers; K also may be expected to vary in roughly the same way for these layers, It is of interest to compare the seasonal variations implied by $\sqrt{\cos\chi}$ and by (9), for a latitude similar to that of Greenwich, say 52°. The ratio $K_{\text{winter}}/K_{\text{summer}}$ as derived from the $\sqrt{\cos\chi}$ formula is $\sqrt{(\cos 61^\circ/\cos 15^\circ)}$ or 0·71, and from the formula $(1+\frac{3}{2}\cos\chi)^2$ is 0·50, implying a considerably larger seasonal variation in the latter case.

23.10. The differences between the S and L fields. The solar daily convection in the ionosphere is unknown, and also we do not know whether S and L are produced in the same ionospheric layer. If we suppose that the solar convection is semi-diurnal, of type P_2^2, like the lunar, and that S and L *are* produced in the same layer, then although we may not know the phase difference between the solar and lunar convections, we may infer that the main terms in the harmonic expressions for S and L will be of the same type, and will have approximately the same *relative* magnitudes and the same phase-differences *inter se* in the two expressions. Actually it is found from the observation and analysis of S and L that the principal harmonics are the same in each, but that their mutual relations are different.

This is shown by the following table of ratios of 'observed' solar E_{n+1}^n to lunar E_{n+1}^n for the annual harmonics P_{n+1}^n (Ch. XX).

	P_2^1	P_3^2	P_4^3	P_5^4
Equinox . .	21·8	10·4	10·0	7·6
Solstice . .	18·5	10·2	10·0	7·3

The ratios for the last three harmonics are in moderate agreement, but the ratios for the first harmonic seem to preclude the conclusion that the solar daily ionospheric convection is purely semi-diurnal. This whole set of ratios might be explained by supposing the solar convective motion to include both a 24-hour and a 12-hour wave, the 12-hour wave having twice the amplitude, and maxima coinciding with the extreme values, of the 24-hour wave.

The *seasonal* harmonics P_n^n in the S field will next be considered. They are, relatively to the annual harmonics, only about one-third as large in S as in L, as is shown by the following table of ratios of solar to lunar E_n^n.

P_1^1	P_2^2	P_3^3	P_4^4
7·6	3·4	4·1	(14·7)

This set of ratios should be compared with those given above for the 'annual' harmonics P_{n+1}^n. In amplitude the S seasonal harmonics agree reasonably well with the theory based on the formula (9) for K. But the difference between S and L in regard to these seasonal harmonics implies that, if the S and L fields are produced in the same layer, the two convective circulations have different seasonal variations; if the lunar tide has no seasonal variation, then K must have a greater seasonal variation than is given by (9) or by the radio data, and the solar daily circulation in the ionosphere must have a seasonal variation which neutralizes two-thirds of the seasonal variation of S due solely to K and to the annual mean daily circulation.

Chapman has maintained that S and L are produced in different layers of the ionosphere, because of their different modes of variation. Besides this difference in their seasonal variations, S and L are unlike in other ways. The variation of S in the course of the sunspot cycle is much greater than that of L (Ch. XVIII); there may also be a difference between their variations with magnetic activity, though some of the evidence on this point brought forward by Chapman is rendered doubtful by the work of Bartels and Schneider (19.7).

The difficulty of explaining the differences between the S and L fields, in a way compatible with a dynamo theory of each, cannot however be overcome merely by saying that the two fields are produced in different layers, and attributing the necessary characteristics to the conductivity and the periodic convection (solar or lunar daily) in each. If possible the supposed S and L layers should be identified with one or both of the E and F layers, or with some other layer, perhaps below the E layer, for which independent evidence is available; and their supposed conductivity, or at least its distribution and temporal variations, should agree with what the radio data indicate. Further, the hypothesis that S and L are produced in different layers must be understood as meaning that each is *mainly* produced in one layer; the lunar tidal convection must presumably occur also in the layer in which S is mainly produced, and this layer must therefore make *some* contribution, though only a minor one, to L. If the solar and lunar daily convections, however, bear the same ratio (q, say) to one another at all heights in the ionosphere, it seems necessary that their magnetic effects should be in the same ratio q, and that S and L should both be produced in the same (the most conducting) layer; the differences between S and L seem to imply therefore that there are important variations in the relative magnitude of the solar and lunar daily convections at different heights in the ionosphere.

In an attempt to overcome some of these difficulties, Chapman was led (partly by his now perhaps doubtful data indicating a large dependence of L on the magnetic activity) to suggest that L is produced primarily in high latitudes, and that the L current-system in the middle belt of the earth is induced there by the primary currents in high latitudes; later work on the theory of electromagnetic induction, by Chapman and Price [22.10], does not favour this hypothesis, and it remains most likely that the L current-system in the ionosphere is produced directly *in situ* by the local lunar atmospheric tide. The difficulties in explaining the different characteristics of S and L on the basis of the dynamo theory therefore remain unsolved, but no alternative theories of S and L are available which seem to offer fewer or less formidable obstacles.

The recent work by McNish (10.3, 15.5) on the augmentation of the S current-system, at times when there are solar flares of ultra-violet light accompanied by radio-fading, seems to indicate that the S currents flow in or below the E layer. If the main L currents do not flow in this layer, they must presumably be situated at a higher level, and the speed of the lunar tidal current in the higher layers must be supposed greater, relative to the solar daily convection, than in the S layer; this is rather different from what the great (solar) daily changes of height of the F_2 layer would suggest; these would seem likely to be accompanied by greater horizontal motions than occur in or below the E layer.

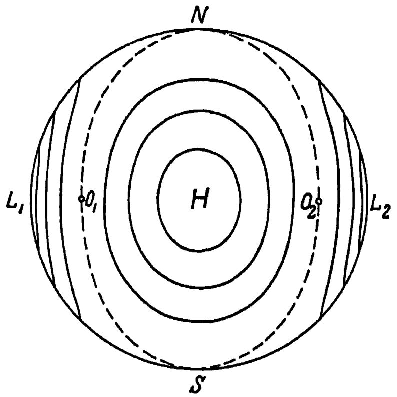
23.11. A qualitative derivation of the L current-system from the tidal motion. In § 12 the mathematical development of the dynamo theory is briefly described, but it seems useful to preface that account by a qualitative and graphical explanation of the theory, in a manner due to Chapman [10] (subject to a correction by Bartels—ref. G 12, p. 636). Fig. 1 illustrates the dynamo theory of L by four schematic diagrams, showing the terrestrial distribution of various atmospheric properties, as seen from the moon; the diagrams showing the distribution over the other hemisphere, as viewed from the opposite direction, would be identical.

The moon is for simplicity supposed to be in the plane of the earth's equator, and on the Greenwich meridian. The north and south poles of the earth are N and S; the earth rotates from terrestrial west to terrestrial east, that is from left to right in the diagrams. The moon is rising for points on the meridian NL_1S, and setting for points on the opposite meridian NL_2S.

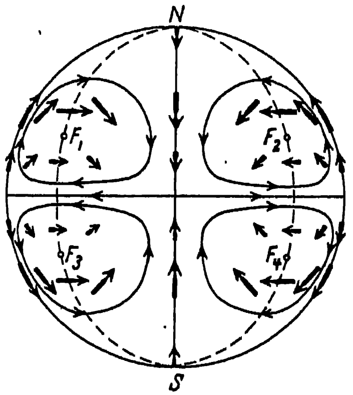
Fig. 1 *a* shows the lunar atmospheric tide as manifested at ground level by the lunar semi-diurnal departure of the barometric pressure from the local mean value. The lines in the diagram are lines of equal pressure-departure, or isobars. The pressure-departure is supposed to reach its maximum (high pressure), at H, and its minimum values (low pressure) at L_1 and L_2. The air is heaped up 'under' the moon, and at the antipodal point H′; at the intervening points L_1 and L_2 there is a corresponding deficiency. In showing H as situated immediately 'beneath' the moon, the observed lag of high tide 'behind' the moon, by about 40 minutes, is disregarded. To make the diagram conform better with observation, the whole pressure system should be rotated about the earth's axis NS, through 10°, so that H moves 10° towards the east (i.e. to the right). In any latitude, as the earth revolves relative to the moon, carrying any station eastwards (to the right) relative to this pressure-distribution, the pressure varies semi-diurnally, from a minimum (low tide) on the meridian NL_1S, to a maximum (high tide) on the meridian NHS, succeeded by another minimum when the meridian NL_2S is crossed, and rising again to another maximum on the meridian NH′S.

This lunar semi-diurnal variation of pressure is associated with a system of horizontal periodic winds, which are illustrated in Fig. 1 *b* by wind-vectors (arrows) and stream-lines. Over the central meridian NHS the winds are westward, and over the meridians NL_1S, NL_2S they are eastward. The winds converge to the point O_1, and diverge from the point O_2. On account of the convergence to O_1, the pressure at a point passing eastward (owing to the earth's rotation) through O_1 will be rising at its maximum rate, from the minimum at L_1 towards the maximum at H; similarly the pressure is falling at its maximum rate as a station passes through the position O_2. Similar remarks apply also to any station not on the equator, as it crosses the meridian NO_1S (rising pressure) or NO_2S (falling pressure).

The distribution of wind strength (illustrated only qualitatively in Fig. 1 *b*) is determined by the pressure distribution of Fig. 1 *a* and by the earth's rotation, which tends to deflect the wind movements. This may be illustrated by comparing the distribution of wind on



a) ISOBARS b) AIR MOTION



c) ELECTROMOTIVE FORCES AND CURRENT CIRCUITS d) HORIZONTAL MAGNETIC FORCES AND LINES

FIG. 1. Diagrammatic views of the earth as seen from the moon, when the latter is in the plane of the earth's equator; N and S are the north and south geographical poles. The diagrams illustrate the dynamo theory of the lunar daily magnetic variation L, as due to lunar tidal action;

(*a*) shows the isobars, or lines of equal barometic pressure at the ground, so far as the pressure is affected by the lunar gravitational tide-producing field—H refers to areas of pressure above (higher than) the average, and L to those of pressure below (lower than) the average: the pressure has its average value along the meridians NO_1S, NO_2S, and the opposite meridians (see also Fig. 3, p. 775);

(*b*) shows by arrows the direction and the relative magnitude of the horizontal air velocity near the ground, in a tidal air wave progressing, with the moon, from east to west; the lines are the stream lines showing at each point the direction of flow there: the greatest westward velocities occur on the central meridian, and the greatest eastward velocities on meridians 90° away: the motion converges towards O_1 and diverges from O_2;

(*c*) shows the horizontal electromotive forces (indicated by arrows) induced by the horizontal air motion across the vertical component of the earth's magnetic field (see also Fig. 4, p. 776); the continuous lines are the lines of flow of the resulting electric currents, which close their circuits, near the equator, against the (small) electromotive forces existing there: the current flow encircles four foci, F_1, F_2, F_3, F_4, in the hemisphere visible from the moon, and similar current-flow round four corresponding foci occurs on the opposite hemisphere owing to the lunar tidal air circulation existing there also;

(*d*) shows by arrows the direction and the relative magnitude of the horizontal component of the magnetic field at the earth's surface due to a current-system similar to that shown in (*c*), and situated in the upper atmosphere: the continuous lines are lines of horizontal magnetic force for this field.

(i) a non-rotating earth and (ii) the rotating earth, due to the same lunar tidal variation of pressure, taken to be

$$0{\cdot}05\sin^3\theta\cos 2(\tau_0+\phi) \tag{10}$$

in millimetres of mercury; here θ denotes the polar distance, ϕ the east longitude from Greenwich, τ_0 the lunar time at Greenwich, and $\tau = \tau_0+\phi$ the local lunar time in longitude ϕ. The southward and eastward velocity components of the wind (in cm./sec.), u and v, may be calculated from (10) by a method described by Gold [11] and Bartels [12]. For a non-rotating earth

$$u = -1{\cdot}7\sin^2\theta\cos\theta\sin 2(\tau_0+\phi), \quad v = -1{\cdot}2\sin^3\theta\cos 2(\tau_0+\phi), \tag{11}$$

and for the rotating earth

$$u = -2{\cdot}9\cos\theta\sin 2(\tau_0+\phi), \quad v = -(1{\cdot}2+1{\cdot}7\cos^2\theta)\cos 2(\tau_0+\phi). \tag{12}$$

On the rotating earth, the wind strength increases towards the poles. For instance, in 45° latitude, the strongest winds towards the north and west are about equal, 2·0 cm./sec., whereas on a non-rotating earth the corresponding values are only 0·6 and 0·4 cm./sec. The time-factors in (11) and (12), however, are the same.

The horizontal components of the electromotive forces (e.m.f.) produced by the air-motion of Fig. 1 *b* across the earth's magnetic field are pictured by the arrows in Fig. 1 *c*. The e.m.f. is the vector product of the velocity and the vertical magnetic intensity. Neglecting the obliquity of the earth's magnetic axis, the vertical intensity is directed downward in the northern, and upward in the southern hemisphere. The law of electromagnetic induction is such that in order to obtain the directions of the arrows in Fig. 1 *c*, for the e.m.f., we have to turn the arrows in Fig. 1 *b* through 90°, anti-clockwise in the northern hemisphere, and clockwise in the southern hemisphere, as viewed from above.

On the equator, where the vertical magnetic intensity is zero, the electromotive forces also vanish, and they are small in adjacent latitudes. In 45° latitude, where the vertical magnetic intensity is about 0·44 Γ, and u and v are each about 2 cm./sec., the components of electromotive force, both north-south and east-west, are about 0·9 e.m.u. (or $0{\cdot}9\times10^{-8}$ volts/cm.). With a total conductivity $K = 10^{-6}$ e.m.u. ($= 10^3$ ohm^{-1}) the corresponding north-south or east-west current across a horizontal distance of 1 cm. would be about 10^{-6} e.m.u. or 10^{-5} ampere; across a breadth of 3,000 km. a total current of 3,000 amperes might flow. This, however, must be regarded as an upper limit, because such a current-intensity could be reached only if the voltage and the conductivity were uniform around a closed circuit.

For simplicity, we consider the conductivity as independent of longitude, decreasing from the equator towards the poles; this is appropriate in considering the mean lunar daily magnetic variation over a number of whole months, in which the solar influences are averaged all round each circle of latitude.

The current-circuits are derived from the arrows for the e.m.f. most easily by considering first the e.m.f. along the central meridian NHS and the two meridians NL_1S, NL_2S; the e.m.f. along NHS is directed towards the equator; along NL_1S and NL_2S it is towards the poles. The current-flow from L_1 to N to H, or from L_2 to N to H, is closed by currents along the equator, where there is no e.m.f. The hemisphere is thus divided into four octants. For the current-lines inside the octant, the circuits are closed by flow in a direction which near the equator is *against* the e.m.f. (which there is relatively small). The currents circulate round four foci, F_1, F_2, F_3, F_4, and there are four similar foci in the opposite hemisphere.

On comparing the current-diagram Fig. 1 *c* with Fig. 8.4, deduced from the *observed* L, we see clearly that, while the general arrangement of the current-circuits is the same in the two diagrams, the current-*directions* are opposite. Hence, if the dynamo theory is true, the air-motion and the pressure-wave in the ionosphere must be reversed as compared with those pictured in Fig. 1 *a* and 1 *b* for the ground-level.

From the overhead current-circuits in Fig. 1 *c*, the directions of the horizontal magnetic forces at any point P near the ground, due to these currents, are easily obtained by turning the direction of the current-lines at P anti-clockwise through 90° (Fig. 1 *d*). The lines of horizontal magnetic force thus obtained converge to or diverge from the foci of the current-circuits. According to Fig. 1 *d*, in the equatorial belt—between these foci—the magnetic north component X, in the lunar semi-diurnal variation, should have its *maximum* about 3 lunar hours after the culmination, and outside this belt it should then have its minimum. For stations north or south of the equator, the east component Y should have its maximum or minimum about the time of the moon's culmination. The actual observations show nearly opposite phases (Ch. VIII)—confirming, of course, the comparison of the theoretical current-circuits of Fig. 1 *c* with those derived from the observations of L.

23.12. Outline of the mathematical development of the dynamo theory. A brief outline of the present mathematical development of the dynamo theory will now be given.

The conducting layer in which the dynamo action is effective is treated as a thin spherical shell, of thickness e and electric conductivity κ uniform throughout the thickness, so that the integrated conductivity K is κe. Actually the value of K depends on the direction relative to the earth's magnetic field (§ 8), but the present theory does not take this complication into account.

The position of a point P on the shell, at the time t' of a standard meridian, is specified (cf. Fig. 2) by the east longitude ϕ relative to that meridian, and by the north-polar angle (or co-latitude) θ; if t' and ϕ are reckoned in angular measure (t' at the rate 2π per day), then $\phi+t'$ will be the local time t at P, reckoned in angular measure.

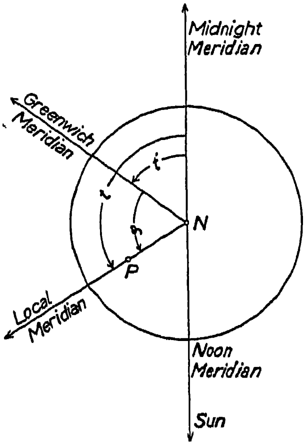


FIG. 2

The motion of a medium with velocity $\boldsymbol{v}$, in the presence of a magnetic field $\boldsymbol{H}$, induces a 'dynamo' electric field $\boldsymbol{E}$, equal to $\boldsymbol{v} \wedge \boldsymbol{H}$; if the southward, eastward, and upward-vertical components of $\boldsymbol{v}$ are u, v, w, and those of $\boldsymbol{H}$ are H_x, H_y, H_z, the corresponding components of $\boldsymbol{E}$ (cf. 1.18) are

$$E_x = vH_z - wH_y, \tag{13}$$

$$E_y = wH_x - uH_z, \tag{14}$$

$$E_z = uH_y - vH_x. \tag{15}$$

In the case of the conducting shell, the vertical component E_z of the electric field has little importance; it merely sets up opposite surface-charge distributions on the outer and inner sides of the shell, which neutralize E_z in the substance of the shell.

If the motion of the air of the shell is supposed horizontal and irrotational, it is associated with a velocity potential Ψ, such that

$$\boldsymbol{v} = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\Psi,$$

and since $w = 0$, Ψ is independent of z; also

$$u = -\frac{\partial \Psi}{\partial x} = -\frac{\partial \Psi}{a\,\partial\theta}, \qquad v = -\frac{\partial \Psi}{\partial y} = -\frac{\partial \Psi}{a\sin\theta\,\partial\phi}, \tag{16}$$

where a is the radius of the shell. Hence

$$E_x = vH_z = -H_z \frac{\partial \Psi}{a \sin\theta\, \partial\phi}, \tag{17}$$

$$E_y = -uH_z = H_z \frac{\partial \Psi}{a\, \partial\theta}, \tag{18}$$

so that the dynamo field depends only on the vertical component of the earth's magnetic intensity.

In general, the dynamo field is not of the right type to impel a steady current-system in the shell, without accumulating electric charge at any point. The existence of a steady current-system, without continued (growing) accumulation of charge, implies that the southward and eastward components (U, V) of the current-intensity (integrated throughout the thickness e of the shell) are associated with a current-function R, such that the current-flow from left to right, across any element of length ds, is $(\partial R/\partial s)\,ds$ (cf. 17.18). Hence

$$U = \frac{\partial R}{a \sin\theta\, \partial\phi}, \qquad V = -\frac{\partial R}{a\, \partial\theta}. \tag{19}$$

The electric force needed to impel this current must have components U/K, V/K, but the dynamo components vH_z and $-uH_z$ are not in general related to any function R in the manner of U/K and V/K. Hence the dynamo field sets up a charge distribution whose field, together with the dynamo field and any additional field due to self or mutual induction, *is* such as to set up a current-system having some current-function R. If S is the electrostatic potential of the charge distribution, the southward and eastward components of the field are

$$-\frac{\partial S}{a\, \partial\theta}, \qquad -\frac{\partial S}{a \sin\theta\, \partial\phi}. \tag{20}$$

If for the present we ignore self or mutual induction, then Ohm's law for the current-flow gives

$$vH_z - \frac{\partial S}{a\, \partial\theta} = \frac{1}{K} \frac{\partial R}{a \sin\theta\, \partial\phi}, \tag{21}$$

$$-uH_z - \frac{\partial S}{a \sin\theta\, \partial\phi} = -\frac{1}{K} \frac{\partial R}{a\, \partial\theta}. \tag{22}$$

From these two equations, by eliminating R or S, an equation for the remaining function, S or R, can be obtained. The equation for R is

$$K\left\{\frac{\partial^2 R}{\sin\theta\, \partial\phi^2} + \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial R}{\partial\theta}\right)\right\} - \left\{\frac{\partial K}{\partial\phi} \frac{\partial R}{\sin\theta\, \partial\phi} + \sin\theta \frac{\partial K}{\partial\theta} \frac{\partial R}{\partial\theta}\right\} = aK^2\left\{\frac{\partial vH_z}{\partial\phi} + \frac{\partial(uH_z \sin\theta)}{\partial\theta}\right\}. \tag{23}$$

This determines R when the vertical magnetic field H_z, the conductivity K, and the motion u, v (or the velocity potential Ψ) are known. If the shell is uniformly conducting, the second term of this equation disappears.

The vertical magnetic field H_z is expressible as a series of surface harmonics over the shell. If the earth's field is regarded as that of a central magnetic dipole, its north axis-pole having the coordinates θ_0, ϕ_0, then (by 18(14) and (17))

$$H_z = C\{\cos\theta + \tan\theta_0 \sin\theta \cos(\phi-\phi_0)\}, \tag{24}$$

where C is a constant whose value is approximately $-\frac{1}{3}$ gauss.

Schuster [3] attempted to infer the air-motion in the shell from the barometric pressure variations observed at the earth's surface; he used the equation

$$\frac{\delta p}{p_0} = -\frac{1}{c_0^2}\frac{\partial\Psi}{\partial t} \tag{25}$$

to connect the velocity-potential Ψ with the proportionate changes $\delta p/p_0$ of the pressure p_0 at ground-level; c_0 here denotes the Newtonian velocity of sound. The relation (25) is obtained from the fundamental equations for small atmospheric motions, by neglecting all forces (including those due to the earth's rotation) other than the pressure gradient, so that these equations become

$$\rho_0\,\partial u/\partial t = -\partial p/a\,\partial\theta, \qquad \rho_0\,\partial v/\partial t = -\partial p/a\sin\theta\,\partial\phi, \tag{26}$$

where ρ_0 denotes the average density of the air near the ground, and the pressure p is $p_0+\delta p$. If we substitute for u and v from (16) and integrate with respect to θ or ϕ, we obtain

$$\rho_0\,\partial\Psi/\partial t = \delta p, \tag{27}$$

or (25), since, in terms of the gas constant R and the absolute temperature T_0, we have $p_0 = \mathrm{R}\rho_0 T_0$, and for the Newtonian velocity of sound (isothermal waves) $c_0^2 = \mathrm{R}T_0$.

Chapman [4, 5] likewise used the equation (25), but it is now certain that it is not applicable. The theory of the atmospheric oscillations (15.12) shows that the motion (and therefore Ψ, if a velocity potential exists) is affected by the earth's rotation even at ground-level, and furthermore varies with the height; hence the air-motion in the ionosphere bears no simple relation with $\delta p/p_0$ at ground-level.

Whatever the relation may be between Ψ and the pressure variation at ground-level, Ψ may be assumed expressed as a series of surface harmonics. If the motion depends only on θ and the local time, a typical term of the series will be

$$\Psi_N^M \equiv k_N^M P_N^M \sin(Mt+\alpha_N^M), \qquad \Psi = \sum_N \sum_M \Psi_N^M. \tag{28}$$

(Here and in similar expressions t must be interpreted in angular measure.) Since the magnetic effects of the different harmonic terms of Ψ are additive, it suffices to determine the effect of the typical term Ψ_N^M. Schuster considered specially the terms Ψ_1^1 and Ψ_2^2; Chapman considered mainly the term Ψ_2^2.

The conductivity K must likewise be expressible as a series of surface harmonics. In discussing the general theory, Schuster and Chapman took K to be of the special form

$$K_0\Big(1+\sum_{s=1}^{\infty} a_s \cos^s\chi\Big), \tag{29}$$

where χ is the zenith distance of the sun at the point (θ,ϕ) considered, so that when the sun has the declination δ,

$$\cos\chi = \sin\delta\cos\theta+\cos\delta\sin\theta\cos t. \tag{30}$$

Schuster considered in detail only the case $a_1 = 1$, $a_s = 0$ $(s > 1)$; Chapman discussed the case $a_1 = 3$, $a_2 = \frac{9}{4}$, $a_s = 0$ $(s > 2)$.

The current-function R, to be determined from (23), must also be expressible as a series of surface harmonics R_n^m, that is,

$$R_n^m = r_n^m P_n^m \sin(mt+\beta_n^m), \qquad R = \sum_n \sum_m R_n^m. \tag{31}$$

Hence, using the equation 17 (42 a), which defines a surface harmonic of degree n, the first term of (23) may be more simply written as

$$-K\sin\theta \sum\sum n(n+1)R_n^m. \tag{32}$$

In order to express later results more clearly, it is convenient to admit, in (31), also negative values of m, using, for this purpose, the definition (when m is a negative integer)

$$P_n^m \equiv P_n^{-m}. \tag{31 a}$$

The range of m in (31) is therefore from $-n$ to $+n$, since $P_n^m = 0$ if m is numerically greater than n. This makes (32) valid for negative m also. Clearly it is possible to combine any two terms involving R_n^m and R_n^{-m} $(m > 0)$ by writing

$$\begin{aligned} R_n^m+R_n^{-m} &= r_n^m P_n^m \sin(mt+\beta_n^m)+r_n^{-m} P_n^{-m} \sin(-mt+\beta_n^{-m}) \\ &= P_n^m\{r_n^m \sin(mt+\beta_n^m)-r_n^{-m}\sin(mt-\beta_n^{-m})\} \\ &= r_n^{(m)} P_n^m \sin(mt+\beta_n^{(m)}), \end{aligned} \tag{31 b}$$

with new values for the amplitudes and phases, as indicated by the notation.

The magnetic field of the current-system, at the earth's surface (at radius a') has a potential Φ, also expressible as a series of surface harmonics

$$\Phi = \sum_n \sum_m \Phi_n^m, \tag{33}$$

and each term is related to the corresponding term in R as follows (cf. 17 (83), omitting the factor 10, because R is reckoned in electromagnetic units):

$$\Phi_n^m = -4\pi \frac{n+1}{2n+1}\left(\frac{a'}{a}\right)^n r_n^m P_n^m. \tag{34}$$

This last equation, when the coefficients r_n^m have been determined, is the solution of the problem, since it gives the external part of the dynamo-produced field at the earth's surface. Our object therefore is to find the coefficients r_n^m and the phases β_n^m from known or assumed data for H_z, K, and Ψ. It should be particularly noted that in general each term Ψ_N^M in Ψ gives rise to many harmonic terms in R or Φ.

Consider first the simplest possible case, assuming K constant, and no obliquity of the earth's magnetic axis (i.e. $\theta_0 = 0$ in (24)). Then in (23) the left-hand side reduces to (32), and the right-hand side becomes

$$aK^2C\left\{\cos\theta\frac{\partial v}{\partial\phi} + \frac{\partial(u\sin\theta\cos\theta)}{\partial\theta}\right\} \tag{35}$$

or

$$-K^2C\left\{\cot\theta\frac{\partial^2\Psi}{\partial\phi^2} + \frac{\partial}{\partial\theta}\left(\sin\theta\cos\theta\frac{\partial\Psi}{\partial\theta}\right)\right\}. \tag{36}$$

To determine the contribution made to this expression by any particular harmonic term Ψ_N^M of Ψ, we transform (36) by using 17 (58 c), which, in our present notation, reads

$$-M^2\cot\theta P_N^M + \frac{\partial}{\partial\theta}\left(\sin\theta\cos\theta\frac{\partial P_N^M}{\partial\theta}\right)$$
$$= -\frac{\sin\theta}{2N+1}\{[(N+1)^2-1][(N+1)^2-M^2]^{\frac{1}{2}}P_{N+1}^M + (N^2-1)(N^2-M^2)^{\frac{1}{2}}P_{N-1}^M\};$$

on comparing with (32), we find that Ψ_N^M contributes the two following terms to the current-function R:

$$r_{N+1}^M P_{N+1}^M \sin(Mt+\alpha_N^M) + r_{N-1}^M P_{N-1}^M \sin(Mt+\alpha_N^M), \tag{37}$$

where

$$r_{N+1}^M = k_N^M KC\frac{N[(N+1)^2-M^2]^{\frac{1}{2}}}{(N+1)(2N+1)}, \tag{38}$$

$$r_{N-1}^M = k_N^M KC\frac{(N+1)(N^2-M^2)^{\frac{1}{2}}}{N(2N+1)}. \tag{39}$$

It should be noted that the phase in each of these two harmonic terms of R is the same as that of the harmonic term in Ψ to which they are due.

If $N = M$, as was the case for both the harmonic terms (Ψ_1^1, Ψ_2^2) of the velocity potential discussed by Schuster, one of these terms in the current-function vanishes, namely the term R_{N-1}^N. Thus, if the velocity potential of the solar daily circulation in an ionosphere of uniform K can be represented solely by harmonics Ψ_1^1, Ψ_2^2,..., Ψ_N^N,..., the current-function (and magnetic potential) will contain only terms R_2^1, R_3^2,..., R_{N+1}^N. Similarly, if the velocity potential of the lunar atmospheric tide is of the form Ψ_2^2, the potential of L should contain only one term, a multiple of P_3^2. The fact that L involves also other harmonics indicates the need to consider a non-uniform distribution of conductivity.

If K is expressible in the form (29), then by substitution from (30), and certain transformations, we may write

$$K = K_0 \sum_{-\infty}^{+\infty} f_s \cos st, \qquad K^2 = K_0^2 \sum_{-\infty}^{+\infty} g_s \cos st, \tag{40}$$

where t denotes local solar time measured from midnight. In (40) f_s and g_s can be expressed as power series in $\sin\theta$ and $\cos\theta$, whose coefficients involve $\sin\delta$ and the coefficients a_s of (29). The expressions (29) or (40) for K are, it should be noted, rather special, in that they assume a conductivity-distribution symmetrical on the two sides of the noon meridian; radio observations and the theory of the ionosphere both indicate an asymmetry, particularly as between the conditions just before dawn and after sunset (Ch. XV). If this were taken into account, (29) and (40) should contain sine terms in χ or t respectively, as well as cosine terms.

If, retaining the assumption that the earth's magnetization is axial $(\theta_0 = 0)$, the expressions (40) for K and K^2 are substituted in (23), and each side is transformed so as to be a series of surface harmonics, then by equating the coefficients of like harmonics on the two sides, we obtain a series of equations, linear on one side in the coefficients r_n^m of R, and on the other in the coefficients k_N^M of Ψ; the factors of the coefficients in these equations involve the numbers f_s and g_s. If every k_N^M except one is made zero, the equations will give the contribution to R made by the single velocity-harmonic Ψ_N^M. The solutions, giving r_n^m in terms of the coefficients a_s in K, are complicated even when the series for K ends with the term a_2.

If K is of the form $K_0(1+a_1\cos\chi)$, a harmonic Ψ_N^M of Ψ contributes to R a set of terms R_n^m for which n may range from $N-3$ to $N+3$, and m from $M-3$ to $M+3$; if K also involves a term $\cos^2\chi$, the range of n and m is further increased. When $N = M$, many of these terms R_n^m are absent because their n is numerically less than their m.

With the above form of K, symmetrical about noon and midnight, the phases of all the terms in R arising from a term Ψ_N^M in Ψ are the same as those of Ψ_N^M, except that the numerical coefficient r_n^m may be opposite in sign to that of k_N^M, implying merely a reversal of phase. This agreement of phase between Ψ_N^M and the terms R_n^m which it contributes to R would not be preserved if the actual asymmetry of K about noon were taken into account. It will also in general not be preserved when harmonics R_n^m and R_n^{-m} are combined, as in (31 b).

We have so far assumed that the time-periodicities of the atmospheric oscillations and the conductivity are commensurate. This is valid in the solar case, but not in the lunar, because the lunar tidal circulation is governed by lunar time (τ), and the conductivity K by solar time (t). The difference may be approximately taken into account by regarding the lunar tide as having the period of a solar day, but with a phase which varies slowly throughout the lunar synodic month (from one new moon to the next). Thus, we write

$$t = \tau+\nu, \tag{41}$$

where ν is a measure of the phase of the moon, and increases from 0 to 2π from one new moon to the next. The time-factor in (28) is

$$\sin(M\tau+\alpha_N^M), \quad \text{or} \quad \sin(Mt+\alpha_N^M-M\nu),$$

so that if, in (28), α_N^M is replaced by γ_N^M, where

$$\gamma_N^M \equiv \alpha_N^M-M\nu, \tag{42}$$

the analysis for the solar case can be at once applied to the lunar one. If K be assumed uniform over the earth, and if the velocity potential of the lunar tidal circulation be supposed represented by the single term Ψ_2^2 (which is not quite accurate), then R will involve the single term $r_3^2 P_3^2 \sin(2t+\gamma_2^2)$ or $r_3^2 P_3^2 \sin(2\tau+\alpha_2^2)$; this represents a current-function (with an associated magnetic potential Φ) consisting of one term only, R_3^2, in phase with the lunar tide. If K is not uniform, but is of the form (29), then R will include terms R_n^m (n and m having the same range as in the solar case for the corresponding solar term Ψ_2^2) in which the time-factor will be

$$\sin(mt+\gamma_2^2)$$

or

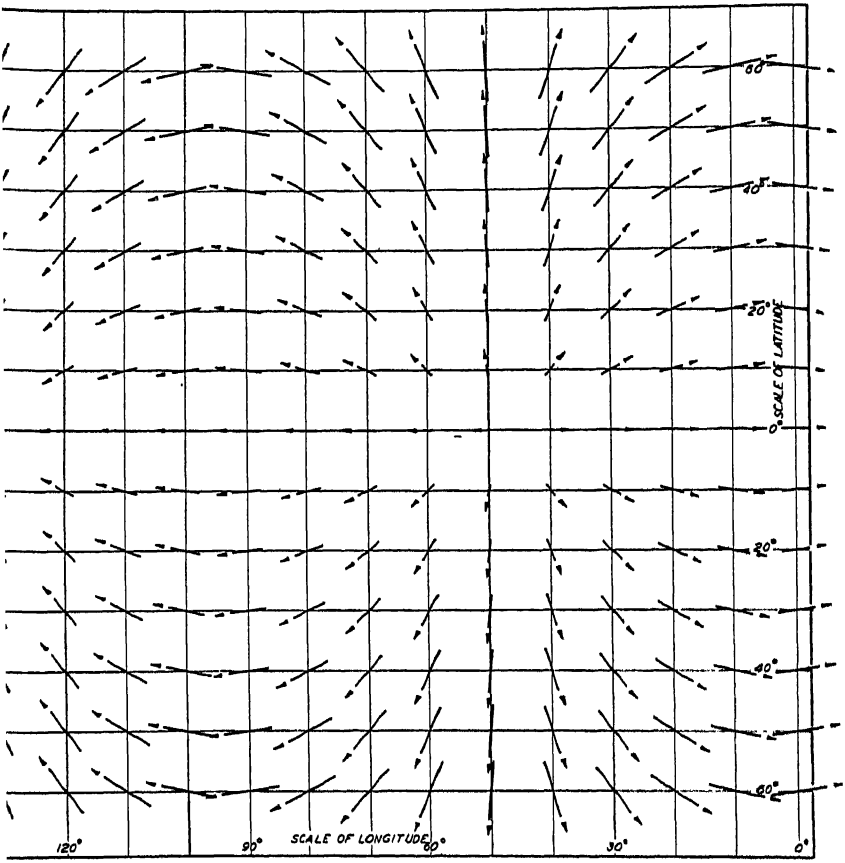
$$\sin(m\tau+\gamma_2^2+m\nu)$$

or (by (42), substituting $M = 2$)

$$\sin\{m\tau+\alpha_2^2+(m-2)\nu\}. \tag{43}$$

Such a harmonic, relative to lunar time τ, is not of constant phase throughout the month; its phase increases by $(m-2)2\pi$ per month.

us, as was explained in (6a), the phase of the semi-diurnal term is nstant, that of the 8-hour term ($m = 3$) increases by 2π, and that of e 6-hour term ($m = 4$) increases by 4π per month; the phase of the -hour term ($m = 1$) decreases by 2π per month, and there is a term $= 0$) which has no daily period, but whose period is fortnightly, since phase increases by 4π per month. In the case of the harmonics R_n^m r which m is negative, the monthly change of phase is still more rapid; en for $m = -1$ it is 6π, so that this term is distinctly different in riod from the term $m = 1$. These inferences as to the phase-changes r $m > 1$) are borne out by analysis of the observations; the harmonics $\leqslant 0$ have not yet been determined from the observations.



. 3. The air-velocities associated with the solar semi-diurnal atmospheric oscillation own for part of the western hemisphere, for the epoch 11^h of the local time of the ridian 75° W., but typical for other regions at the corresponding local times; see also . 1 *b*). Arrow-scale: 50 cm./sec. for a length equal to the side of the small squares. (After A. G. McNish)

23.13. The intensification of S and L at Huancayo. McNish [7.23] has pointed out that in South America the magnetic equator (the line along which the vertical intensity is zero) reaches its greatest southward deviation (about 15°) from the geographic equator. This means that the vertical intensity Z is directed downwards in the intervening regions south of the geographic equator; hence the electromotive forces (Fig. 1 *c*) derived from the air-motion (Fig. 1 *b*) will there be directed opposite to the arrows shown in Fig. 1 *c*, that is, *parallel* to the direction of the currents instead of *against* it. This is illustrated

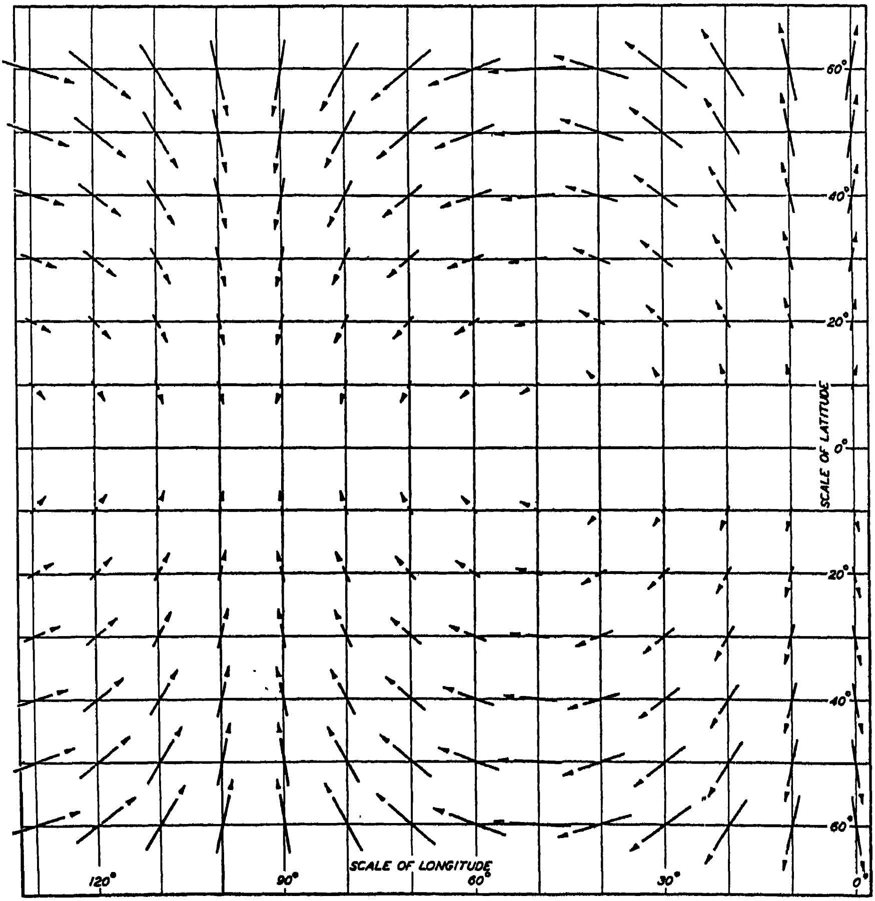


FIG. 4. The electromotive forces induced by the air-motions shown in Fig. 3, assuming that the earth's magnetic field is that of the centred dipole, alined along the *geographical* axis (see also Fig. 1 *c*). Arrow-scale: 0·023 volt/km. for a length equal to the side of the small squares. (After A. G. McNish)

by Figs. 3, 4, 5, due to McNish; the first shows the terrestrial distribution of air-velocity corresponding to a semi-diurnal tide, and represents facts already portrayed (in a slightly different way, and on a different map-projection) in Fig. 1 *b*. Fig. 4 shows the electromotive forces which

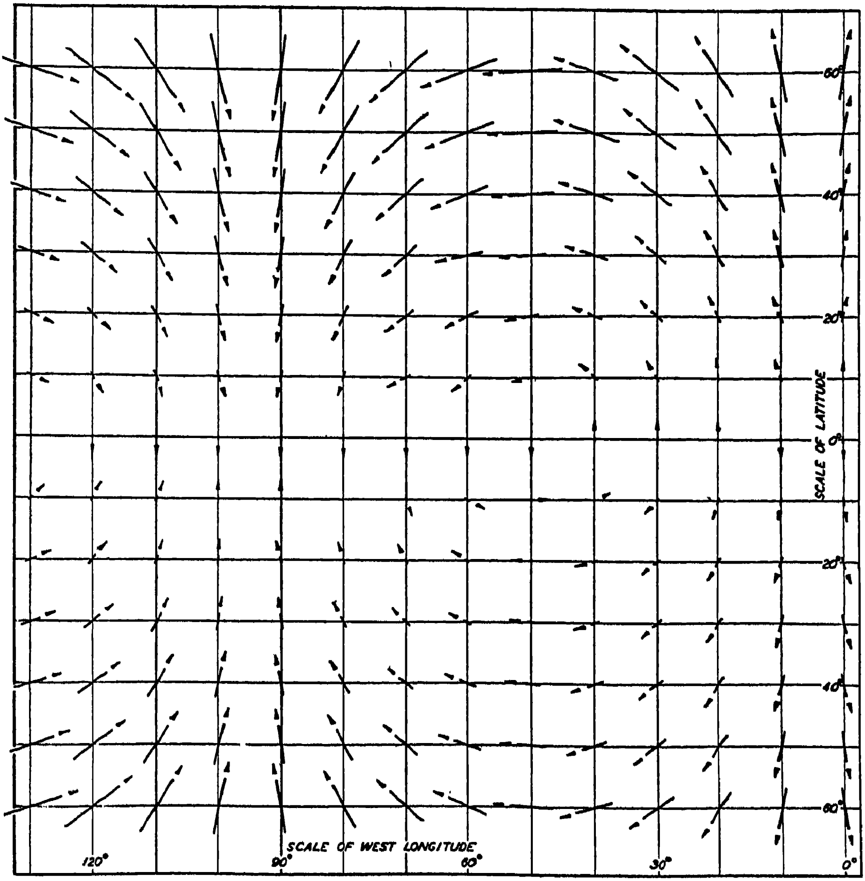


FIG. 5. The electromotive forces induced by the air-motions shown in Fig. 3, at the epoch there stated, in the presence of the actual geomagnetic field (arrow-scale as before). (After A. G. McNish)

would result from these air motions, if the earth's field were that of a central dipole alined along the geographical axis; this represents a distribution already shown, in less detail, by the arrows in Fig. 1 *c*. Fig. 5 shows the electromotive forces corresponding to the air-motions of Fig. 3, and to the *actual* terrestrial distribution of vertical magnetic force. At about 10° S., the e.m.f. in Fig. 5 is opposite to that in Fig. 4. It is in this region that, according to Fig. 1 *c*, the S current-flow is opposite to the e.m.f. Consequently, according to McNish, when the S current-circuits are crossing the northerly part of South America, on their daily passage round the earth, the opposition between the e.m.f. and the current is changed to agreement between their directions, and the southern circuit becomes intensified. An indication of how great this intensification is may be seen by comparing Figs. 6 and 7; Fig. 6 is a reproduction of the average form of the S current-system at the equinox, already shown in 7.6; Fig. 7 is a representation of the S current-system, when the main circuits are passing across America,

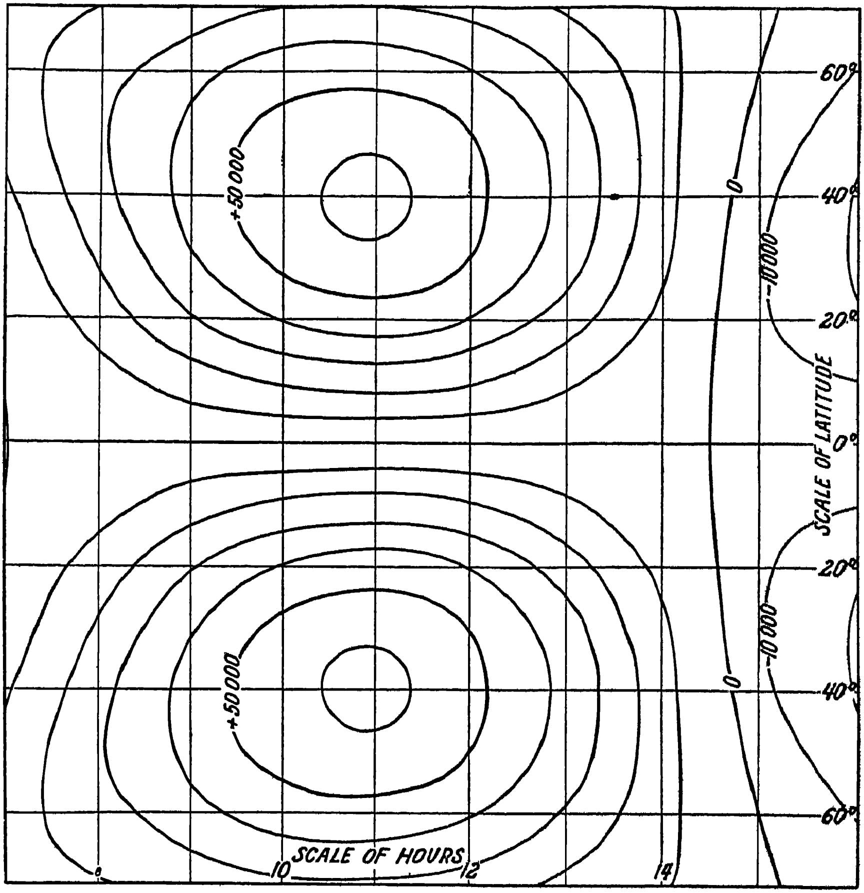


FIG. 6. Current-diagram for the overhead S_q current-system, drawn by Bartels, using the results of Chapman's spherical harmonic analysis; it represents an averaged current-system, based on equinoctial data for many widely-distributed stations (it corresponds to part of Fig. 7.15, p. 229, otherwise shown in Fig. 20.1, p. 696; it is here drawn for the area and local time to which Fig. 3 refers). (After A. G. McNish)

and was constructed by McNish from a spherical harmonic analysis based on five American stations.

This qualitative argument by McNish seems likely to furnish the interpretation of the large amplitudes of S and L at Huancayo. Although the computed electromotive forces in the equatorial regions are small, because of the small values of Z, the transverse electric conductivity (in the direction of the current there) can be expected to be higher there than elsewhere, since the transverse magnetic field is smaller than in higher latitudes. A quantitative discussion of these outstanding phenomena is desirable.

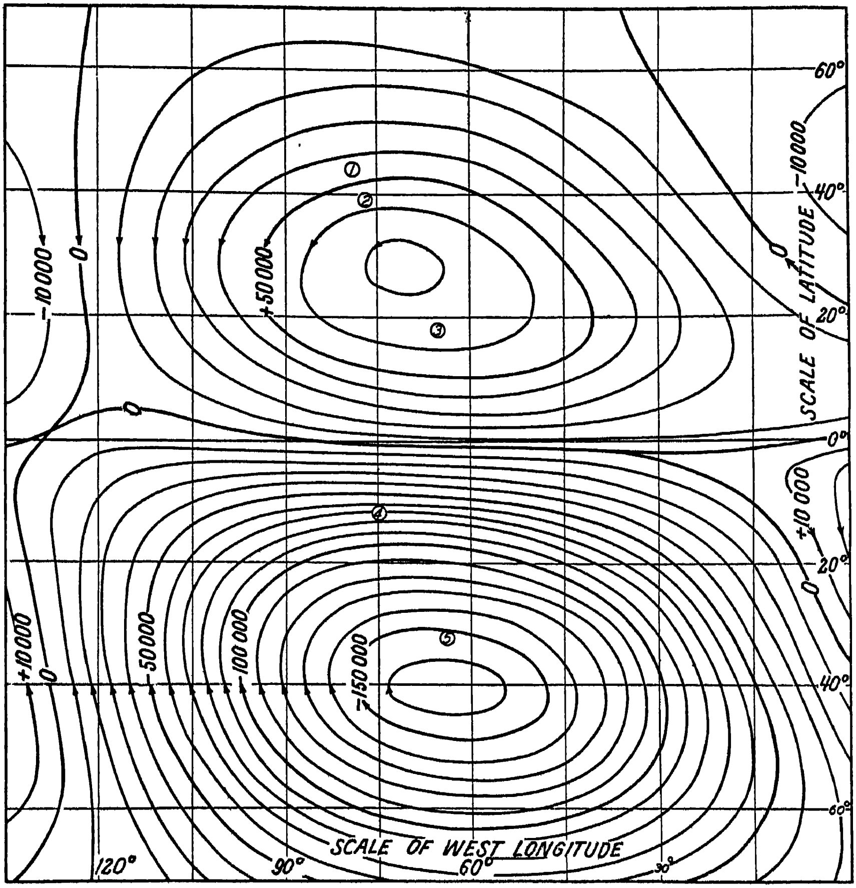


FIG. 7. The current-lines for the overhead S_q current-system, over the area and for the epoch to which Fig. 3 refers, based on a spherical harmonic analysis of equinoctial data (1923) for S_q at five American stations, Agincourt (1), Cheltenham (2), Vieques (3), Huancayo (4) and Pilar (5). (After A. G. McNish)

B. THE DIAMAGNETIC AND DRIFT-CURRENT THEORIES OF S

23.14. The diamagnetic theory. If a rare ionized gas is in the presence of a magnetic field, the electrons and ions execute spiral motions (15.10); the influence of the magnetic field upon the electric conductivity of such a gas was first explained by Townsend; its importance in relation to the dynamo theory of S and L was recognized in 1927 by Pedersen [6], and shortly afterwards independently, by Gunn [17] (1928). Later Gunn [18] and Chapman [19, 20] attributed other electromagnetic effects in geophysics and solar physics to the spiral motions of charges in magnetic fields.

Gunn pointed out that the circular part of the spiral motion of the electrons and ions around the lines of magnetic force H makes these moving charges equivalent to small magnetic doublets with their axes opposed to the field. The magnetic moment due to a charge e (e.m.u.) which describes a circular orbit of area A in time t is the same as that of an electric current of intensity $i = e/t$; hence, by 1.13, the magnetic moment is

$$eA/t. \tag{44}$$

The radius r of the circle (or the 'spiral radius' considered in 15.10) is mv/eH, where v is the circumferential velocity of the charge and m its mass. (The motion along the axis of the spiral can be neglected in the present discussion.) Also

$$\frac{A}{t} = \frac{\pi r^2}{2\pi r/v} = \frac{rv}{2} = \frac{\frac{1}{2}mv^2}{eH}.$$

If the velocity v is regarded as the component, normal to the magnetic force H, of the random thermal velocity of the charges, corresponding to a temperature T (absolute), then the mean value of $\frac{1}{2}mv^2$ is two-thirds of the total mean translatory kinetic energy of a particle in a gas at that temperature, namely $\frac{2}{3}\times\frac{3}{2}kT$ or kT. Hence the mean magnetic doublet per charge, due to the spiral motion round the field, is

$$\frac{kT}{eH}. \tag{45}$$

This does not depend on the mass of the charge, and therefore its magnitude is the same for electrons as for ions. The sign of e is different for positive and negative charges, but so also is the direction of their circular motion; consequently their magnetic moment has the same sign, and its direction is easily seen to be opposite to that of the field, as in a diamagnetic body.

The intensity of magnetization of the gas, due to n free charges (electrons and ions together) per unit volume, is consequently

$$\frac{knT}{eH}, \tag{46}$$

in the direction $-\boldsymbol{H}$. This calculation applies only to the gas in regions where the density is so low that all the charges are free to spiral with little or no disturbance by collisions. It implies that the outer part of the ionosphere is a magnetized medium, and indicates how the intensity and direction of magnetization are dependent on $\boldsymbol{H}$ and on the distribution of nT. Assuming a reasonable qualitative geographic distribution of nT (dependent on local time) over the ionosphere, Gunn made an approximate calculation of the magnetic field at the earth's surface due to such a magnetized layer. He found that qualitatively the field would resemble the S field, and he considered also that the absolute magnitude of nT, and the thickness of the layer, were probably sufficient to permit the S field to be explained by this diamagnetism. Considering also (as he did) that the dynamo theory was insufficient to account for S, because of the reduction of the transverse conductivity by the magnetic field, he proposed this *diamagnetic theory* as an alternative.

In his discussion Gunn referred to a theory of diamagnetism in metals, which had been proposed in 1912 by Schrödinger [21], and which was based on the same idea that the spiral motion of the free charges, in the presence of a magnetic field, renders them equivalent to magnetic doublets. He mentioned that an examination of this theory by Bohr [22] in his doctor's thesis, and later by van Leeuwen [23], had led to the conclusion that if reflections of the electrons at the boundary of the metal are taken into account, then the net diamagnetic effect outside the metal would be zero. Gunn contended that these objections would not apply to the ionosphere, where the free paths are very long, and where the thickness of the transition layer, between the non-diamagnetic region of short free paths and the region of almost free spiral paths, is thousands of times longer than the spiral radius of even the heaviest ion. This point is further discussed below.

In 1929 Chapman [24] improved the mathematical calculation of the field of Gunn's diamagnetic layer of the ionosphere. Except for a slight difference of phase he confirmed Gunn's conclusions that the diamagnetic field had the same form as the S field. He concluded [24, 19], however, that the total ion-content of the ionosphere was inadequate to account for the observed *intensity* of the S field, on the diamagnetic hypothesis.

23.15. The drift-current theory. In two papers on the sun's magnetic field [19 *a*] and on S [20], Chapman considered the drift-velocity of charges spiralling in a magnetic field, when a force oblique to the field acts upon the charges (15.10); this drift-velocity is not along the direction of the force, but is perpendicular both to the force and the magnetic field. The charges in an ionized atmosphere are subject to the vertical force g of gravity, and also to a vertical electric field E (recognized by Pannekoek [25] and Rosseland [25 *a*] and often called by their names), which arises from a slight separation of the electrons and the ions, due to the tendency of the electron gas to rise above the heavier ionic gas. This separation of the positive and negative charges is extremely small, but suffices to maintain the number of ions per unit volume (or the *number-density*) almost exactly equal to that of the electrons, in spite of their widely different masses (m_e and m_i). It does this by equalizing the combined gravitational and electric forces on each type of charge, so that (almost exactly)

$$m_i g+eE = m_e g-eE.$$

Thus
$$eE = -\tfrac{1}{2}(m_i-m_e)g, \tag{47}$$

and the resultant force on an electron or an ion is

$$\tfrac{1}{2}(m_i+m_e)g,$$

or, since m_e is so small, approximately $\tfrac{1}{2}m_i g$.

This resultant force, in the presence of a magnetic field, should produce equal horizontal drift-velocities of the ions and electrons in opposite directions, to the magnetic east for the ions, and to the magnetic west for the electrons. These drift-motions would convey an electric current, whose intensity would depend on n, and would therefore, in the earth's atmosphere, vary with the local time and the latitude. The inequality of the current-flow along any circle of magnetic latitude would tend to accumulate positive and negative charge in certain regions, and a charge-distribution would thus be set up (cf. § 12), which in conjunction with the drift-currents would maintain a (quasi-) steady current system.

Without giving detailed reasons or calculations, Chapman [20] suggested that these current-systems would resemble those of the actual S field, as drawn by Bartels (Figs. 7.15, 7.16, p. 229, or Figs. 20.1, 20.2,

p. 696), except for a difference of phase. He inferred that the drift-currents might account qualitatively for a large part of the S field, though even so, on account of the difference of phase, an appreciable part would still have to be explained by dynamo action. Whether or not these drift-currents are the main cause of the S field would depend on the ion-content of the ionosphere, just as in the case of Gunn's diamagnetic theory. Chapman stated, however, that the drift-current contribution to S, for any given ion-content, would be many times as great as the diamagnetic contribution (250 times if $T = 300°$, or 80 times if $T = 1{,}000°$), implying that diamagnetism could at most play an insignificant part in the production of S, even if the drift-currents were the main cause of S. Chapman gave the name *drift-current theory* to the hypothesis that the drift-currents are the main cause. He did not adopt this hypothesis, but indicated that, on the contrary, the total ion-content of the ionosphere was probably inadequate to enable the drift-currents to account for the major part of S; in any case, the choice seemed to lie between the dynamo theory and the 'drift and dynamo' theory.

In his discussion Chapman took account of the degree to which both the drift-current and the diamagnetism would be reduced by collisions, as represented by the factor $\omega^2/(\omega^2+\nu^2)$ affecting both of them (15 (89)). This factor, it should be noted, is different for ions and electrons, so that the region of almost free spiral paths of electrons extends lower in the ionosphere than does that for ions (15.11).

23.16. Criticisms of both theories. At this stage the importance to be attributed to diamagnetism and drift-currents in the ionosphere seemed to depend on the value of the total ion- (and electron-) content (and also on the temperature) of the ionosphere. But Cowling [27] soon afterwards raised much more fundamental considerations relating to both effects.

As regards diamagnetism, he recalled the criticisms of Schrödinger's theory of diamagnetism in metals; Gunn had regarded these criticisms as not affecting the diamagnetism of the ionosphere. Bohr and van Leeuwen had shown that if the Schrödinger diamagnetic effect exists (for example, in a metal in a uniform magnetic field), it must be a boundary effect. Schrödinger himself had proved that in the interior of the metal the mean electronic velocity at any point is zero (and the velocity-distribution Maxwellian) so that their motion should produce no resultant magnetic field there. Hence the supposed diamagnetism would depend on the electronic motions near the boundary. There would be no resultant diamagnetism, if the Maxwellian velocity-distribution is maintained up to the boundary, as would be the case, for example, if the boundary of the metal were conceived to be perfectly plane and smooth, so that the electrons impinging on it rebounded elastically.

In the latter case, we may perhaps visualize as follows the reason for the disappearance of the diamagnetism that should *apparently* result from the existence of the individual magnetic doublets due to the spiral motion of each electron. The doublets are the equivalent of small current-circuits in the same planes as the circular motions of the electrons. Each such current-circuit is equivalent to a circular magnetic shell; the intensity would be the same for every such shell if the spiral velocity were the same for all. The distribution of small circular magnetic shells can be thought of as replaced, without essential modification, by a series of uniform equal shells in each of a series of parallel planes spaced at the mean molecular distance apart; and the shells in each such plane can be supposed reduced in intensity, increased correspondingly in area, and changed in shape, so that their boundaries become contiguous, and their areas combine into one area, bounded by the non-overlapping parts of the boundaries of the outer shells in each plane. The shells in each plane can then be replaced by the equivalent currents round their boundaries. As in the usual proofs of Stokes's theorem on surface and line integrals, the currents along the interior boundaries will cancel out, and there remains only the outer circuit of all the amalgamated shells in each plane; this circuit is practically the same as the section of the boundary of the metal, by the plane in question, normal to the magnetic field. Thus the whole collection of circular electronic orbits in the interior of the metal is magnetically equivalent to a series of currents (or, again, to a continuous surface-distribution of current flowing transverse to the magnetic field) round the boundary of the metal, in planes normal to H. But the electrons which are sufficiently near the boundary to collide with it will, in the presence of the magnetic field, be caused (jointly by their collisions and by their spiral motions between collisions) to travel round the boundary in the sense opposite to that of the free circular motion. This is illustrated in Fig. 8, for a metal whose section normal to the field is a rectangle. The sense of the circular motions in the interior is indicated by the small circle within the rectangle. The combined circular motions of all the interior free electrons within a thin layer of the metal (of thickness equal to the average distance between the free electrons) will be equivalent to the motion of a single electron round the boundary, in the same sense, and with a speed which bears to the electronic speed the same ratio as the ratio of the combined area of the orbits of all the electrons in the layer, to the total cross-sectional area of the metal. The diagram shows also how an electron near the boundary would travel round the boundary as a result of successive collisions; this direction is opposite to the direction of the free circular motion. It may be shown that the currents due to such boundary electrons neutralize the current due to the boundary motion of the single equivalent electron corresponding to the interior undisturbed electric orbits.

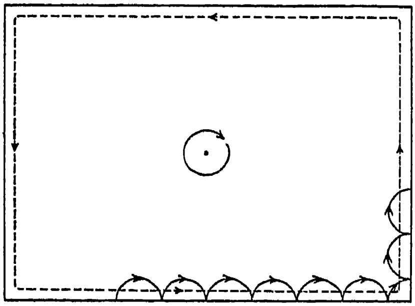


FIG. 8. The curve in the centre illustrates the circular (or spiral) motion of a free electric charge, in the presence of a uniform magnetic field directed perpendicularly to the plane of the figure; if the charge is negative (e.g. an electron) the direction of motion, as drawn, corresponds to a magnetic vector *into* the paper. The series of arcs round the edge of the figure illustrate the path of such a charge near the (rectilinear) boundary of the gas, where the charge continually collides with, and is reflected from, the boundary; the magnetic field bends the path round again so as to produce further collisions, with the result that the charge travels round near the boundary, in a sense (as illustrated by the dotted line) opposite to the sense of the circular motion of the charges (such as the one near the middle of the diagram) that do not meet the boundary. This motion of the boundary charges neutralizes the diamagnetism due to the central charges.

This illustrative argument may at least help one to understand how a compensation is brought about, which precludes the existence of diamagnetism due to this supposed cause.

This treatment can hardly be extended to the case of the ionosphere, in which the density has a continuous vertical gradient, due to gravity. The magnetic effect of the spiral motion must then be discussed by the methods of the kinetic theory. Such a treatment necessarily includes at the same time the drift-velocity to which Chapman attributed geophysical importance, and to which also he ascribed the radial limitation of the sun's magnetic field. If, in addition, the magnetic field is not uniform, the treatment must involve also another type of drift-current, discovered by Gunn [18], who regarded it as playing an important part in the mechanism of sunspot magnetic fields.

Cowling's first paper [27] on this subject was mainly concerned with Chapman's theory of the radial limitation of the sun's magnetic field by drift-currents [19 *a*], but the theoretical points involved apply equally to Chapman's discussion of drift-currents in the ionosphere [20].

Chapman had treated the force acting on each electron as $m_e g + eE$, where E is the Pannekoek-Rosseland electric field already referred to. The electric force eE is, according to (47), practically equal to half the combined weight of an electron and an ion; it acts downwards on the electrons, and upwards on the positive ions. Thus both the electrons and ions (only *positive* ions were considered in the discussion of the sun's magnetic field) are acted on by a force practically equal in each case to half the weight of an ion. This would cause the electrons and ions, if free, to drift with equal speeds in opposite directions, so that the currents due to the electronic and ionic drifts would be equal both in direction and magnitude. Owing to the mass-difference between the electrons and ions, and the correspondingly smaller spiral radius of the electrons, the latter are free to spiral in regions where the ions are not free; that is to say, for the two kinds of charge, there are different transition layers from the regions of no drift, no diamagnetism, and full electric conductivity, to the regions of drift, diamagnetism, and reduced conductivity.

If there is a resultant drift velocity of ions or electrons, in the transition layers and above, it will involve a mean electromagnetic force on the volume of gas containing these free charges; this force will be perpendicular to the drift, and to H, and since the drift is horizontal, and H is not vertical, the force will have a vertical component. This was not considered by Chapman in his equation for the statical equilibrium of the atmosphere. Cowling pointed out the necessity to take this term into account, and also introduced the equation of dissociative equilibrium between the ions, electrons, and neutral particles, which must hold in a state of thermodynamic equilibrium; in such a state the ionization is due entirely to the temperature of the gas—a condition possibly valid in the sun's atmosphere, but clearly inapplicable to the ionosphere. For this reason Cowling's discussion does not bear directly on the terrestrial importance of drift-currents.

23.17. A fundamental error in the simple drift-current theory. On correcting Chapman's discussion (of the sun's magnetic field) in these two respects, Cowling was led to a contradiction, which pointed to the need for still further modification. By a detailed argument too long to be reproduced here, he located the remaining error in the assumption that, in the presence of a pressure gradient, the mean values of $\dot{x}_0$ and $\dot{y}_0$ in 15(77, 78) are zero.

He showed that in an actual atmosphere, owing to the density gradient, more charges enter (and leave) any thin horizontal layer of gas through the lower boundary surface than through the upper surface; those charges coming from below will be near the top of their curved paths, and those from above will be near the bottom of their paths. But the velocity in the free motion represented by 15(77–9) is not the same at the top and bottom of the paths; hence, though each individual charge may have an average drift in one direction, the mean motion of the charges in any particular layer may be zero. Cowling's detailed discussion indicated that this is so.

Such an apparent paradox is not unfamiliar in the kinetic theory. Even in the case of a simple gas at rest in isothermal equilibrium subject to gravity, a somewhat similar paradox appears. Every individual particle has a downward acceleration (the same for all, that of gravity), yet every element of the gas as a whole is at rest. Within any thin horizontal layer of the gas, the average downward velocity of those particles that are descending exceeds the average upward velocity of the ascending particles; but the former are fewer than the latter, and the over-all average velocity is zero.

Cowling's conclusion that there is no drift-current in a gas in thermodynamic equilibrium can also be viewed in another way. In a paper on the sun's general magnetic field and the chromosphere [19 *b*], where calcium ions are subjected to a succession of radiative impulses, which help to support the chromosphere, Chapman showed that the succession of impulses on an ion can be treated as a continuous force, in calculating the drift-velocity of the ions. It seems likely that the proof can be extended to the succession of impulses due to collisions with other ions and with neutral molecules, and therefore that, when the charges are not diffusing upwards or downwards, the continuous force replacing the impulses just balances the vertical gravitational and electric force on a charge; the resultant vertical force thus being zero, so also is the corresponding drift-velocity. This would agree with Cowling's conclusion for the case of a gas in thermodynamic equilibrium. In the case of the ionosphere, which is not in such a state, and in which the presence of free charges is due to external causes (the absorption of ultra-violet light or of ionizing particles), vertical diffusion of the charges will in general occur. But the unbalanced vertical force causing drift-currents will be much less than that (approximately $\frac{1}{2}m_i g$) assumed by Chapman in his discussion of the drift-currents in the ionosphere; moreover, it is unlikely to have the same sign at all levels.

Chapman concluded that if the drift-currents in the earth's atmosphere are to have the same intensity as the actual S current-system [20], the number N' of 'drift-effective' charges per square centimetre column of atmosphere above the equator at noon must be of the order 10^{14}; by 'drift-effective' is here meant that each charge is counted not as one but only as $\omega^2/(\omega^2+\nu^2)$, thus allowing for the reduction of the drift-current by collisions. He estimated the actual value of N' as of order 10^{12}; the radio data underlying this estimate now need revision.

To obtain an upper limit for N', we may suppose that all the *electrons* in both the *E* and *F* layers are fully effective (i.e. that ν_e/ω_e is small enough to permit us to reckon $\omega_e^2/(\omega_e^2+\nu_e^2)$ as unity). The total number of electrons in the *E* layer may be estimated as less than 10^{12}, and in the *F* layer as less than 10^{13}. For the ions in the *E* layer it was estimated in 15.11 that ω_i/ν_i is about 1/30, so that $\omega_i^2/(\omega_i^2+\nu_i^2)$ is of order 10^{-3}. Taking the maximum possible ratio of ions to electrons in this layer as $3{\cdot}7\times10^3$, the contribution of the ions in the *E* layer may be estimated as $10^{-3}\times3{\cdot}7\times10^3$ or 3·7 times that of the electrons, and therefore less than 4×10^{12}. In the *F* layer, reckoning each ion as fully effective, and assuming that there the ions are equal in number to the electrons, the ionic contribution to N' will not exceed 10^{13}. Thus, in all, we may estimate N' as less than 3×10^{13}.

Thus, even on the basis of the faulty equations used by Chapman, the drift-currents seem inadequate, by a factor between 3 and 10, to account for the intensity of the S field.

When the upward impulses on the charges, at collisions, are taken into account, the inadequacy of the drift-currents appears still greater. Hence it seems possible to conclude that though drift-currents are likely to occur in the ionosphere, in association with general upward and downward motions of the charges, their magnetic field will be small compared with the S field.

23.18. The inadequacy of the diamagnetic theory of S. In a later paper [28] Cowling re-examined by different methods the problem of an ionized atmosphere in thermodynamic equilibrium in a magnetic field, especially in order to test Gunn's assumption that the arguments advanced against Schrödinger's theory of diamagnetism in metals were inapplicable to diamagnetism in our atmosphere (see also p. 52 of [25.9]). Cowling's conclusion was as follows:

> 'It appears, then, that in the transition layer in which electrons are becoming free to spiral under the action of the magnetic field there are drift currents similar to those' (cf. Fig. 2) 'which mask the diamagnetic effect of spiralling in a metal, and that these completely mask the diamagnetic effect in the gas.'

This conclusion, which does not depend on the detailed nature of the interactions at collisions, has an important bearing on Gunn's theories [18] of the radial limitation of the sun's magnetic field, and of the maintenance of magnetic fields in sunspots, which did not depend explicitly on an assumption that the state departs from one of thermodynamic equilibrium. In the case, however, of the diamagnetic theory of S, since the ionosphere is not in thermodynamic equilibrium, the precise importance of diamagnetism is less easily estimated. It seems likely to be less than Gunn's simple theory suggests, after allowing for the effect of collisions in reducing the diamagnetism (in the ratio $\omega^2/(\omega^2+\nu^2)$). But even without allowing for any reduction by boundary or transitional effects, the value of $N'T$ needed to account for the observed S field is $1{\cdot}6\times10^{19}$ (according to Chapman's calculation of the diamagnetic field; Gunn's calculation required the still higher value $5{\cdot}4\times10^{19}$); N' is here the same number as occurs in § 17, where it was called the number of drift-effective charges. Since T is unlikely to exceed 1,000°, N' must be at least 10^{16} to justify the diamagnetic theory; actually it seems to be not more than 3×10^{13}.

It thus appears that the free charges in the ionosphere are too few for their diamagnetism and drift-currents to produce a magnetic field of the same order as S, even taking the theoretical expressions used by Gunn and Chapman as correct; the inadequacy is still greater if their expressions are to be further reduced by compensating terms, such as are suggested by Cowling's discussion. The inadequacy without such reduction is by a factor between 3 and 10 for the drift-currents, and about 10^3 for the diamagnetism; the former factor, at least, is thought to be small enough, in view of the many uncertainties attaching to the upper atmosphere, to justify the above detailed discussion of drift-currents and diamagnetism.

McNish [7.23] has drawn attention to another argument adverse to the diamagnetic and drift-current theories. Both these theories are simple in the sense that they depend rather directly on the magnitude and distribution of one quantity, N' (or $N'T$); this possibly attractive simplicity has the disadvantage that it implies a certain rigidity in these theories, as compared with the dynamo theory, which involves not only the ion-content, but also the convection of the ionosphere. The considerable changes in S which sometimes occur from day to day (7.8) (as Moos and especially Hasegawa have pointed out) seem on this account more easily explicable on the dynamo theory than on the other two theories; they undoubtedly imply some change either of ion-content or motion in the upper atmosphere, of whose cause and nature we are still ignorant; but it is perhaps likely that the change depends at least partly on variations of motion as well as of ion-content.

Further, the remarkable intensity of S and L at Huancayo seems difficult to account for on the diamagnetic or drift-current theories, but may be explicable (§ 13) on the dynamo theory, as a consequence of the obliquity of the earth's axis (leading to terms in S dependent on the longitude as well as on the local time).

C. ATMOSPHERIC IONIZATION AND THE DAILY MAGNETIC VARIATIONS

23.19. Conclusions from the radio and magnetic effects of solar flares. In recent years radio methods (15.5) have determined the daily variation and the geographical distribution of the ionization in the different layers of the ionosphere; also the simple theory of ion-production and recombination (15.6, 15.7) has given a satisfactory qualitative explanation of these facts, at least as regards the E and F_1 layers. The daily variation and the geographical distribution of the ionization had previously been inferred from the discussion of the S_q and (more directly) of the L current-systems, on the basis of the dynamo theory; the inferences thus made are in good general agreement with the radio determinations, although the latter refer to the maximum specific electron-density in each layer, whereas the deductions from S and L refer to the integrated conductivity K in the conducting layer.

The magnetic data do not indicate the height of the conducting layers, and we are still ignorant as to the layer with which L is chiefly associated. The radio data furnish clear proof of the height and character of distinct ionized layers, and the recent discussion (10.3, 15.5) of the magnetic variations associated with brief solar flares or outbursts suggests that the S current-system flows mainly in or below the E layer (or possibly also in the F_1 layer).

The argument may be briefly repeated here: the connexion between the solar outbursts, the radio fade-outs, and the magnetic variations, as shown by their frequent coincidence in time, which is far closer than could be attributed to chance.

The radio data indicate that the fade-out is due to fleeting ionization in a layer below the E layer, at a height between 60 and 100 km.; electronic collisions are more frequent there than in the E layer, so that this new low ionized layer strongly absorbs radio waves which could otherwise be reflected downwards again by the upper ionized layers. The simultaneity in the production of this layer, and the observation of the visible solar flare, shows that the ionizing agent travels with the speed of light, and is therefore presumably ultra-violet radiation. In this respect it is similar to the ionizing agents mainly responsible for the E and F_1 layers, as has been shown by the study of the decrease of ionization in these layers during solar eclipses (15.5, 24.8); the radiation during the brief solar outburst is, however, more penetrating than the normal ultra-violet radiation that produces the E and F layers. The sudden ionization below the E layer produces magnetic effects which, as Fleming [10.9] suggested, consist of an augmentation of the S field over the sunlit hemisphere, including (as McNish [10.10] showed) any peculiarities in the S field that may exist on the day concerned. This is most naturally explained, on the dynamo theory, by supposing that the convection currents and the associated electromotive forces which impel the S current-system are always present, not only in the layers which ordinarily conduct the S current-system, but also in adjacent layers; so that, if these become suddenly ionized, new and similar currents are set up in them which augment the regular S field. Taking the location of these extraordinary S currents to be known, as below the E layer, a presumption is created that the ordinary S currents flow in an adjacent regularly ionized layer, and therefore in the nearest such layer, namely the E layer, and possibly also in the F_1 layer. It seems unlikely, in view of the great daily variation of height of the F_2 layer, that the daily convection currents in this layer are similar to those in and below the E and F_1 layers.

The rapid increase and decay of ionization in the abnormal sub-E layer, during a solar flare, will prevent the current-system in this layer attaining the intensity which would correspond to the existing electromotive forces, acting in a layer constantly ionized to the maximum degree occurring during the brief solar outburst. This must be allowed for in inferring the intensity of the electromotive forces from the

ctivity K of this temporary layer, if the value of K can be deduced
radio observations. In this way it may be hoped that some know-
may be gained as to the variation with height of the electromotive
and convection currents, between the level of the sub-E layer,
hat of the E layer itself.

20. The effect of the sunspot cycle on S and L. The long-
l changes of intensity of the S field, during the course of the sun-
ycle (7.3), are naturally ascribed to changes in either or both of
wo factors that govern S (according to the dynamo theory),
ly the atmospheric conductivity K and the daily convection cur-
We cannot say *a priori* that the latter remain constant and that
K changes with the sunspot epoch. Radio observations show
the ionization of all the three main layers varies in synchronism
the sunspot cycle; according to a recent report from the Depart-
of Terrestrial Magnetism of the Carnegie Institution of Wash-
n [32], the ionization of the E and F_1 layers was more than 45 per
higher near the 1937 sunspot-maximum than at the previous
ot-minimum (1933), whereas the F_2 layer shows an increase of
t 250 per cent. If the integrated conductivity K for each layer
ges in a similar ratio, this change alone, in the E layer (or E and F_1
s) would account fairly closely for the change in S between these
epochs; one might therefore conclude that this is the sole or main
e of the long-period changes in the intensity of S, and that the daily
ection in the E layer (or E and F_1 layers) does not undergo any
ked change in the course of the solar cycle. Such a conclusion
s not unreasonable, but must be regarded as only tentative, to be
ner examined as our knowledge of the ionosphere and of S and L
nces.

ne heating effects associated with the ionization of the upper atmo-
re appear to be much greater in the F_2 than in the E and F_1 layers.
change in the intensity of the F_2 ionization, in the course of the
pot cycle, may thus produce a long-period change in the average
perature-distribution with height, in this layer. The height-distri-
on of the intensity and phase of the atmospheric oscillation at high
ls appears, from the calculations of Pekeris, to be sensitive to the
ht-distribution of the mean atmospheric temperature. It is there-
possible that though the lunar tidal potential is almost independent
eight, and is certainly independent of the sunspot cycle, yet the
ar tidal circulation in the upper atmosphere may vary both with
ht and with the sunspot epoch, on account of the varying thermal

structure of the upper atmosphere. Until this possibility is better investigated, it is not legitimate to conclude from the known variation of L with the sunspot epoch (a variation which is less than that for S) that L is produced in a layer whose integrated conductivity K varies by only 20 or 30 per cent. in the course of a sunspot cycle. If L and S are produced in different layers, and if S is produced in or below the E layer (or in the E and F_1 layers), it might seem necessary to conclude that the L current-system flows mainly in the F_2 layer. But the ionization of this layer appears to vary more with the sunspot cycle than does that of the E and F_1 layers. On account of the daily variation in height and thickness of the F_2 layer, the daily variation of ionization and of integrated conductivity K in this layer are probably less parallel to the daily variation of maximum ion-content than is the case for the lower layers; but there certainly seems a difficulty in associating L chiefly with the F_2 layer, unless the lunar tidal convection is subject to large changes in the course of the sunspot cycle. These considerations at least suggest that the phenomena of the ionosphere are probably much more complicated than may appear at first sight, and that they cannot be expected to yield up their secrets readily. They give promise, however, of adding much to our knowledge not only of geophysics but also of solar physics.

23.21. The day-to-day changes in S. The day-to-day changes in S, sometimes remarkable both in intensity and type (7.8), suggest day-to-day changes not only of ionization, but also of convection, in the S layers of the ionosphere. Changes of ionization may mainly govern the day-to-day changes of *intensity* of S, and will somewhat affect the *type*, because their effect will not disappear at once, and will to some extent be carried round with the rotating earth; but it is hard to see how a beam of radiation, proceeding from a distant body like the sun, can produce irregular local inequalities of ionization over the earth (other than those depending merely on the sun's zenith angle). Hence the day-to-day changes of *type* of the S current-system seem to imply passing irregularities in the convection system. It is not easy to conceive likely causes for such large-scale irregularities of convection, though changes of cloud-distribution over large areas in the lower atmosphere may appreciably modify the distribution of outflowing radiation in the ionosphere, and so affect the convective circulation. If such low-level phenomena should prove to be mainly responsible for the changes in the ionospheric convection system from day to day, there should be little or no long-period change (11 years' cycle) in the average ionospheric convection system, because low-level meteorological phenomena show practically no correlation with the sunspot cycle.

As a possible cause of irregular local inequalities of ionization, the corpuscular solar radiation (causing polar lights) must be mentioned. The deflexion of this radiation by the earth's magnetic field (Ch. XXIV) makes its distribution more or less local in character. It seems certain that—although some of its effects are visible only on the night side of the earth—part of it will also impinge on the day side, possibly increasing the ionization in the S current-layer. The fact that the day-to-day changes of S occur also in very quiet times does not preclude a partial dependence on variations of corpuscular radiation, because aurorae are very frequent in higher latitudes, and near the auroral zone some sort of polar aurora is seen every second night on the average.

23.22. The effect of a solar eclipse on the S current-system. It is known from radio observations that during a solar eclipse the ionization of the E, F_1, and F_2 layers is reduced; this is in accordance with theoretical expectation, on the assumption (thus confirmed by the observations) that a main or important part of the cause of the ionization is ultra-violet radiation, travelling from the sun with the speed of light. The observations indicate about 50 per cent. reduction of ionization at the centre of the eclipsed area in the E layer of the ionosphere (this centre may be at some distance horizontally from the centre of the eclipsed area on the ground, if the sun is at a low altitude as viewed from the eclipsed area); the reduction in the F_2 layer appears to be less than 50 per cent., as would be expected in view of the slower recombination of free charges at the higher levels.

The eclipse-reduction of ionization implies also a reduction in the integrated conductivity K for each layer; this must affect the distribution of any electric currents flowing in the layer.

Many attempts have been made, particularly under the influence of Bauer, to determine whether during solar eclipses there are magnetic variations that can reasonably be associated with the eclipses. A brief account of the results is given in 10.8. Opinions have been divided as to whether eclipse effects on the magnetic field have been demonstrated: Bauer [33] was convinced that they had been established, Chree [34] left judgement in suspense. The question at issue, so far as the observations go, concerns quite small changes in the field, of magnitude 20γ or less. This is natural, since the eclipse-reduction of K can only affect the small part of the field that is due to upper-atmospheric current-systems; this includes the 'external' parts of the S, L, and disturbance

(D) fields. Any eclipse effect on the D current-systems will be mixed up with D variations that have no relation with the eclipse, and it will scarcely be possible to disentangle the two types of variation. The L field, on the other hand, is too small to be isolated on a single day, and an eclipse effect upon it is practically beyond our ability to determine from the observations. Hence the only eclipse effect that has any chance of being observed is one affecting the S current-system; an eclipse occurring on a day of magnetic calm may conceivably modify the S field appreciably. The intensity of the external part of the S field, at the earth's surface, is about 20 or 30γ, and an eclipse may be supposed to increase or decrease this by some moderate fraction of the whole.

Chapman [35] has discussed the nature of the eclipse effect to be expected consequent upon a temporary reduction of K over the eclipsed area. The reduction will vary from zero at the edge of the area, to a maximum near the centre. This continuously varying reduction of K occurs in a (curved) ionospheric region over which K also varies (independently of the eclipse) with the latitude and longitude. In his mathematical calculations, Chapman substituted for this rather complicated problem the simpler picture of a plane conducting sheet, in which a circular area has a uniform conductivity K', less than that (K) of the rest of the sheet. It is a simple matter to calculate the change which such an inequality of conductivity would produce, in a system of current flow due to a uniform unidirectional external electric field: such a field, in a plane sheet of uniform conductivity throughout, would of course produce a uniform unidirectional current flow; this is modified by the non-uniform conductivity over the circular area, representing the eclipsed area of the ionosphere. Fig. 9*a* shows the modification of the uniform current-system, for the case when $K' = \frac{1}{2}K$; the current-lines are drawn at intervals corresponding to equal current-flow between adjacent lines. It is clear that even a 50 per cent. reduction of K over the circular area (shown by a dotted line) does not greatly distort the current flow. The distortion shown is equivalent to that which would result if the current-system in Fig. 9*b* were superposed on the uniform current-system which would be present if the conductivity of the sheet were everywhere uniform. The current-lines in Fig. 9*b* are drawn at intervals such that between each adjacent pair the current-flow is only *one-quarter* of that between adjacent lines in Fig. 9*a*. The magnetic field of this additional current-system represents the magnetic disturbance due to the reduction of conductivity over the circular area.

This illustrative calculation should at least give a fair representation of the qualitative change in the distribution and intensity of the S current-system, when the eclipsed area happens to lie over a part of the S system (cf. Figs. 20.1, 20.2, p. 696, or Figs. 7.15, 7.16, p. 229) in which the current-flow is fairly uniform. The size of the area over which the ionization is materially reduced during an eclipse is fairly small compared with the area covered by each of the two great current-circulations over the sunlit hemisphere, as shown in the figures cited; the diameter of the penumbral eclipse region is about 4,000 miles, but the area over which the reduction of ionization is one-third or more is only about 1,000 miles; this may be called the inner eclipse area. It is only about one-tenth as large as the area of the current circulations over the day hemisphere.

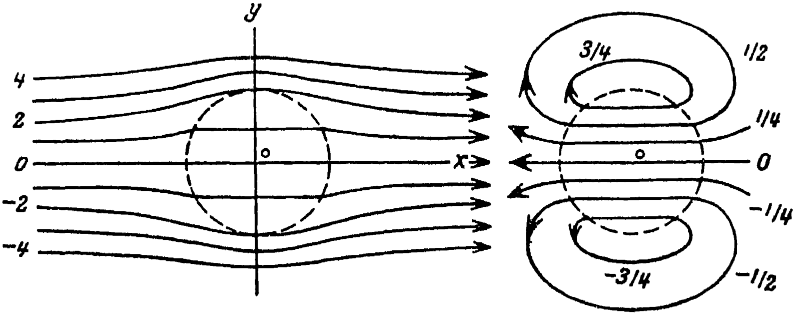


FIG. 9. Illustrating the effect of a circular area of reduced conductivity, in a plane conducting sheet carrying a current-flow which, apart from this area of reduced conductivity, would be uniform; the left-hand diagram (*a*) shows the distorted current-flow, and the right-hand diagram (*b*) shows the 'distorting' current-system which is superposed on the uniform current-flow. (This illustrates the effect of a solar eclipse on the current-flow in the ionosphere)

Taking the diameter of the circular area in Figs. 9*a*, 9*b* to be 1,000 miles, Chapman calculated approximately the magnetic field, at ground-level, of the disturbing current-system of Fig. 9*b*, taking this to be situated at a height of either 60 miles (the E layer) or 150 miles (the F layer); his calculation referred only to points on the ground along the line Oy, perpendicular to the direction Ox of the undisturbed current-flow in the layer.

Let H' denote the field of the (horizontal) uniform current-flow; this field is likewise horizontal, perpendicular to the current, and the same at all points, since the current-flow is supposed plane and uniform. Vertically below the centre O of the eclipsed area of the current-layer, the disturbance due to the current-system of Fig. 9*b* has its maximum value, and corresponds to a reduction of H' by the amount $0{\cdot}28H'$ (if the S currents flow in the E layer) or $0{\cdot}22H'$ (if their height is 150 miles), supposing $K' = \frac{1}{2}K$ in either case. At points along Oy, at a distance from O greater than the radius of the circle of reduced conductivity, the disturbance in H' will be smaller, and of the opposite sign (an increase); between such points and O the disturbance will vanish and change sign.

The reduction of H' at the point below O, as here calculated, will be increased to a small extent by the field of the current-system induced in the earth by the system of Fig. 9*b*; it will be decreased, however, owing to the fact that the system of Fig. 9*b*, as calculated, refers to the steady state, whereas in an actual eclipse K' varies from K to (say) $\frac{1}{2}K$ and back to K again, and the 'distorting' current-system (Fig. 9*b*) will not attain its full value corresponding to a steady state in which $K' = \frac{1}{2}K$. Moreover, the eclipsed area is not stationary, but moves rapidly; this fact is not taken into account in the above discussion. If the motion should happen to be nearly transverse to the local direction of the S current-lines, the fractional reduction in H' should be experienced successively at different points along the track of the eclipse centre.

The value of H' is not necessarily given by the departure of the horizontal force and of the magnetic declination from the *mean* value for the day, but is more likely to be given by the departure from the values of these elements shortly before dawn, when the atmospheric ionization, and presumably the S currents, are least. Thus at points near the centre of the eclipsed area, when this centre does not fall too near a focus of the S current-system, the magnetograph curves for horizontal intensity or declination may be expected to deviate from their normal course by less than one-third the way towards their night values. The expected deviation is therefore of amount 10 or 20γ at most. The effect should decrease with increasing distance from the centre of the eclipse, and be reversed beyond about 500 miles from the centre (along a line in the direction transverse to that of the S current-flow at the time of the eclipse); the reversed changes will be only a small fraction of the central changes, and will be difficult to establish with certainty.

If the S currents flow in the F_2 layer (as Chapman considered not improbable at the time of his discussion), the eclipse changes will be still smaller; but the E layer, or below, is perhaps the more likely location of the S currents. It would therefore appear that the eclipse changes will not be too small to be observable under favourable conditions, when the eclipse occurs on a magnetically calm day, and when the track crosses the region *between* the current-foci, where the current-flow is fairly uniform in direction.

In 24.8 an account is given of another possible effect of a solar eclipse, due to the interception by the moon of *neutral* corpuscles from the sun; this would affect the ion-content of the atmosphere in regions different from those over which the sun is visibly eclipsed, so that if S were partly produced in the layers thus affected, the S field would be modified slightly over these additional regions.

XXIV

CORPUSCULAR EMISSIONS FROM THE SUN, AND GEOMAGNETIC DISTURBANCE

24.1. Corpuscular streams, or bursts of ultra-violet light? In Chapter XXIII the S and L fields were discussed, and attributed to dynamo action in one or more layers of the ionosphere, ionized mainly, if not wholly, by ultra-violet light from the sun. The conclusion that this is the main source of the ionization is based partly on the much greater ion-content over the day than over the night hemisphere, which shows that the ionizing agent travels from the sun along a straight path; furthermore, observations of the ionosphere during solar eclipses (§ 8) indicate that the ionizing agent travels with a speed indistinguishable from that of light, and is therefore most probably light, and, because of its high ionizing power, ultra-violet light (15.5). It is possible, however, that neutral solar particles also contribute to the ionization of the upper atmosphere.

The phenomena of aurorae and geomagnetic disturbance (which in this chapter will together be referred to as the D phenomena) show many close relationships with the sun, where their primary cause is certainly located. How the sun acts to produce the D phenomena is, however, still in doubt. Schuster [1] thought that these phenomena result from a sudden intensification of the ionization of the upper atmosphere, by an increase in the ultra-violet radiation from the sun; it is now known, however, that the short flares of intense ultra-violet light occasionally observed, and accompanied by radio fade-outs, only cause a temporary augmentation of the S field (10.3, 23.19)—an effect quite different from the D phenomena.

The contrast of intensity of the S and L fields, as between the day and night hemispheres, is not shown by the D phenomena; hence these suggest action by some solar agent that can reach the dark hemisphere, turned away from the sun, as well as the sunlit hemisphere. Such an agent must be able to travel along a curved path, at least in the neighbourhood of the earth. Birkeland [2] concluded from this that electric charges are the cause of the D phenomena, since such charges, in motion near the earth, will be deflected by the earth's magnetic field, and so can get round to the dark hemisphere. Such deflexions may also explain another outstanding property of the D phenomena, namely, that aurorae are most frequent in the polar regions, particularly in the auroral zones centred at the earth's *magnetic axis-poles*, and that geomagnetic disturbance is most intense and most locally differentiated in these regions. This may be called the *polar* property of the D phenomena.

It is extremely difficult to account for this polar property if the cause is regarded as ultra-violet light, falling only on the sunlit hemisphere. Hulburt and Maris made a spirited attempt to construct an (ultra-violet) light hypothesis of the D phenomena (25.8), but with doubtful success. As McNish [10.10] has pointed out, the observed magnetic effects of the solar flares associated with radio fade-outs tell heavily against the light hypothesis of the D phenomena.

Another property of these phenomena which is unfavourable to a light hypothesis of their origin is the 27-day recurrence tendency (Ch. XII). This, which will be called the *recurrence*-property, has been very fully discussed by Maunder [11.34]. He showed that it can be most naturally explained by supposing that the sun emits streams of corpuscles from disturbed areas throughout intervals of time often extending over more than one rotation period of the sun; the D phenomena are supposed to be caused in some way when the stream encounters the earth. As the sun rotates, the direction of emission changes continuously, and the stream will sweep round, like water from a fire-hose whose direction of emission is rotated. If the stream is suitably directed, and the emission continues over a sufficiently long period, a stream which once encounters the earth will sweep on and may encounter it again, once or more, after successive rotations of the sun. This accounts for the 27-day recurrences; the fact that there is only a probability or tendency for recurrence, and not a certainty, can be explained in several ways; the emission of the stream may be intermittent or brief; the direction of emission may change; or the earth may move along in its orbit out of the track of the stream. This explanation fits well with many observed properties of disturbed areas on the sun. These often endure for more than one rotation period, but rarely for many such periods; old disturbed areas die away, and new ones appear elsewhere.

This interpretation of the recurrence property of the D phenomena implies that the solar agent travels outwards in laterally limited streams; these may pour out from the sun continuously, and not only during periods of geomagnetic disturbance; times of magnetic calm may occur either because there are no streams, or because the streams do not pass near the earth.

It is difficult, though perhaps not quite impossible, to suppose that bursts of ultra-violet light from the sun can be laterally confined, like a lighthouse beam. If they came out of deep holes on the sun, or were in some way focused into a beam before leaving the sun, they might be as compatible with the recurrence property of the D phenomena as the corpuscular streams; but this is hardly conceivable. It seems likely that a burst of ultra-violet light occurring anywhere on the visible hemisphere of the sun will affect the earth, though doubtless with greater intensity if the outburst occurs near the centre than near the edge of the disk. If such bursts are associated with disturbed solar areas, as it is natural to suppose, then they will affect the earth only when these areas are on the visible hemisphere. Their terrestrial effects may therefore be supposed to show the influence of the sun's rotation, by being more frequent in certain half-rotation periods than others, namely, in those during which the more disturbed hemisphere of the sun faces towards the earth. If the disturbed areas emitted ultra-violet light (with special intensity) throughout their lifetime, the terrestrial effects of this light should show the same weak and blurred influence of the solar rotation that is seen on the 27-day time patterns of sunspot numbers or areas for the whole disk of the sun (see Table I at the end of this book); each sunspot contributes to the daily sunspot figure for all the days of its visibility on the sun. If, however, the outbursts occur only sporadically during the lifetime of the disturbed area, the influence of the solar rotation on their statistics, and likewise on the statistics of their terrestrial effects, will be further weakened. Hence the rather marked recurrence property of the D phenomena seems unfavourable to the light hypothesis of their origin. It seems to favour some form of corpuscular hypothesis.

(Incidentally it may be remarked that this argument suggests the improbability of a marked 27-day recurrence tendency for the solar flares associated with radio fade-outs. Dellinger [10.7], had indeed suggested that these show even a 55-day recurrence tendency; but the data are as yet insufficient to establish any definite conclusion on the point.)

24.2. The emission of corpuscular streams. Eclipse photographs of the sun sometimes show great extensions of the solar corona in particular directions, and these seem to be usually associated with disturbed areas which are near the limb of the sun at the time of the eclipse. On a few occasions, far-reaching and laterally limited coronal streams have been photographed, extending outwards much beyond the ordinary limits of the corona. There is consequently some visible evidence for the occurrence of corpuscular streams from the sun. Uprushes of solar material above disturbed areas are frequently observed, but often the material falls back again to the surface, perhaps in the form of an arched prominence. Occasionally thin wisps of gas are seen to be impelled upwards, apparently travelling right away from the sun [8]; one such has been observed [9] to have a velocity of over 700 km./sec.

Such phenomena observed near the edge of the solar disk indicate emissions directed nearly at right angles to the direction from the sun to the earth. Allen [10] has observed a corpuscular emission travelling towards the earth with a velocity of 750 km./sec., as indicated by the displacement towards the ultra-violet of spectral lines for the light from dark 'markings' on the sun. But solar physicists have not yet succeeded in identifying the occurrences on the sun's surface which can with reasonable certainty be regarded as the cause of magnetic storms. One of the remarkable features of solar and geomagnetic statistics is the frequent great dissimilarity between the 27-day time patterns of magnetic activity and those of sunspot numbers or other indices of visible solar disturbance; all the solar indices appear to be much more closely correlated with one another, from day to day, than with the magnetic activity. If and when it becomes possible to observe the emission, from the sun, of the agencies that cause geomagnetic disturbance, it will become merely a question of fact, based on observation, whether the emission proceeds continuously, and is terrestrially effective only when the disturbed area on the sun is suitably directed in relation to the earth, or whether the emission is confined to periods corresponding (perhaps with some time-lag) to those of magnetic disturbance.

Little is known at present of the way in which the sun ejects matter from its atmosphere; Pettit [8] and his collaborators, who have closely studied the vertical motion of solar prominences, conclude that the material is subject to repeated sudden impulses rather than to a continuous acceleration. Milne [5.14] showed theoretically that, at high levels in the solar atmosphere, upward-moving atoms might be continually accelerated away from the sun; if the chief support of the atmosphere at such levels is provided by selective radiation pressure, by absorption of sunlight in a particular line or lines of the spectrum, the upward-moving atom, owing to the Doppler effect, may become able to absorb in the neighbouring spectral region outside the absorption line. It will then be subject to a greater radiation pressure than the other atoms, and be driven right away from the sun. Milne showed that the velocities likely to be thus obtained, at a great distance from the sun, do not much depend on the mass of the atom or on the particular absorption line; they are of the order 1,600 km./sec. The process may affect either neutral or ionized atoms. If for some reason such emission occurs with special intensity from a particular region of the sun, a corpuscular stream of neutral atoms, or positive ions, or both, will proceed outwards from this area. While this type of emission appears likely to occur, no definite observational evidence indicating its occurrence seems to have been obtained. The subject of solar emissions has been further discussed by Pike, Kiepenheuer [34, 35], Waldmeier [36], and others.

Pike [11] showed that the vertical instability due to the Doppler effect, on which Milne's process of emission is based, will have also a horizontal counterpart; this may be brought into play by local temperature inequalities on the sun's surface, or merely by an abnormal horizontal component of atomic motion in the absence of such temperature inequalities. On this basis Pike gave a general explanation of prominence motions, showing that sunspots, owing to the deficiency of radiation pressure from them, must act like centres of attraction to prominence matter in their vicinity, causing the matter to be drawn towards them, as is often observed. Sunspots are often thought to be associated with magnetic storms, but if so this would seem to be due to their association with neighbouring bright areas, which are able to eject matter away from the sun, in spite of the 'attraction' exerted by the spots themselves.

Pike showed also that atoms which in some manner gain a moderately abnormal *horizontal* velocity component will be *vertically* as well as horizontally accelerated; unless drawn back into the sun by neighbouring dark areas or sunspots, they may leave the sun altogether.

Particles ejected from the sun with a speed which, over a considerable fraction of the distance between the sun and the earth, is of the order 1,600 km./sec., would take about 26 hours to travel to the earth's orbit. This is about equal to the interval found by Greaves and Newton, and by Abetti, for the interval between the passage of notable sunspots across the sun's central meridian, and the beginning of great magnetic storms apparently associated with the spots (Ch. XI). Maurain found a rather longer interval (about 2·5 days) between days of notable geomagnetic disturbance and the epoch of maximum sunspot number in the neighbouring period. These statistical results are generally regarded as favouring a corpuscular speed of 1,000 to 1,600 km./sec.

Some writers, however, such as Birkeland [2] and Dauvillier [7], have for various reasons suggested that the sun, by some unexplained process, is able to eject streams of charged particles with a speed far greater than these, indeed approaching that of light, and have attributed [3–6] a great variety of phenomena, terrestrial and extra-terrestrial, to their agency. Auroral and geomagnetic disturbance due to such particles should begin after only a few minutes from the time of their emission.

24.3. The geometry of corpuscular streams. Chapman [12.17] has discussed the geometry of a stream of corpuscles emitted continuously from a small region P on the rotating sun (12.9). The velocity of emission was supposed to have a constant radial component v_0 and a transverse (solar horizontal) component $\omega_0 a$, where a ($= 7\times 10^5$ km.) denotes the sun's radius. The particles were supposed subject after emission only to radial forces, so that their angular momentum about the sun's centre O remains constant, and the transverse velocity at distance r from the centre is ωr, where $\omega r^2 = \omega_0 a^2$. The radial velocity at distance r is supposed to be $v = f(r)$. The analysis will not be reproduced here; only the main results will be described.

The particles emitted prior to any time t will lie along a certain curve, and each point on this curve will correspond to an earlier instant of emission (t_0), a parameter of the point. If the initial transverse velocity of the particle is merely that of the sun's rotating surface at the point of emission (this velocity at the sun's equator is only about 2 km./sec.), and if either v_0 or v, beyond a distance of not more than a few solar radii from the centre, is of order 1,000 km./sec. or more, then the path of any individual corpuscle will be approximately along the radius OP drawn through P at the instant of its emission; actually the path will diverge slightly from this radius in the 'forward' direction, 'forward' here meaning in the direction of the sun's rotation. Even if the initial transverse velocity is as great as 100 km./sec., the path of the particles, beyond a distance of a few solar radii, will be nearly along a straight line through O; it will be inclined slightly to this line, either forward or backward according as the horizontal component of the velocity of emission is forward or backward.

The curve of the stream, however, will bend in the backward direction, lagging behind the sun's rotation. This curve will change its position but not its form as t changes; it will sweep forward with and round the sun, like the water from a moving fire-hose, as already mentioned. Some confusion of thought has at times arisen on this point, the rotation of the curve of the stream being supposed to imply rotation of the matter of the stream round the sun, like the rotation of a rigid body; this idea is of course in direct opposition to the law of conservation of angular momentum. The material of the stream moves almost radially outwards at all distances, and therefore advances obliquely to the curve of the stream. This curve is merely the changing locus of a line of moving particles.

The initial transverse velocity of the particles can probably be neglected, in the case of most streams that reach the earth, if the average radial velocity is of order 1,000 km./sec. or more. If v is constant, the angle subtended at the sun's centre O by the curve of the stream, between the point of emission P (supposed situated on the sun's equator) and the point where r is equal to the radius R ($= 1{\cdot}5\times 10^8$ km.) of the earth's orbit, will be 24° if $v = 1{,}000$ km./sec., 15° if $v = 1{,}600$ km./sec., and 12° if $v = 2{,}000$ km./sec. These angles are also very nearly equal to the inclination of the curve of the stream at P to the radius from O.

If we ignore the obliquity of the sun's axis to the pole of the ecliptic (the plane of the earth's orbit), then a stream emitted radially from a point on the sun's equator will sweep round across the earth's orbit. The particles advance nearly radially, and the curve of the stream sweeps onwards round the sun, while the earth moves forward in the same direction; but whereas the earth traverses its orbit only once in a year, the sun's equator rotates once in 25 days, so that the stream will overtake the earth. Successive passages of the stream across the earth will, owing to the earth's motion, take place at intervals of about 27·3 days (the sun's synodic rotation period).

This is illustrated by Fig. 1, in which the stream-curves are drawn for two corpuscular speeds, one very small (200 km./sec.) on the left, and the other, on the right, 1,000 km./sec. The dotted lines show the stream-curves or lines of particles previously emitted from a point P on the sun, at the instant when P is on the centre of the sun's disk as seen from the position A of the earth in its orbit. When the stream overtakes the earth, this has moved on to the point B; the time elapsed is 9·3 days or 1·7 days in the cases illustrated; this exceeds the time of travel of the particles from the sun to the earth's orbit, because of the orbital motion, the difference being 16 and 3 hours in the two cases shown. The radial lines in the diagram on the left are drawn to the particles emitted 1, 2,... days after the epoch when the earth was at A.

The stream will overtake the earth on its rearward side (looking along the direction of orbital motion); which, since the earth's rotation about its axis is in the same sense as its revolution in its orbit, is its *post meridiem* hemisphere. The first contact of the stream with the earth will be near the evening twilight meridian, but actually a little on the sunlit side of this meridian, owing to the curvature of the stream.

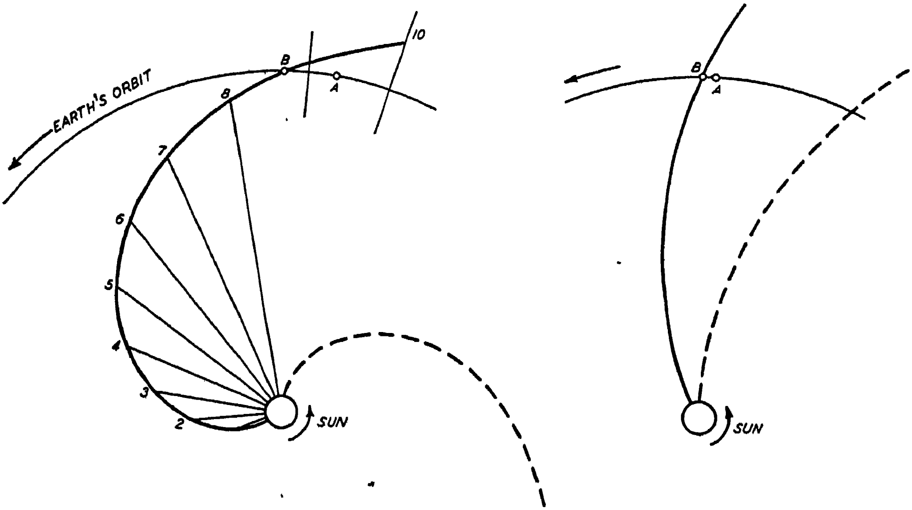


FIG. 1. Illustrating the curvature of a stream of particles, continuously emitted radially from a point on the sun, owing to the sun's rotation. The radii of the sun and of the earth (indicated by the small circles A and B, at two points along its orbit) are shown magnified tenfold, in relation to the distance (the orbital radius) between them. On the left the speed of the particles is taken to be 200 km./sec., and on the right, 1,000 km./sec. On the left the continuous heavy line represents the particles, and the numbers along the line indicate the positions of particular particles emitted at equal successive intervals of time; each such particle has, since its emission, travelled along the radial line shown

If the obliquity of the earth's equator to the ecliptic is ignored, the first contact of the stream with the earth will occur on the equator at about $4^h\,40^m$ p.m., 5 p.m., or $5^h\,12^m$ p.m. if $v = 1{,}000$, $1{,}600$, or $2{,}000$ km./sec. respectively. These results take no account of the possible magnetic deflexion of the stream near the earth.

The time taken for the line of the stream to sweep across the earth is independent of the speed v; it is governed only by the sun's synodic rotation period, and by the angle $17\cdot5''$ subtended by the earth's diameter at the sun; it is found to be about 30 seconds (cf. 9.24).

If the stream, instead of being merely a mathematical line, proceeding from a *point* P on the sun, is emitted radially from a small but finite *area* on the sun, it will, at any time t, occupy a volume shaped like a curved diverging tube. This tube will be bounded by the surface described by the line of particles (for the time t) emitted from a point P on the *boundary* of the solar area, as P is made (in imagination) to travel round this boundary. The diameter of cross-section of the stream, at the earth's distance R, will be R/a times the diameter of the emitting area; $R/a = 216$. The area of cross-section at the distance R will be $(R/a)^2$, or 46,000, times the area of emission. The density of the stream at the distance R will conversely be only 1/46000 times the density at emission, supposing v constant; if v is not constant, the ratio is $(v_0/v)(a/R)^2$. If the emitting area on the sun is a circle whose diameter subtends only 1° at the sun's centre O, the diameter of the stream at the earth's distance will be 200 times that of the earth. Thus a stream emitted from even so small an area on the sun would completely envelop the earth; it would take 200×30 seconds, or $1^h\ 40^m$, to sweep right across the earth, from the first contact of its forward surface with the earth's p.m. hemisphere, to the last contact of its rearward surface with the a.m. hemisphere.

If the duration of a magnetic storm (say 1 day) be regarded as indicating the time during which the earth is enveloped in such a solar stream, the diameter of the stream near to the earth must be much greater, nearly 3,000 times the earth's diameter; this would correspond to an emitting area subtending an angle of about 15° at the sun's centre, if the emission is radial.

It is not likely, however, that the emission is strictly radial. The gas emitted is likely to have the solar temperature, which, taken to be 5,000°, corresponds to thermal velocities of 11 km./sec. for hydrogen (H) atoms, and 1·8 km./sec. for calcium (Ca) atoms. The lateral component of these thermal velocities will cause a sideways expansion of the stream. This is in addition to the expansion above described, due merely to the divergence of the radii from the sun's centre O to the various points of the emitting area. If the stream is uniformly accelerated from a small radial velocity at the sun to the velocity 1,600 km./sec. near the earth, so that its time of travel from the sun to the earth is two days, the thermal broadening of the stream during this interval will be $3{\cdot}8 \times 10^6$ km. or $6{\cdot}2 \times 10^5$ km., for streams composed of H and Ca atoms respectively. This will represent a greater or less ratio of expansion, according to the size of the emitting area; if the angular diameter of the area, viewed from the sun's centre, has the values 1°, 5°, 10°, 15°, then at $r = R$ the breadth of the stream, apart from the thermal expansion, will be respectively $3{\cdot}8 \times 10^6$, $1{\cdot}9 \times 10^7$,

$3{\cdot}8\times10^7$, $5{\cdot}7\times10^7$ km. The ratio of the thermal broadening to this breadth is, in the respective cases, 1, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{15}$, for hydrogen atoms, and $\frac{1}{6}$, $\frac{1}{31}$, $\frac{1}{61}$, $\frac{1}{92}$, for calcium atoms. Clearly the thermal broadening is of importance only for narrow-angle streams, and more particularly for those composed of hydrogen or other light atoms.

24.4. The influence of the heliographic latitude of the source. The sun's axis is not normal to the ecliptic, as has been supposed, for simplicity, in the above discussion; it is inclined at 7·3° to the normal. The earth attains its maximum northerly and southerly heliographic latitudes (+7·3° and −7·3°) on about September 5 and March 5; it crosses the plane of the sun's equator on about June 5 and December 5 (5.1).

Moreover, the emitting areas, if they are supposed to lie in the same latitudes as sunspots, will occur most often between heliographic latitudes 10° and 15° (N. or S), though they appear at times in all latitudes between ±30°, or even beyond.

If the emission is radial, and the emitting centre does not lie on the sun's equator, the curve of the stream will not lie in a plane, but will describe a surface of revolution about the sun's axis, approximating to a cone at distances of order R. The position and extent of the stream at such distances depend mainly on the direction of emission rather than on the precise situation of the emitting centre. In the absence of evidence to the contrary, it seems reasonable to assume that the emission from any point is on the average along the radius through the point, though there may well be many deviations from this mean direction.

Consider a conical stream of semi-angle α, emitted so that its axis is inclined at the angle l to the sun's equator; if the emission is radial, the emitting point must be in heliographic latitude l. The stream will sweep over any distant object of which the heliographic latitude lies between $l\pm\alpha$; the latitude (l') of the earth must lie between these limits if a magnetic storm is to occur. When $l'=0$, that is, near the solstices, streams radially emitted from areas in the latitudes $\pm12\frac{1}{2}^\circ$ must be of angular breadth not less than 25° in order to affect the earth. When $l'=7{\cdot}3^\circ$ N., the angular breadth need only be 11° for streams radially emitted from areas in north latitude $12\frac{1}{2}^\circ$, though for streams from an equal southerly latitude the breadth must be at least 40°. It would therefore seem likely that in September magnetic disturbance is commonly due to northern solar areas, and in March to southern solar areas. Attempts have been made to test whether the sunspot and magnetic data are in agreement with this expectation (cf. p. 415), but no final conclusion has been reached.

It is well known that the frequency and intensity of terrestrial magnetic disturbance have two annual maxima, at about March and September, with intervening minima near the solstices (11.9).

These maxima and minima are the same for the earth's northern and southern hemisphere, showing that they are not due, like the ordinary terrestrial seasons, to the annual changes in the *orientation* of the earth relative to the sun.

The changes of magnetic disturbance refer to the earth as a whole, and recur year by year as the earth revolves round the sun; hence they may reasonably be attributed to the changing position of the earth as a whole, being determined by its heliographic latitude (and longitude); this accords with the suggestion made by Cortie [13], that the reason why the maxima of disturbance occur near the equinoxes is because then the radius from the sun to the earth is nearest to one or other maximum sunspot zone. If on the average the solar emissions are radial, it is necessary, for this explanation to be satisfactory, that streams of which the angular breadth exceeds 11° shall be more than twice as numerous as those of which the angular breadth exceeds 25°; for at the equinoxes all the former, but from one zone only (except the few of breadth greater than 40° from the other zone), will sweep over the earth, while at the solstices streams from both zones equally will sweep across the earth, but only if their angular breadth exceeds 25°; and the conjunctions are more frequent at the equinoxes than at the solstices. The restriction is not accurately stated, because of the 'spread' in the latitudes of the emitting areas, and in the directions of emission relative to the radius through the point; but it is clear that the equinoctial maxima of magnetic disturbance imply a restriction of this kind. The restriction itself, as above stated approximately, does not seem at all improbable.

24.5. The penetration of the corpuscles into the earth's atmosphere. Chapman's geometrical discussion of a corpuscular stream, summarized in § 3, is essentially independent of any particular hypothesis as to the mode or velocity of emission of the solar stream, though the illustrative calculations have mostly referred to velocities of the order (1,600 km./sec.) suggested by Milne's theoretical process of emission. This velocity seems to agree well with the statistical estimates of the time of passage of the corpuscles from the sun to the earth, though these estimates cannot be completely relied on, so long as we cannot identify the emitting areas on the sun.

There is, however, a difficulty, to which Milne himself [5.14] drew attention, in explaining how particles with no greater speed than this, even if they are as massive as calcium atoms, can penetrate so far down into the earth's atmosphere as the observed lower level of auroral rays, which is often at 90 or 100 km. height, but sometimes distinctly lower, even down to 70 or 65 km. height.

After reviewing the experimental data bearing on the penetration of air by calcium atoms with this velocity, Milne concluded that the equivalent air-range of such particles, that is, their range in air of the density existing in the atmosphere near the ground, is about 0·15 cm. He also pointed out that, according to the calculations by Chapman and Milne [15.6 *a*], the equivalent air-ranges down to 100 km. and 80 km. height respectively, along a vertical path, are about 0·4 cm. and 5·6 cm. Even the former is thrice too large to be consistent with the supposed velocity of the calcium atoms. Actually, however, the difficulty is much greater, because there can be little doubt that the densities calculated by Chapman and Milne for heights above 80 km. are too small (Ch. XV); the exact values are still uncertain, but their estimates of the equivalent air-ranges down to 100 and 80 km. respectively should probably be increased by factors of about 4 and 2, to about 1·5 and 10 cm. The latter is beyond the range (7 cm.) of the fastest particles from natural radio-active atoms, and even the former is ten times as large as the range of the calcium atoms moving with the velocity estimated by Milne.

Unless some undiscovered mechanism exists which imparts much greater velocities to the solar corpuscles (cf. § 10), possibly only in the near neighbourhood of the earth, we must conclude that the earth's atmosphere is more penetrable, down to 100, 80, or even 70 km., than is indicated by our present information concerning the upper air and concerning the range of atomic particles moving with high speeds; or else we must conclude that the lower limits of auroral luminosity do not mark the depth of penetration of solar corpuscles. The latter solution of the difficulty seems improbable; it would imply that the low aurorae correspond to electric discharges induced by corpuscular precipitation at higher levels; if so, it would seem difficult to account for the often rather definite form of the auroral rays.

24.6. The detection of the corpuscular streams by solar observations. The density of the solar streams is not yet known, either by inference from the solar conditions, or from any established theory of the geomagnetic and auroral phenomena which are attributed to such streams. It seems of interest to point out that, if the emission process is as suggested by Milne, there is some possibility [12.17] of determining the density directly and of detecting the presence of the stream between the earth and sun, by comparing the intensity of the solar spectrum near the main absorption lines, at times of magnetic quiet and of magnetic disturbance. At the latter times the earth is probably enveloped in the stream for many hours, or even for days, and the whole of the sun's disk must then be viewed through a great length of the stream. If the density is sufficient, the spectrum of the light from any part of the disk should show an absorption band on the violet side of the absorption lines 'out of which' the atoms, according to Milne's theory, have 'escaped'. The band will be wide because of the great variation of velocity along the stream; the supposed extreme velocity of $1{\cdot}6 \times 10^8$ cm./sec. corresponds to a Doppler shift of the K line of calcium of about 22 A, so that the shallow absorption band will extend 14 A beyond the normal boundary of the line (of which the usual half-width is 8 A) on the violet side.

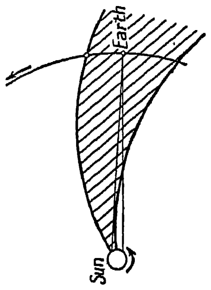


FIG. 2. Illustrating two lines of sight from the earth to points on the sun's disk, when the earth is enveloped in a solar stream. The absorption of light along the line of sight will be greater for the left-hand than for the right-hand line

This band will be a continuous extension of the normal absorption line, if the line of sight from the earth traverses the whole length of the stream; but if it traverses only a part of it, as in Fig. 2, namely a part extending from the earth (within the stream) up to a distance r from the sun, the band may be separated from the boundary of the absorption line. If the stream is approaching the earth but has not yet enveloped it, the regions of the sun to the (solar) east of the emitting area will not show the absorption band at all, while those to the (solar) west of the area will show the band separated from the line, and also stopping short of its full extent (14 A) in the violet direction.

Owing to the expansion of the stream as it travels from the sun, the number of atoms in it per cm.^{3} at any distance r will be approximately $(R/r)^2 n'$, where n' is the number per cm.^{3} at the earth's distance R. The stream will be densest near the sun, where the velocity of the particles is least; Milne has shown that the atoms just get clear of the normal absorption line (if this is the K line of calcium their velocity is then $6{\cdot}1 \times 10^7$ cm./sec.), at $r = \frac{4}{3}a$. From his formula for v as a function of r, combined with the above approximate expression for the density of the stream at distance r, it is easy to deduce the law of variation of intensity in the absorption band, corresponding to a line of sight passing either right through the stream, or through only a part of it.

A line of sight which traverses the stream from distance r to R will pass over

$$R^2\left(\frac{1}{r}-\frac{1}{R}\right)n' \tag{1}$$

atoms per sq. cm. of the beam. If $r = a$, so that the line passes through the whole length of the stream, the number is approximately $3\times 10^{15}n'$. Eddington [14], discussing absorption by diffuse matter in interstellar space, on the basis of Milne's theory of chromospheric absorption, estimated that a moderately dark absorption line in the spectrum of a star would be produced by $1{\cdot}5\times 10^{13}$ calcium$^+$ atoms per sq. cm. Hence if all the atoms in the stream were calcium ions, and if the density n' near the earth were only 1 atom per 100 c.c., a measurable amount of absorption should result; the absorption would, however, be made more difficult to measure, owing to its being spread over a width of 14 A. If $n' = 1$, the absorption would be 100 times as great; at present we have no knowledge at all as to the value of n', e.g. whether it is 100, 1, or 0·01. Study of the sun's spectrum during times of magnetic disturbance, in the manner here suggested, might enable an upper limit, at least, to be set to n'.

In making the observation it might be best not to look through the whole length of the stream, because when this is done the spectrum examined is that of the emitting area, which is likely to be abnormal just because it is a centre of emission. The line of sight to other regions on the sun, where conditions are normal, will pass out of the stream at some distance from the sun, thus cutting off the most intense part of the absorption band due to the stream; but the separation between this band and the normal absorption line may, if not too great, enable the stream absorption band to be more easily detected.

Further suggestions relating to this proposed method of observing the presence of the stream by solar observations are given in Chapman's paper already cited [12.17]. Should it be found that such absorption bands are observable during times of magnetic disturbance, a great extension of our knowledge of the solar emissions would become possible. Not only the density but also the velocity and the composition of the stream might be determined (the velocity by the width of the bands, and the composition by the absorption lines which are thus bordered by absorption bands).

24.7. The composition of the solar stream. According to Milne the process of emission which he described should eject atoms of all the kinds which show evidence of being supported by radiation pressure in the chromosphere. These include ionized atoms such as Ca^+, Ti^+, Sr^+, and neutral excited atoms such as H, He, Na, Fe, Mg. The limiting speeds for the different kinds of atoms will be different, but all are likely to be of the same order of magnitude.

The neutral atoms will be undeflected by the earth's magnetic field, and will therefore impinge directly on the sunlit hemisphere of the earth, and contribute to the ionization in some layer of the atmosphere, at a height determined by the speed and penetrating power of the particles, and by the height-distribution of density in the atmosphere.

The neutral particles impinging on the sunlit hemisphere may include some atoms that on emission were ionized, but have recombined with an electron during their journey from the sun. Lindemann [15] showed in 1919 that the amount of recombination will probably be small, in the sense that a large percentage of the ionized atoms will remain ionized; but even a small percentage of recombinations may be of terrestrial importance.

The proportion of *recombined* neutral atoms to the ionized atoms can be calculated (very roughly) by using a modified Saha formula. Assuming a balance between recombination and ionization at the distance R, we have

$$\frac{N_i^2}{N} = \frac{(2\pi mkT)^{\frac{3}{2}}}{wh^3\delta} e^{-\chi/kT}, \tag{2}$$

where N_i and N are the number of ionized and recombined atoms per cm.^{3}, χ denotes the ionization energy, w is a weight factor, assumed equal to 2, and δ is a 'dilution factor' $\frac{1}{4}(a/R)^2$;

$$m = 9\times 10^{-28}, \qquad h = 6{\cdot}54\times 10^{-27}, \qquad k = 1{\cdot}37\times 10^{-16}.$$

For calcium atoms, with an ionization potential of 6·1 volts, at $T = 6{,}000^\circ$ the value χ/kT is 11·8. Hence $N_i^2/N = 2{\cdot}4\times 10^{10}$, or $N/N_i = N_i/2{\cdot}4\times 10^{10}$. If we assume that N_i is not greater than 10^5, which is perhaps much too great, this indicates that the ionized atoms outnumber the neutral ones in the ratio $2{\cdot}4\times 10^5$ at least. The above formula is likely to underestimate the neutral atoms, but unless N_i exceeds 10^5, the recombination will hardly amount to 1 per cent. The calculation does not refer at all to the proportion of neutral atoms originally ejected from the sun. If N_i were so large as 10^5, there should be intense displaced calcium absorption lines in the sun's spectrum during magnetic storms (§ 5).

Another treatment of the recombination, based on Kramers's theory of absorption and emission, has been given by Ferraro [16].

Milne's emission process may be expected to operate continuously over the whole solar disk, as well as being possibly the cause of the emission of specially intense streams of particles from disturbed areas. The regular emission of *neutral* atoms, which travel undeflected by the earth's magnetic field, will contribute to the regular daily ionization of the sunlit hemisphere, and any special streams including neutral particles will temporarily intensify such ionization over the sunlit hemisphere.

24.8. Solar eclipses, and ionization by neutral corpuscles. Solar eclipses afford a clear test of the question whether any of the known ionized layers of the atmosphere owe an appreciable part of this ionization to the *regular* emission of neutral corpuscles from the sun.

An eclipse occurs when the moon comes between the earth and the sun, so as to cut off some of the sunlight that would otherwise fall on the hemisphere of the earth that faces towards the sun (the 'sunward hemisphere').

This cut-off affects the light of all wave-lengths, from the extreme ultra-violet to the far infra-red. The astronomers are concerned only with that part of this light which normally traverses the whole of our atmosphere and so reaches the ground; it is by such light that they are able to see or to photograph the sun. The interest of the eclipse for them is that, from points in the belt of totality, where for a few minutes the sun's visible disk is wholly covered by the intervening moon, they are able to see phenomena, such as solar prominences or the corona (extending beyond the disk), which at other times it is difficult to observe.

The partial or complete cut-off, during the eclipse, of light that is ordinarily intercepted in the higher layers of the atmosphere, has local effects that are only indirectly observable. Part of this light is able to ionize the air which absorbs it, and when such light is cut off, the ions and electrons begin to recombine, and the ion- and electron-content decreases; this interrupts the normal increase of the ion-content, if the eclipse occurs in the forenoon, or accelerates the normal decrease, if the eclipse is in the afternoon. Radio observers are able to follow the daily changes of electron-content in the various layers of the ionosphere, and can also observe any abnormalities in these changes, such as an eclipse may produce.

The reduction of the ion- and electron-content in the ionosphere during an eclipse also reduces the electrical conductivity of the various layers, and (as shown in 23.22) will thus modify the course of the current-systems that produce magnetic variations at the earth's surface. It may be possible to determine the resulting change in the solar daily variation S in the eclipsed and neighbouring regions, but this is a much more difficult matter than the observation of the change of electron-content by radio methods; and the magnetic changes, even if they are determined, are much less easy to interpret than are the radio data. They are nevertheless worthy of attention, because they can contribute some information different from and supplementary to that which the radio methods afford.

If the sun sends out neutral corpuscles in all directions, which also help to ionize the earth's upper atmosphere, their influx will likewise be interrupted when the moon comes between the sun and the earth. Since they do not penetrate to the ground, their cut-off during an eclipse can be observed *only* by radio methods, or less directly by the magnetic effects.

Owing to the probable great difference between the speed of light and the (supposed) speed of the corpuscles, the time and place of the cut-off of the corpuscles from the ionosphere will differ considerably from that of the ultra-violet light; this was shown by Chapman [15.37], who gave the name *corpuscular eclipse* to distinguish this phenomenon from the *optical eclipse* observed by astronomers.

In considering the geometry of an optical eclipse, the speed of light may be treated as practically infinite. The geometry of the region of space which is shaded by the moon from the light of the sun then becomes very simple, and depends, at any instant, only on the positions then occupied by the sun and moon. Two parts of the shadowed region can be distinguished—one wholly and the other partly in shadow. Their boundaries are the two conical surfaces which can be drawn to touch both these spherical bodies. Since the diameter of the sun exceeds that of the moon, one of these cones tapers from the sun towards the moon, and beyond the moon to a vertex V. The region within this cone, between V and the moon, is called the umbral region; the sun is completely invisible from any point P within this region, because it lies wholly within the cone drawn with V as vertex and touching the moon (as shown schematically—not at all to scale—in Fig. 3*a*). The other cone that is tangent both to the moon and sun has its vertex V′ between them, and beyond the moon it diverges. The region within this part of the cone, beyond the moon but outside the umbral region, is called the penumbral region. From any penumbral point P′ the sun is partly visible, because the tangent cone to the moon, with vertex at P′, intersects the sun (Fig. 3*b*), so that part can be seen and the rest is shaded from P′.

Relative to the earth the moon describes an approximately circular orbit in a plane which is inclined by a few degrees to the earth's orbital plane, called the ecliptic; the latter is practically fixed, but the moon's orbital plane is not; its line of intersection with the ecliptic revolves nearly uniformly in the ecliptic. The moon comes between the earth and the sun only when two conditions are fulfilled; the moon must be passing through this line of intersection (where it is met by the moon's orbit), and this line must at the same time lie along the line from the earth to the sun; if these conditions are not both approximately satisfied, the shadow cones cast by the moon will not intersect the earth—if the moon is not near the ecliptic plane its shadow cones will pass too far above or below the ecliptic, and even when the moon is in the ecliptic, its shadow cones will not meet the earth unless the sun, moon, and earth lie nearly along a straight line. Naturally also the moon must be *between* the sun and the earth; if it lies on the other side of the earth, it will be itself eclipsed by the earth.

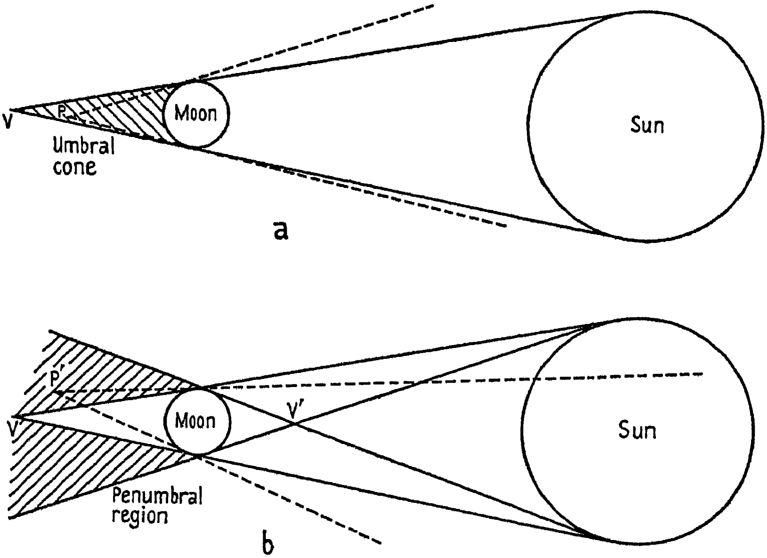


FIG. 3. Diagrammatic representation of the umbral and penumbral cones from the sun to and beyond the moon; these bodies are drawn much magnified (the moon more than the sun) in relation to the distance between them. Lines of sight from points P and P′ in the umbral and penumbral shadowed regions are shown

Since the angular diameters of the sun and moon, as seen from the earth, are nearly equal, the vertex V of the umbral cone is near the earth's surface at the time of an eclipse; it is sometimes above the earth, in which case there is no total eclipse, but only an annular one. A total eclipse occurs only when V is 'below' the earth's surface; the umbral cone then includes a roughly oval area of the earth's surface, which is the region of totality, and the penumbral cone includes a much larger area, over which there is a partial eclipse. Since the earth and moon are both in orbital motion relative to the sun, these eclipsed areas are not stationary; the shadow cones sweep across the earth, and trace out a narrow band over which the eclipse is total, and a much broader band over which it is partial. These bands are inclined to the ecliptic plane, because the moon is passing from above to below it, or vice versa, at or near the time of eclipse.

Since the moon accompanies the earth in its orbital motion round the sun, its average speed along the ecliptic must be the same as that of the earth, say u. The orbital speed of the moon relative to the earth, which for our present discussion may be considered as constant, and equal to u', is much smaller than u (approximately $u = 1{,}110$, $u' = 37$, in miles per minute). The direction of revolution of the moon round the earth is the same as that of the earth round the sun; hence, when the moon is between the earth and the sun, its velocity is $u - u'$, or about 1,070 miles/minute, in the same direction as the motion of the earth (apart from the small component of u', which we may now ignore, which is at right angles to the ecliptic plane). Before the eclipse the moon and its shadow cones are in advance of the earth, and are sweeping along owing to the forward motion of the moon; their speed is $u - u'$ at the moon, and is very little greater at the earth's orbit. The earth gains on the forward motion of the shadow cones, or, relative to the earth, these sweep backwards with the velocity u' in the direction opposite to the earth's motion. They first meet the earth along the dawn meridian, and sweep across it with an eastward component of motion (combined with a northward or southward component according to the sign of the inclination of the lunar orbit at the time); the earth's surface is also travelling eastwards, owing to the daily rotation, but the speed of the shadow cones is much greater than this surface speed of the earth. If the moon were moving in the plane of the ecliptic, the shadow cones would travel from west to east across the earth in about $8{,}000/u'$, or about 216 minutes; during this time the rotation of the earth is considerable, the peripheral speed at the equator being about

17 miles/minute; consequently places which were near the twilight meridian when the eclipse begins will have moved on into the earth's shadow before the eclipsed area reaches the twilight meridian. In general the axis of the shadow cones (the line joining the centres of the moon and sun) will be inclined to the vertical direction at the eclipsed stations, so that the eclipsed area in the ionosphere will be displaced (in some cases considerably) relatively to the area eclipsed on the ground. For this reason the eclipsed belt of the ionosphere is now calculated specially for the E and F layers separately [40].

We next consider the nature of the shadow regions shielded by the moon from the neutral (and other) corpuscles emitted from the sun. The neutral corpuscles are of more interest than the charged ones (ions or electrons) in the present connexion, because the latter will in any case not travel along a (nearly) straight path when they enter the earth's magnetic field, so that the eclipse effect upon them will be more complicated, and probably difficult to detect by magnetic or radio observations.

We shall suppose, for definiteness, that the velocity v of the corpuscles in the direction radial from the sun, when they approach the earth-moon system, has the value suggested by Milne's theory, namely 1,600 km./sec.—or 60,000 miles/minute. They will have a small component of motion along the direction of the earth's orbit, due to the solar rotation or the non-radial velocity component at emission, but this can here be ignored. Since, at and near the time of eclipse, the moon is travelling nearly parallel to the earth's orbital velocity, in the same direction, with the speed $u-u'$, the axis or centre-line of the region shielded from the corpuscles will be inclined to the moon-sun axis at the small angle $(u-u')/v$ (in circular measure), or, with the adopted values of u, u', and v, by about 1°. This line will 'lag behind' the motion of the moon, just as the region shaded from vertically falling raindrops by an umbrella carried along by a pedestrian slopes rearwards, opposite to his direction of travel (the corresponding angle of lag in the case of the optical eclipse, due to the finite velocity of light, is about 1/200 of a degree).

Owing to this inclination, to the sun-moon line, of the axis of the shadowed regions, the centre of the shadow lags behind this line, at the distance d of the earth from the moon, by an amount $(u-u')d/v$, or, since $d = 240{,}000$ miles, by about 4,300 miles.

This shadow moves forward with the moon in the direction of the earth's orbital motion, but relative to the earth it is moving in the opposite direction with the velocity u', so that its centre-line will meet the earth at a time $(u-u')d/u'v$, or about 116 minutes (roughly 2 hours), before the centre of the optical shadow does so.

On account of this great difference in time between the corpuscular and optical eclipses, and the rotation of the earth during the interval, there is also a great difference in the regions affected by the two eclipses, the central line of the corpuscular eclipse track being to the east of the optical eclipse track, by a distance of about 2,000 miles. For each eclipse special calculations of the corpuscularly eclipsed area of the ionosphere, and of the times of the eclipse there, are necessary (they should be calculated for one or more assumed values of v). The first calculation of a corpuscular eclipse chart was made by Miller for the eclipse of August 1932 [15.38]. The British Nautical Almanac Office has calculated such charts [40] for later eclipses. Fig. 4 is a chart for the corpuscular eclipse of 1940 October 1 ($v = 1000$ miles/sec.).

Fig. 5 *a* illustrates the succession of the two shadow regions; the angles and dimensions are incorrectly drawn in order to emphasize the points it is desired to bring out.

Taking a section of the shadow regions at the distance of the earth's orbit, the distances in the direction of motion are shown approximately in Fig. 5 *b* for a typical case. The distances are given in miles and also in minutes, at the rate of 35 miles/minute, representing the rate of travel of the shadow system relative to axes through the earth's centre but not rotating with the earth.

Chapman has suggested that the umbral area in corpuscular eclipses will be much larger than in optical eclipses, and the penumbral area smaller. In order to explain the reasons for this suggestion, we first consider the so-called 'ideal' or 'cosine' law of emission of light by a luminous surface; this signifies that the intensity of light emitted in the direction inclined at the angle θ to the normal to the surface is proportional to $\cos\theta$. This implies that the light received at a point P from a small plane luminous surface, of area δA, with centre C, when CP is large compared with the linear dimensions of δA, and is inclined at the angle θ to its normal CN, is expressible as $I\cos\theta\,\delta A/\mathrm{CP}^2$, where I is a constant which depends on the intrinsic brightness or rate of emission of light from the surface, *per unit area*. Since $\cos\theta\,\delta A$ is the projection of δA on a plane normal to CP, it follows that the light received at P is the same as would come from an area $\delta A'$, equal to $\cos\theta\,\delta A$, situated at C and having the same intrinsic brightness I as the actual element. Consequently, a body of any form (with centre C,

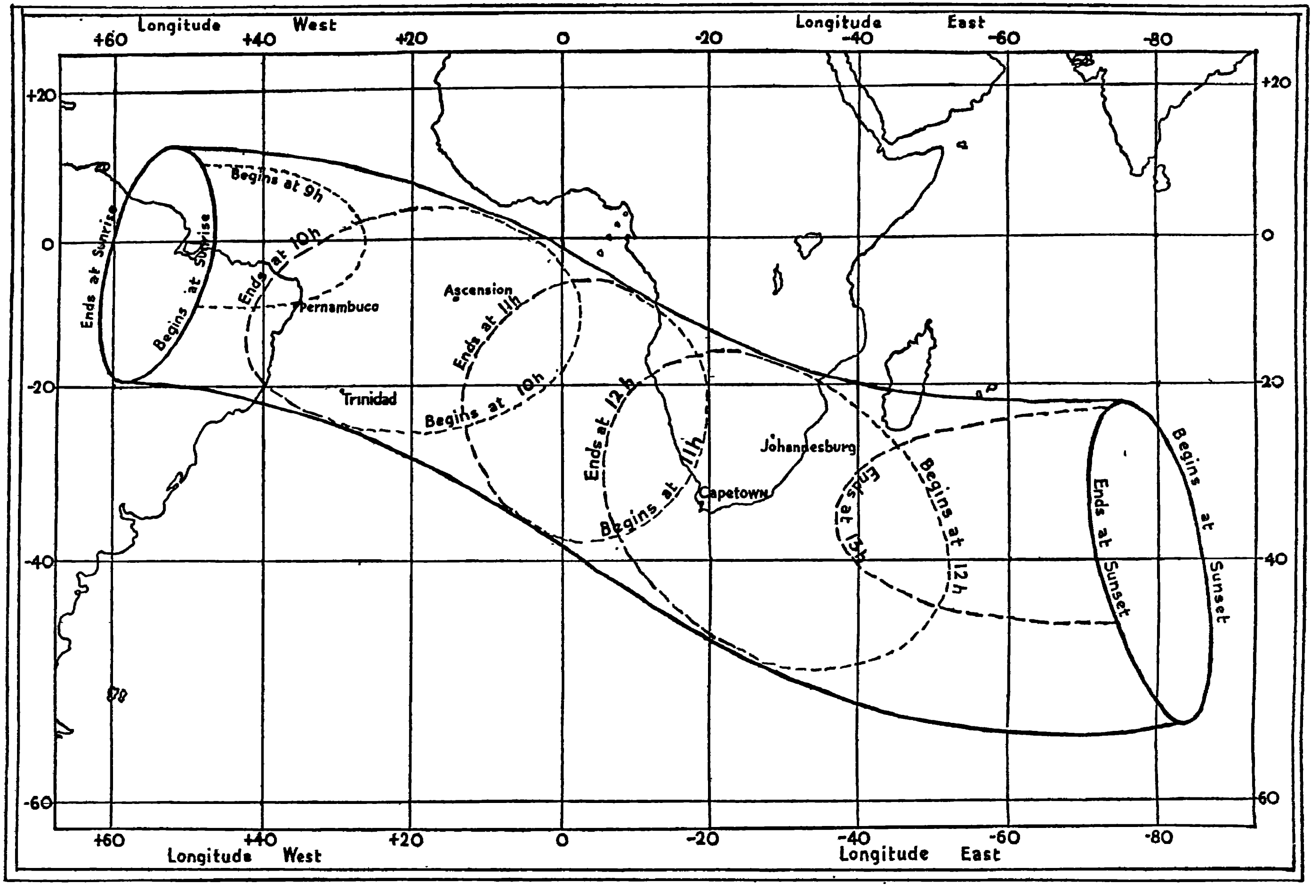


FIG. 4. The track of the corpuscular eclipse of 1940 October 1, indicating also the boundary of the corpuscular shadow at particular times (10, 11, 12 o'clock, Greenwich mean time)

say), whose surface emits light according to the cosine law, with uniform intrinsic brightness I, will appear uniformly bright when viewed from a point P whose distance CP is great compared with the dimensions of the whole body; the light received at P will be the same as would come from a uniformly bright plane disk, normal to the line of sight, shining with the intrinsic brightness I.

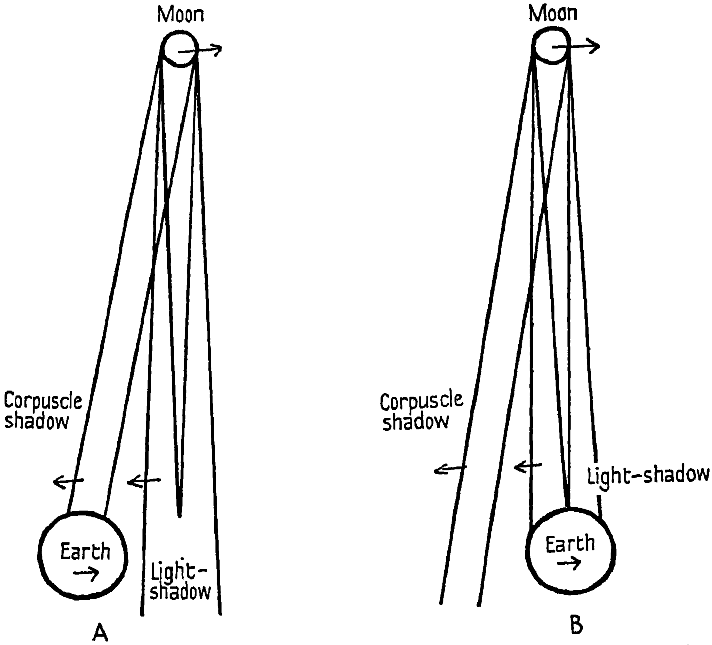
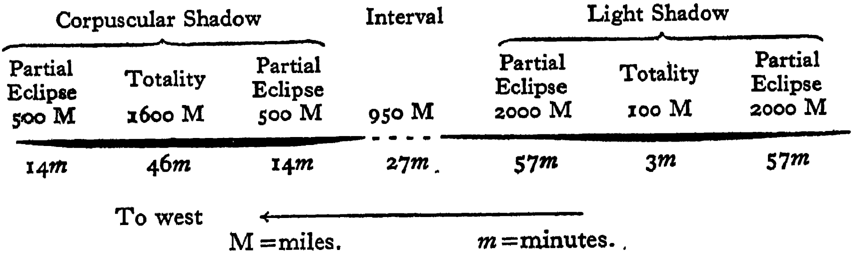


Fig. 5*a*. A, the corpuscular eclipse in progress, before the light eclipse begins. B, the light eclipse in progress, after the corpuscular eclipse is over



Corpuscular Shadow			Interval	Light Shadow		
Partial Eclipse 500 M	Totality 1600 M	Partial Eclipse 500 M	950 M	Partial Eclipse 2000 M	Totality 100 M	Partial Eclipse 2000 M
14*m*	46*m*	14*m*	27*m*	57*m*	3*m*	57*m*

To west ←

M = miles. *m* = minutes.

Fig. 5*b*

If, however, the ratio of the emission of light in the direction θ to that in the normal direction is not $\cos\theta$ but $f(\theta)\cos\theta$, where $f(\theta) = 1$ (like $\cos\theta$) when $\theta = 0$, then the apparent brightness of a uniformly luminous body, as seen from a great distance, will not be uniform; it will be proportional to $f(\theta)$.

The sun's disk appears almost uniformly bright except near the edge or limb of the disk, where there is a perceptible darkening. This implies that the cosine law of emission is approximately fulfilled, or $f(\theta)$ is nearly 1, until θ becomes nearly 90°, when $f(\theta)$ becomes materially less than 1. It is not surprising that the emission does not follow the cosine law exactly, since the sun's light is not emitted from a definite surface but comes through an atmosphere which modifies it by selective absorption and scattering.

Observation of prominence motions at the sun's limb shows that much of the matter that rises from the surface is deflected back to the surface again, often forming an arched prominence. The few prominences that have been seen to rise with high velocity and acceleration, such as to make it likely that they will escape altogether from the sun's attraction, seem to have been moving nearly radially when lost to view. Pike's calculations (§ 2) suggest this as a theoretical probability on certain assumptions. Interpreted in accordance with the above discussion, this may be expressed by saying that the law of corpuscular emission from the sun probably corresponds to a factor function $f(\theta)$ which decreases rapidly from the value unity, as θ increases from 0. If so, and if, from outside the earth's atmosphere, we could view or photograph the sun by reception of the corpuscular emission instead of by the sun's light, then the darkening of the sun's disk towards the limb would be more rapid, and would begin nearer the centre, than is actually observed. If the darkening was such that along the outer half (say) of the radius of the visible disk the corpuscular emission was negligible, then the angles of the umbral and penumbral cones would be approximately halved. Hence the distance of the vertex V of the umbral cone, from the moon, would be almost doubled. The consequence would be that if the corpuscular track crossed the earth's equator at noon, the diameter of the totally eclipsed area would be half that of the moon, or about 1,000 miles. The diameter of the corresponding penumbral area would be reduced from about 4,000 miles to about 3,000 miles. If the existence and extent of the corpuscular eclipses should in time be confirmed by radio observations, we shall be able to infer the nature of the emission law or of the function $f(\theta)$.

At any point along the track of the eclipse (optical or corpuscular), the interception of the ultra-violet ionizing components of the sun's light, or of the ionizing neutral corpuscles, will cause a temporary drop in the ionization of the layer concerned, since recombination will outweigh fresh ion-production. Figs. 6*a*, 6*b* give tentative graphs (*a*) of q, the rate of ion-production, as reduced during the two types of eclipse, and (*b*) (below) of n, the ion-content, for layers each ionized solely either by light or by corpuscles.

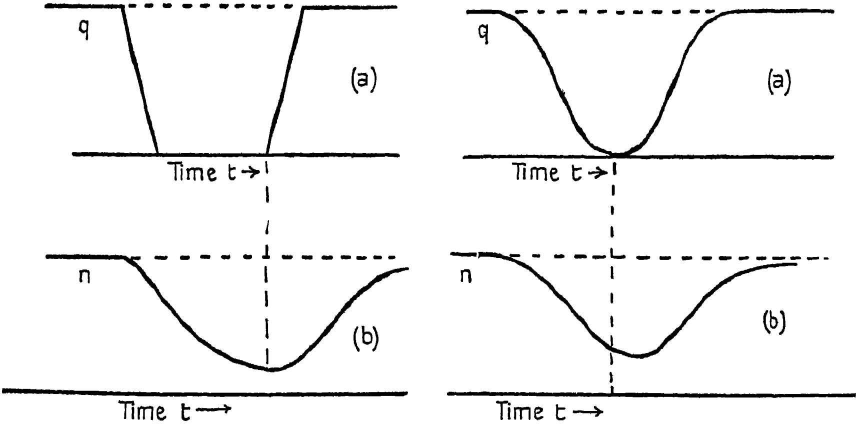


Fig. 6. *Left*, the variation (*a*) of the intensity of the corpuscular beam falling on the earth, and (*b*) of the ion-content n, at a point on the centre of the eclipse track, during the corpuscular eclipse, and, *right*, the variation (*a*) of the light intensity, and (*b*) of n, during the light eclipse

Radio observations during eclipses have shown that the eclipse effect in the E and F_1 layers occurs at the time of the optical eclipse, indicating that these layers are ionized by light (or by corpuscles with nearly the speed of light); for the F_2 layer the evidence favours some reduction of ion-content during the optical eclipse, but is not yet conclusive as to whether or not there is also an indication of reduced ion-content during an earlier period, which might be interpreted as due to a corpuscular eclipse [15.36].

24.9. The electrical state of corpuscular streams on emission from the sun. The selective radiation pressure on upward-moving atoms, which according to Milne's theory is the cause of their emission from the sun, is more powerful than the general radiation pressure on the electrons in the sun's atmosphere. Hence the primary tendency is for the emission of neutral atoms and (positive) atomic ions, so that streams or clouds of particles issuing from disturbed solar areas will initially be positively charged. Such a charged mass, however, rising out of the sun's atmosphere, would exert a powerful electrostatic attraction on the electrons below, which are present in abundance. Hence electrons would be drawn into the stream of neutral and positively charged atoms, tending continuously to render the stream electrically neutral.

Milne suggested that the stream, if of limited length, might have a positively charged head and a negatively charged tail, because the ions (of mass m_i, impelled with acceleration f_i) are subject to a greater acceleration than the electrons (of mass m_e and acceleration f_e). The electrons could lag behind until the separation of positive and negative charge sets up a compensating electrostatic force able to equalize the accelerations of the positive and negative particles. It is easy to show that a very small separation is enough for this purpose [12.17].

It is sufficient, in order to estimate the magnitude of the separation, to consider a spherical cloud instead of a cylindrical or conical stream. The electrostatic separation will be over-estimated if we ignore the 'external' accelerating force ($m_e f_e$) on the electrons. When the accelerations on the ions and electrons have been equalized to the value f, the equations of motion of an ion and an electron are

$$m_i f_i - Ee = m_i f, \qquad Ee = m_e f, \tag{3}$$

where e is the positive magnitude of the electronic charge and E is the electrostatic force due to the separation. Hence

$$Ee = \frac{m_i m_e}{m_i + m_e} f_i \doteqdot m_e f_i. \tag{4}$$

The separation of the charges is equivalent to a polarization in the direction of motion (x) of the cloud; a uniform separation produces no volume charge in the cloud, but only a surface density; at a point P such that OP, the radius from the centre O of the cloud, makes an angle θ with the x direction, the surface-density will be proportional to $\cos\theta$, say $\sigma\cos\theta$. Inside the cloud E then has the constant value $\frac{4}{3}\pi\sigma$, and is in the x direction. Consequently

$$\sigma \doteqdot \frac{3m_e}{4\pi e} f_i. \tag{5}$$

In Milne's theory the maximum value of f_i occurs when the ion has just got clear of its absorption line, and is still near the sun's surface; Chapman and Ferraro [12], in their discussion of the present problem, infer from Milne's theory that $f_i < 8g$, where g denotes the surface acceleration of solar gravity, $g = 2{\cdot}74 \times 10^4$ cm./sec.^{2} Hence

$$\sigma < 6m_e g/\pi e = 10^{-13} \text{ electrostatic units.}$$

This corresponds to 2×10^{-4} elementary charges (positive or negative) per sq. cm. of the front or rear of the cloud; for a cloud of density N ions and N electrons per c.c. this would result from a relative separation of the ions and electrons by $2\times10^{-4}/N$ cm. We have no definite knowledge of the value of N, but it is clear that the separation could not amount to more than a few centimetres unless N were 10^{-5} or less. It seems, from Chapman and Ferraro's discussion of the action of an ionized stream on the earth (Ch. XXV), that appreciable geomagnetic disturbance could be produced only if the initial density of the stream, at the sun, is of order 10^{+5} or more; this would correspond to a quite negligible separation of ions and electrons, less than 2×10^{-9} cm. Even if the estimate of f_i were increased by several powers of 10, corresponding to a much higher acceleration of the cloud (and to a much shorter transit time from the sun to the earth), the electrostatic separation in the cloud, important as it is in keeping the electrons with the positive ions, would remain very small.

In the above calculation it was assumed that the attraction of electrons into the cloud of positive ions, from the solar atmosphere below, is fully effective in rendering the cloud electrically neutral. Chapman and Ferraro showed, however, that even if this were not so, any excess charge (of either sign, though presumably it would initially be positive) would very rapidly be dispersed by its own self-repulsion; if initially distributed throughout the volume of the cloud, it would first be repelled to the surface, and then away from the surface.

As in the preceding calculation, they considered a spherical cloud instead of a stream, taking the radius (r_0) to be between 1,000 and 100,000 km. (the latter being one-seventh the radius of the sun). For simplicity they ignored the possible presence of neutral atoms; it seems likely that these would not greatly outnumber the ions, and that they would at most reduce (by a factor not exceeding 10) the speed with which the excess charge would diffuse from the centre to the surface of the cloud. A positive volume-charge would be dispersed to the surface mainly by electrons being drawn inwards, leaving an outer layer of positive ions; a negative volume-charge would be dispersed mainly by outward motion of the electrons. The time taken for the volume-charge to reach the surface depends on the coefficient of diffusion (D) for a gas composed of ions and electrons. Assuming that the cloud has the temperature 6,000°, D is roughly proportional to $1/N$, where $2N$ now signifies the combined number of electrons *and* ions per c.c.; actually ND is not quite independent of N, but is approximately

$$3{\cdot}7\times10^{18}/(1-0{\cdot}048\log_{10}N),$$

so that $10^{-18}ND$ varies only from 3·6 to 6·5 as N increases from 10^{-1} to 10^9. The mean free path of an electron is about $2\times10^6/N$ km.

The time taken for the volume-charge to reach the surface, leaving the interior neutral to a high degree of approximation, depends on a number τ (a time-interval), equal to $kT/8\pi e^2ND$, where $k = 1{\cdot}37\times10^{-16}$; for $T = 6{,}000°$, and $ND = 5\times10^{18}$, τ is 3×10^{-14} sec. The time of diffusion is a moderate multiple of τ, say 100τ. Thus the movement of the volume-charge in a cloud, to the surface, is almost instantaneous in clouds of the density considered ($0{\cdot}1 < N < 10^9$); moreover, it is practically independent of the initial radius r of the cloud. This is because the electrical force at a point within the cloud, at a radius r from the centre, depends only on the charge within the concentric sphere of this radius, and is unaffected by any charge in the outer layers beyond r; for a given volume-density of charge within the cloud, the radial electrostatic force at r, and consequently also the rate of flow of charge per sq. cm. across the sphere of radius r, is proportional to r. The average distance which the charge in the sphere r must traverse, to pass beyond r, is likewise proportional to r. This affords a rough explanation of why the time of passage of the charge to the surface is almost independent of the radius of the cloud.

The total charge Q, having rapidly become concentrated at the surface, will further expand outwards under its own repulsion; this expansion will naturally be slower if the excess charge is positive than if it is negative, because negative electrons, having so small a mass, will be very readily accelerated away.

If σ is the initial surface-density, so that $\sigma = Q/4\pi r_0^2$, there will be an electric force $4\pi\sigma$ at the outside of the layer, decreasing to zero on passing through the layer to its inner boundary. The rate of expansion of the layer may be considered as follows: let r denote the radius which is attained, after a time t from the beginning of the expansion of the surface-layer, by the particles originally lying at a depth within the layer, such that below their level there is a fraction θ of the total charge Q of the layer. The electric intensity at this radius is $\theta Q/r^2$, and the equation of motion of the charges at this distance from the centre of the cloud is therefore

$$m\dot{v} = e\theta Q/r^2, \tag{6}$$

or, since $dv/dt = (dv/dr)(dr/dt) = v\,dv/dr$,

$$dv^2/dr = 2e\theta Q/mr^2, \tag{7}$$

where m is the mass of the surface-charges ($= m_e$ or m_i according as the charge is negative or positive). The solution is

$$v^2 = v_0^2+(2e\theta Q/m)(1/r_0-1/r), \tag{8}$$

where v_0 is the velocity of the charges initially, when they are in the thin surface-layer; the original thickness of this layer can be neglected in comparison with its initial radius r_0. A lower limit for the rate of dispersal of the layer is obtained if we write $v_0 = 0$; the speed v then increases, as $r \to \infty$, to the value $(2e\theta Q/mr_0)^{\frac{1}{2}}$.

By writing

$$r = r_0 \cosh^2 u, \tag{9}$$

so that $u = 0$ when $r = r_0$, (8) can be integrated (taking $v_0 = 0$) in the form

$$r_0(\sinh u \cosh u+u) = (2e\theta Q/mr_0)^{\frac{1}{2}}t, \tag{10}$$

measuring t from the time when the layer begins to expand, when $u = 0$ and $r = r_0$. This equation enables r to be found for any value of t. When t and u are small, we have approximately

$$r-r_0 = r_0 u^2 = (\tfrac{1}{2}e\theta Q/mr_0^2)t^2. \tag{11}$$

When t is large enough for u and r/r_0 to have become large, the appropriate approximate result is

$$r = (2e\theta Q/mr_0)^{\frac{1}{2}}t \tag{12}$$

or

$$r/r_0 = (2e\theta Q/mr_0^3)^{\frac{1}{2}}t.$$

This formula implies that when r/r_0 has become large, each level (θ) of the original layer is expanding with constant velocity, proportional to $\theta^{\frac{1}{2}}$; comparing the values of r for $\theta = 1$ and $\theta = 0{\cdot}01$, it appears that 99 per cent. of the charge on the cloud lies within a shell of which the inner radius ($\theta = 0{\cdot}01$) is one-tenth the outer radius ($\theta = 1$). The charge-density within a thin shell at a given radius r, at a (sufficiently large) time t, so that at that time and radius $\theta = (mr_0/2eQ)r^2/t^2$ and consequently $d\theta/dr = 2\theta/r$, is $Q\,d\theta/4\pi r^2\,dr$ or $\theta Q/2\pi r^3$ or

$$\frac{mr_0}{4\pi ert^2}; \tag{13}$$

this is proportional to $1/r$ and $1/t^2$, and also to the initial radius r_0 of the cloud, but is independent of the total charge Q.

If Q is expressed in terms of the fractional excess (k) of the number of charges of one kind over the number of the other kind (at the time of emission from the sun), then

$$Q = \tfrac{4}{3}\pi r_0^3\,\mathrm{k}Ne,$$

and

$$r/r_0 = (\tfrac{8}{3}\pi \mathrm{k}\theta N/m)^{\frac{1}{2}}et; \tag{14}$$

hence r/r_0, for given values of k, θ, and N, is proportional to t (when r/r_0 has had time to become large).

Suppose, for example, that the cloud is positive, and that the ions are calcium ions (singly charged); then after 10 minutes (= 600 sec.) 99 per cent. of the excess charge (corresponding to $\theta = 0{\cdot}01$) will be *beyond* a radius such that $r/r_0 = 10^4(\mathrm{k}N)^{\frac{1}{2}}$; thus if, for example, k is 1/100, and N is 10^5, r/r_0 will be about 3×10^5; this represents a reduction in the density of the charge, supposed initially spread throughout the sphere of radius r_0, and after the time t distributed between the spheres of radii r and $10r$, to a fraction less than 10^{-16} of the original value.

The time (10 minutes) used in this illustration is approximately that taken by light to travel from the sun to the earth; if instead we take $t = 1$ day, to correspond with the time of travel to the earth for a cloud having a mean speed of 2,000 km./sec., then r/r_0 is increased 144-fold to about 4×10^7.

It is also of interest to consider a spherical cloud initially uniform, and composed solely of charges of one sign, N per c.c. The above analysis for the expansion of the surface-layer is applicable to this case, for the charges at any original level r', if we replace Q by $\frac{4}{3}Ner'^3$ and r_0 by r'. Then at times such that r/r' is large, the approximate formula (12) must be replaced by

$$r/r' = (\tfrac{8}{3}\pi N/m)^{\frac{1}{2}}et; \tag{15}$$

this is independent of r'. If $N = 10^5$, and the charges are electrons, then after 10 minutes r/r' will be about 10^{10}; this indicates how rapidly such a cloud would expand during the short time taken by light to travel from the sun to the earth.

Ferraro [39] has shown that there is little possibility of neutral streams being emitted from the sun, with different speeds for the opposite charges, so that the stream carries an electric current and produces a direct magnetic field.

24.10. The magnetic field produced by the moving stream. Only if the electrons were ejected with a speed differing so little from that of light that their mass was relativistically greatly increased, could a purely electronic stream reach the earth without great dispersion.

This case is of interest because several writers (for example, Birkeland [2] in 1896, and more recently Dauvillier [7]) have considered that a stream of very fast-moving electrons could reach the earth from the sun, with little dispersion and of such density that the stream could have a magnetic field at its surface, sufficient to account by direct action for geomagnetic disturbance. Supposing that such a stream can come quite close to the earth (but without, at present, considering the distortion of the stream by the earth's own field), let us consider the case when its own magnetic field H at its surface is of order 100 γ or 10^{-3} gauss.

As before, it is mathematically convenient to discuss a spherical cloud rather than a cylindrical stream. The problem is best treated by making a relativistic transformation from axes fixed with respect to the sun, to axes moving with the centre O′ of the cloud of fast-moving charges. Let accented symbols refer to the latter set of axes, and unaccented symbols to the 'stationary' or solar axes. Let the direction of motion be taken as the x direction, and suppose that the cloud starts from the sun, at $x = x' = 0$, at the time $t = t' = 0$, with velocity v (nearly equal to c, the velocity of light). Then, if

$$\beta = (1-v^2/c^2)^{-\frac{1}{2}}, \tag{16}$$

the cloud is at x ($= vt$) at time t, or at time

$$t' = \beta(t-vx/c^2) = x/v\beta. \tag{17}$$

The earth is reached at time

$$t = T \equiv R/v, \qquad t' = T' \equiv R/v\beta. \tag{18}$$

Relative to the cloud there is no magnetic field, i.e. H'_x, H'_y, H'_z are all zero. The equations of transformation between the electric and magnetic fields relative to the two sets of axes are

$$E'_x = E_x, \qquad E'_y = \beta(E_y-vH_z/c), \qquad E'_z = \beta(E_z+vH_y/c), \tag{19}$$

$$H'_x = H_x, \qquad H'_y = \beta(H_y+vE_z/c), \qquad H'_z = \beta(H_z-vE_y/c). \tag{20}$$

From these we have

$$H_x = 0, \qquad H_y = -vE_z/c, \qquad H_z = vE_y/c, \tag{21}$$

$$E'_y = cH_z/\beta v, \qquad E'_z = -cH_y/\beta v. \tag{22}$$

Thus, if H is the magnetic intensity (a function of x and the distance from the x-axis) and E_1 is the electric component normal to Ox,

$$E'_1 = c^2H/\beta v, \tag{23}$$

where an additional factor c has been inserted, in order to obtain E'_1 in electrostatic units. This relates the electric field of the cloud, relative to its own axes, in the direction transverse to its motion, with the magnetic field of the cloud relative to the 'fixed' axes.

In the plane $x' = 0$, $E' = E'_1 = Q/r^2$, where Q is the charge (in e.s.u.) on the cloud.

If v is nearly equal to c, so that β is large and T' small, the expansion of the surface of the cloud ($\theta = 1$) in the time T may be only a small fraction of the original radius r_0. In this case, by (11),

$$\begin{aligned} r-r_0 &= (\tfrac{1}{2}eQ/mr_0^2)T'^2 \\ &\doteqdot \tfrac{1}{2}eE_1' R^2/mv^2\beta^2 \\ &= \tfrac{1}{2}ec^2HR^2/mv^3\beta^3. \end{aligned} \tag{24}$$

For electrons, m being the mass at rest, $e/m = 5 \cdot 3 \times 10^{17}$ e.s.u.; also $R = 1 \cdot 5 \times 10^{13}$ cm.; if $v = c$ approximately, and we suppose that H, the magnetic field at the surface of the cloud, is 100 γ or 10^{-3} gauss, we have

$$r-r_0 = 10^{30}/\beta^3.$$

If the cloud is to be only slightly dispersed on its journey from the sun, so that $r-r_0$ is a small fraction of r_0, say $\frac{1}{10}r_0$, then

$$\beta^3 = 10^{31}/r_0;$$

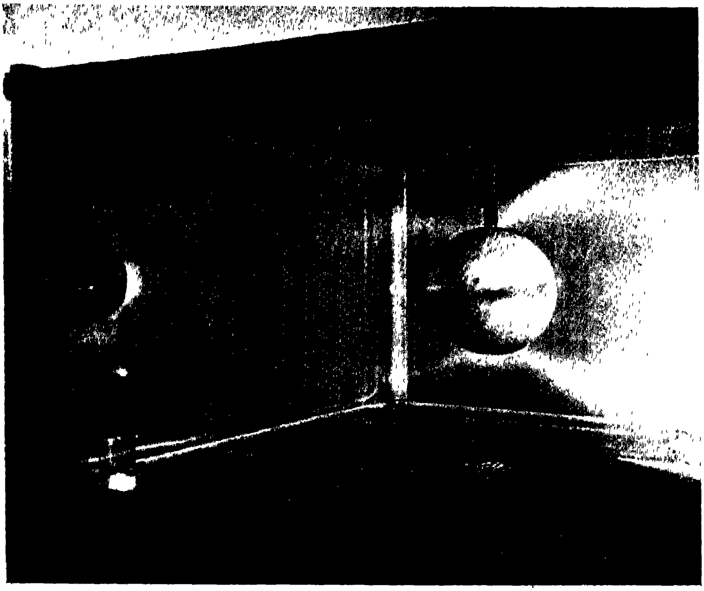
thus, even if r_0 is 10^{10} cm. (an upper limit for the initial size), β must be of order 10^7. This corresponds to a speed differing from the velocity of light by only $1 \cdot 5 \times 10^{-4}$ cm./sec., to an electronic mass βm_e, and to an energy (5×10^{12} electron volts) comparable with the energy of cosmic rays. The electrons could penetrate far below the level of aurorae. It seems reasonable to exclude this as a serious hypothesis of the origin of geomagnetic disturbance.

These illustrations show that any excess charge on a cloud emitted from the sun is likely to be dispersed to an extremely low density before the cloud reaches the earth's orbit. If the cloud originally has only a slight excess charge, this will rapidly pass to the surface, and thence disperse from the neutral body of the cloud, which travels on, and expands, in the way described in § 3.

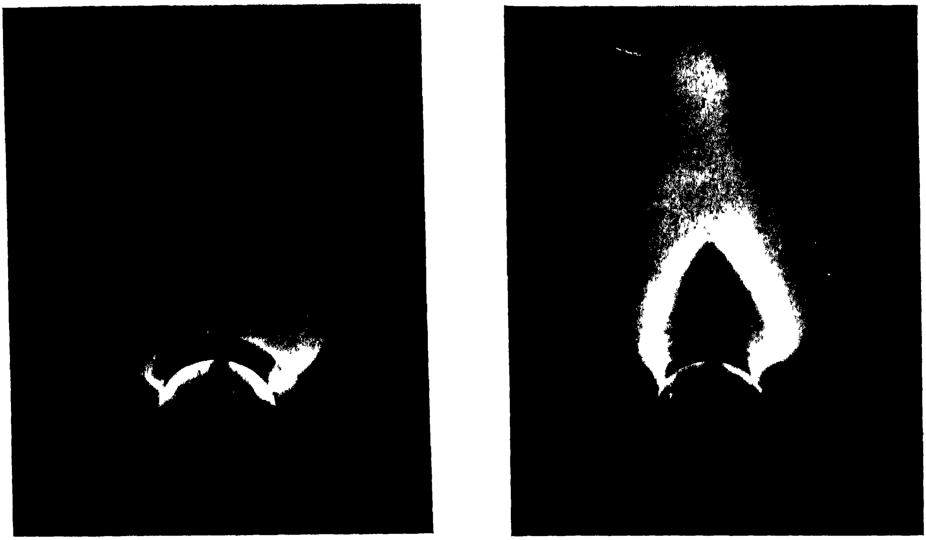
The cloud will still retain, however, a slight and positively charged layer at the surface, which adjusts itself so as to retard the outward flow of electrons across the surface (owing to their thermal velocities) to the rate of outflow of positive ions. If such a positive charge were not present, the electrons, which have much the higher thermal velocities, would escape preferentially until the surface-layer acquires the positive charge requisite for the equalized outflow. In this state the electrostatic potential V at the surface is approximately given by

$$eV = \tfrac{3}{2}kT, \tag{25}$$

so that for $T = 6{,}000°$, $V = 2 \cdot 5 \times 10^{-3}$ e.s.u. $= 0 \cdot 8$ volt. The resultant positive charge Q' on the cloud is Vr, and therefore slowly increases as the neutral body of the cloud expands (owing to the thermal velo-



a. Birkeland's terrella experiment, showing the toroidal space round the earth. The magnetic equator is horizontal



b. *c*.

Brüche's experimental verification of the effect of a current ring on the polar-light zones; (*b*) without, (*c*) with, the ring-current. The magnetic equator and the ring are shown vertical cities and the initial divergence of the cloud). This expansion, and the consequent increase in Q', imply a slow excess leakage of electrons from the surface; the rate of increase of Q' is Vv, where v denotes the radial velocity of the boundary; this is likely to be roughly constant, and to have a value between 1 and 100 km./sec., according to the initial divergence of the cloud. When the cloud has reached the earth's orbit (taking the radius r of the cloud to be then less than 10^{13} cm.), Q' will not exceed $2{\cdot}5\times10^{10}$ e.s.u. The ratio of excess charge to the mass of the whole cloud, taking N to be 10^5, r_0 to be 10^9 cm., and the ions to be calcium ions, will be less than $3\times10^{-15}e/m_H$.

This charge will be situated near the surface of the cloud, where the ratio of charge to mass will be much greater than this. Chapman and Ferraro showed, however, that it is not likely to exceed $10^{-7}e/m_H$. They also showed that the density of the thin charged layer will decrease at a rate similar to that of the neutral body of the cloud.

The above discussion suggests that the only corpuscular streams from the sun that are likely to be able to arrive near the earth's orbit with a density of more than 1 particle per c.c., and a radius of cross-section of order 10^{10} or more, are streams composed wholly of neutral atoms, or streams that electrically are almost completely neutral, but are partly or wholly ionized.

Birkeland in 1896 pointed out that electrons travelling towards the earth would be deflected by the earth's magnetic field. He made experiments in which cathode rays were projected towards a small magnetized sphere, and showed that their distribution near the sphere was largely governed by the field, which, in particular, guided many of the electrons towards the polar regions. This gave the first promising suggestion as to the cause of the polar property of the D phenomena (§ 1). Since then the theories of aurorae and geomagnetic disturbance have been closely linked together. Birkeland developed his ideas in three extensive memoirs of 1901, 1908, and 1913 [refs. G 97, 97*a*]; these form the most detailed and comprehensive theoretical discussion of magnetic storms hitherto published.

The upper part of Plate 35 shows a photograph of one of Birkeland's chief experimental results; the cathode rays are projected from the right towards the magnetized sphere, whose magnetic field deflects them specially towards the north and south polar regions, leaving a middle belt, and a toroidal space around it, free from the particles; this space is shown on Plate 35 by the dark areas to the right and left of the sphere.

E. Brüche [18] has repeated some of Birkeland's experiments, and extended his results. The lower part of Plate 35 shows photographs of two of his experiments, in which a ring was placed round the magnetized sphere; on the left the experiment was made with no electric current in the ring; it shows the polar precipitation of the particles, and the free space round the equator, as in Birkeland's photograph; the right-hand lower picture shows that when an electric current flows in the ring this free space is extended outwards, and the polar zones of precipitation move slightly nearer to the equator and to each other.

This experiment verified a theoretical calculation made by Störmer [19–26], who developed Birkeland's theory mathematically, and extended it, with important results both for the theory of the aurora and, as has since appeared, for the theory of cosmic rays.

Also in other ways Birkeland's work has been continued and developed in Norway, by Störmer (§ 12), Vegard, Harang, and others (Ch. XIV), but with chief reference to aurorae rather than to magnetic storms. Birkeland supposed the corpuscles that are responsible for aurorae and magnetic storms to be electrons of very high velocity, nearly equal to that of light. Störmer and Vegard also considered the possibility that they may be slower electrons or (positive) particles.

Schuster [1] criticized Birkeland's theory of magnetic storms [2] on the ground (among others) that without imparting improbably high velocity and energy to the particles, any stream possessing an appreciable magnetic field would be dispersed (as described above) in its passage from the sun to the earth. To meet Schuster's objection Birkeland suggested that the speed of his supposed solar electrons might differ only very little from that of light. But the above discussion of a cloud of very fast-moving electrons indicates that if it is to get from the sun without serious dispersion, and yet to exert an appreciable magnetic field, its energy must be quite improbably great, and its penetration into the atmosphere far deeper than is observed.

Lindemann [15] in 1919 urged similar electrostatic objections to a theory of magnetic storms proposed by Chapman in 1918 [9.2*a*], which was based on a charged corpuscular stream from the sun; he added to his criticism the suggestion that Chapman's theory might be substantially preserved if the supposed charged stream were replaced by a neutral ionized stream. He showed how a cloud of originally neutral atoms near the sun might be impelled outwards by radiation pressure, and also gradually ionized; he proved that it could travel to the earth without appreciable recombination, and with a mean velocity of about 800 km./sec. He did not consider its terrestrial effects in detail.

24.11. General account of the Birkeland-Störmer auroral theory. In so far as the solar streams consist of neutral corpuscles, no new considerations arise concerning their motions when they come near the earth, until they actually penetrate its atmosphere; this will occur exclusively over the sunlit hemisphere. The charged particles, however, are affected by the earth's magnetic field. The determination of the motion of such particles under this influence involves problems different from, and more difficult than, those which have been solved, in the preceding discussion, relative to the motion outside the earth's sphere of action. This branch of the subject includes the electric corpuscular theory of the aurora and geomagnetic disturbance, which was first proposed by Birkeland. Birkeland sought the solution of this problem by experimental means. The theory has subsequently been developed mathematically by Störmer, who has devoted much labour to the theoretical investigation of auroral problems, and has obtained many beautiful and suggestive results. It seems likely, however, that the theory is still in its initial stages.

Störmer's theoretical work deals almost exclusively with the possible paths of a solitary charged particle. He has shown in a relatively simple manner that any such particle, issuing from the sun and travelling into the earth's magnetic field (regarded as that of an elementary magnetic dipole), can reach the earth only within two narrow zones, centred respectively at the two poles of the earth's magnetic axis; the angular radius of the zones is determined by the values of e/mv for the particles (e charge, m mass, v velocity).

Owing to a certain difficulty in explaining the observed angular radius of the zones, Störmer also considered the effect of a great circular ring of electric charges revolving round the earth in the equatorial plane; the magnetic field of such a ring, if of the intensity and direction required to explain the second phase of a magnetic storm, was shown to be capable of modifying the motion of a solitary charge projected from the sun, in such a way as to enlarge the angular radius of the auroral zones. This result is of interest in connexion with the observed great increase in the radius of the auroral zones at times of magnetic disturbance.

Störmer has calculated a great variety of possible paths of a corpuscle, using numerical methods. He considered, among other problems, a set of paths having initial directions of projection lying within a cone of small solid angle, and showed that those which finally meet the earth may do so within a band lying along the auroral zone, of a breadth very small compared with its length. He pointed out that this may explain the frequent appearance of aurorae in the form of curtains or draperies; these curtains are almost two-dimensional sheets of light, being very thin compared with their length and height.

Interesting and important as these results are, their relation to actual auroral phenomena is at present uncertain, because aurorae must be due to large numbers of charges travelling together; if, as Störmer has supposed, these were of the same electrical sign, their mutual electrostatic influence would be at least comparable with the deflecting effect of the earth's magnetic field, particularly since he showed that the path of a single corpuscle may be profoundly modified by a slight perturbation. If, however, as seems more likely in view of the previous discussion, the aurora is due to an almost neutral ionized stream of electrons and positive ions, the problem of the influence of the earth's magnetic field must be quite different from the problem of the motion of a single particle, considered by Störmer. This problem has been discussed by Chapman and Ferraro, but their work must probably be linked up in some way with that of Störmer, before a proper solution of the auroral problems can be obtained. It is certain that the solution will prove very difficult and complicated, both because of the essentially three-dimensional character of the motion of the particles in the earth's magnetic field, and also because of the observed complexity of the aurora and its rapid changes of form.

24.12. Motion of a single charged particle in the geomagnetic field. Before considering the motion of a neutral ionized stream in the earth's magnetic field, a brief account of Störmer's auroral theory will be given. Let the charge, mass, and velocity of the single moving particle be denoted by e (in electromagnetic units), m, and $\boldsymbol{v}$. Let $\boldsymbol{H}$ denote the magnetic intensity at a point of which the position vector is $\boldsymbol{R}$, relative to a set of axes fixed in the earth with their origin at its centre. The z-axis will be taken along the direction of the earth's magnetic axis. If M denotes the magnetic moment of the earth, the magnetic potential Ω, such that

$$\boldsymbol{H} = -\operatorname{grad}\Omega, \tag{26}$$

is given by 1 (15),

$$\Omega = Mz/R^3. \tag{27}$$

The equation of motion of the corpuscle is

$$m\,d\boldsymbol{v}/dt = e\boldsymbol{v} \wedge \boldsymbol{H}. \tag{28}$$

where $\wedge$ denotes the vector product symbol. Multiplying scalarly by $\boldsymbol{v}$ we find $\dot{v} = 0$, i.e. the speed of the corpuscle is constant; this is evident since the force on the corpuscle, being perpendicular to the direction of motion, does no work.

Let $\boldsymbol{s}$ denote the unit vector along the tangent to the trajectory at any point, so that $\boldsymbol{v} = v\boldsymbol{s}$. Since $d/dt \equiv v\, d/ds$, (28) may be written

$$mv^2\, d\boldsymbol{s}/ds = ev\boldsymbol{s} \wedge \boldsymbol{H} \tag{29}$$

or

$$d\boldsymbol{s}/ds = -(e/mv)\boldsymbol{s} \wedge \operatorname{grad}\Omega, \tag{30}$$

where s denotes the distance measured along the path from some fixed point.

If s is this parameter for the point P or $\boldsymbol{R}$ on the trajectory,

$$\boldsymbol{s} = d\boldsymbol{R}/ds, \tag{31}$$

so that (30) is equivalent to

$$\begin{aligned} \frac{d^2\boldsymbol{R}}{ds^2} &= -\frac{e}{mv}\left(\frac{d\boldsymbol{R}}{ds} \wedge \operatorname{grad}\Omega\right) \\ &= -\frac{eM}{mv}\left(\frac{d\boldsymbol{R}}{ds} \wedge \operatorname{grad}\frac{z}{R^3}\right) \\ &= -l^2\left(\frac{d\boldsymbol{R}}{ds} \wedge \operatorname{grad}\frac{\pm z}{R^3}\right), \end{aligned} \tag{32}$$

where

$$l = (|e|M/mv)^{\frac{1}{2}}; \tag{33}$$

l has the dimensions of a length; the sign of z in (32) is that of e, so that the paths of oppositely charged particles (for any given l) are mirror images of each other in the plane $z = 0$. If l is taken as the unit of length, the factor l^2 disappears from (32). This indicates that the whole family of trajectories is the same, except in scale, for different particles (of the same sign) and for different magnetic dipoles. It is therefore sufficient to consider the one equation (for a negative charge)

$$\frac{d^2\boldsymbol{R}}{ds^2} = \frac{d\boldsymbol{R}}{ds} \wedge \operatorname{grad} U, \tag{34}$$

where

$$U = z/R^3. \tag{35}$$

The function U satisfies Laplace's equation, and has axial symmetry with respect to Oz. In cylindrical polar coordinates r, ϕ, z (Fig. 7) Laplace's equation $\nabla^2 U = 0$, for a function U of axial symmetry $(\partial/\partial\phi = 0)$, has the form

$$r\left(\frac{\partial^2 U}{\partial r^2} + \frac{\partial^2 U}{\partial z^2}\right) + \frac{\partial U}{\partial r} = 0 \tag{36}$$

or
$$\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{\partial}{\partial z}\left(r\frac{\partial U}{\partial z}\right) = 0, \tag{37}$$

showing that there must be a function $V(r,z)$ related to U by the equations
$$\frac{\partial V}{\partial z} = r\frac{\partial U}{\partial r}, \qquad \frac{\partial V}{\partial r} = -r\frac{\partial U}{\partial z}. \tag{38}$$

It follows from these that
$$\frac{\partial U}{\partial r}\frac{\partial V}{\partial r}+\frac{\partial U}{\partial z}\frac{\partial V}{\partial z} = 0, \tag{39}$$

showing that in any meridian plane (ϕ = constant) the curves U = constant and V = constant cut orthogonally. Since the former curves are dipole equipotentials, the latter curves must be the lines of force.

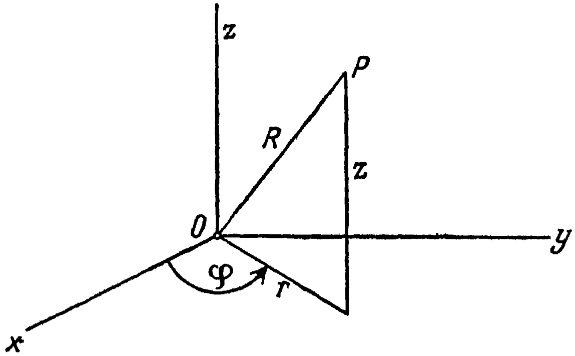
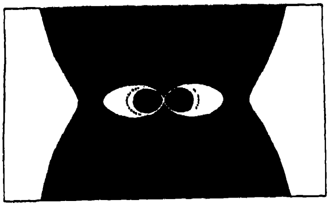


FIG. 7. Illustrating cylindrical polar coordinates

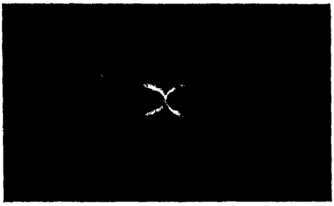
The equation of these lines is (by 1.17)
$$r^2/R^3 = \text{constant}. \tag{40}$$

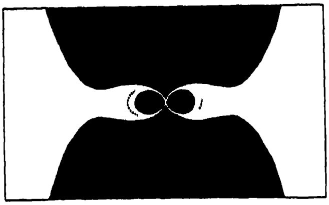
Hence
$$V = r^2/R^3+\text{constant}. \tag{41}$$

If in (34) we interpret s as the time (to which it is in fact proportional or, with a change of units, equal), the left-hand side of (34) is clearly the vector acceleration of a particle with position vector $\boldsymbol{R}$; its three cylindrical polar components, in terms of the coordinates r, ϕ, z of the point $\boldsymbol{R}$, are $d^2r/ds^2-r(d\phi/ds)^2$, $(1/r)(d/ds)(r^2\,d\phi/ds)$, and d^2z/ds^2 (the calculation of these may be found in any treatise on three-dimensional mechanics); the corresponding components of $d\boldsymbol{R}/ds$ and grad U, on the right-hand side of (34), are dr/ds, $r\,d\phi/ds$, dz/ds and $\partial U/\partial r$, $\partial U/r\,\partial\phi$, $\partial U/\partial z$ respectively. Since U is independent of ϕ, it is readily seen that

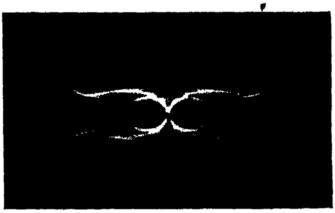


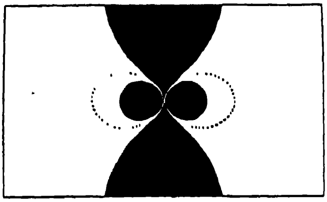
$\gamma = 1{\cdot}016$



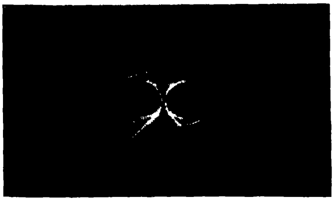


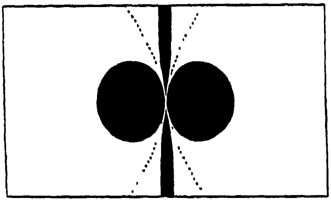
$\gamma = -0{\cdot}97$



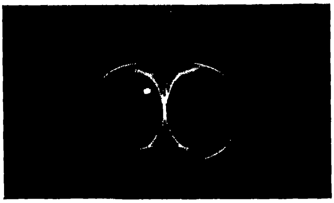


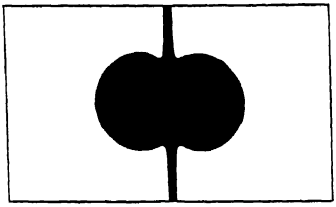
$\gamma = -0{\cdot}5$



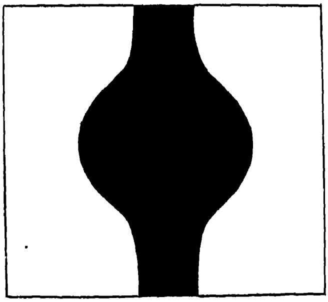


$\gamma = 0{\cdot}05$





$\gamma = 0{\cdot}03$



$\gamma = 0{\cdot}2$

cannot pass (for different values

the three component equations corresponding to (34) are as follows:

$$\frac{d^2r}{ds^2}-r\left(\frac{d\phi}{ds}\right)^2 = r\frac{d\phi}{ds}\frac{\partial U}{\partial z}, \tag{42}$$

$$\frac{1}{r}\frac{d}{ds}\left(r^2\frac{d\phi}{ds}\right) = \left(\frac{dz}{ds}\frac{\partial U}{\partial r}-\frac{dr}{ds}\frac{\partial U}{\partial z}\right), \tag{43}$$

$$\frac{d^2z}{ds^2} = -r\frac{d\phi}{ds}\frac{\partial U}{\partial r}. \tag{44}$$

Substituting from (38) in (43) the latter becomes

$$\frac{d}{ds}\left(r^2\frac{d\phi}{ds}\right) = \frac{dV}{ds}, \tag{45}$$

of which the integral is

$$r^2\frac{d\phi}{ds}-V = \text{constant}, \tag{46}$$

or, by (41),

$$r^2\frac{d\phi}{ds} = \frac{r^2}{R^3}+2\gamma, \tag{47}$$

where γ is an arbitrary constant.

Störmer has shown that interesting conclusions may be derived from (47). Let θ denote the angle between the tangent to the trajectory at the point P, and the meridian plane through P; then

$$\sin\theta = r\,d\phi/ds. \tag{48}$$

Consequently, (47) may be written in the form

$$\sin\theta = \frac{2\gamma}{r}+\frac{r}{R^3}. \tag{49}$$

Since $|\sin\theta| \leqslant 1$, this provides a means of determining certain regions to which the trajectories are restricted. Störmer has investigated these regions very fully, and has shown that the boundaries are of three distinct types, corresponding to the following three intervals for γ:

(*a*) $\gamma < -1$,

(*b*) $-1 \leqslant \gamma \leqslant 0$,

(*c*) $\gamma > 0$.

These three types are shown for typical values of γ within the three ranges (*a*)–(*c*) by the diagrams in Plate 36; the shaded parts are the regions which the corpuscles cannot enter.

To obtain the actual three-dimensional space the figures on the left, showing the excluded regions (in black) in any axial plane, should be rotated about their axes, as shown on the right of Plate 36. From these diagrams it is clear that corpuscles can pass through the origin (the centre of the earth) only if their initial position and direction of motion are such that γ is negative. This can also be seen from (49), written in the form

$$r\sin\theta = \frac{r^2}{R^3}+2\gamma; \tag{50}$$

excluding the case when $r = 0$ and $R \neq 0$, i.e. excluding paths along the z-axis, it is clear that as $R \to 0$, so that also $r \to 0$ and (since $\sin\theta$ is finite or zero) $r\sin\theta \to 0$, it is necessary that $r^2/R^3+2\gamma \to 0$. This is possible only if γ is negative.

Suppose $$\gamma = -\gamma_1,$$
where γ_1 is positive; then as $r\sin\theta \to 0$, the trajectory must approach the line of force

$$\frac{r^2}{R^3} = 2\gamma_1. \tag{51}$$

Störmer has shown that the trajectories which pass through O have a high order of contact with the lines of force at this point; this is because the asymptotic expansion near the origin is found to be

$$R-\frac{\sin^2\psi}{2\gamma_1} = -\tfrac{3}{4}\gamma_1 s^5-\tfrac{33}{32}\gamma_1^2 s^6-\dots, \tag{52}$$

where the arc length s along the trajectory is measured from O, and ψ is the angle between Oz and the position-vector R, so that $r = R\sin\psi$; in terms of R and ψ, (51) takes the form

$$R-\frac{\sin^2\psi}{2\gamma_1} = 0. \tag{53}$$

Störmer calculated l from (33) for various kinds of electric particles, adopting for M the value (cf. p. 645)

$$M = 8{\cdot}52\times10^{25}.$$

For electrons, $e/m = 1{\cdot}76\times10^7$; for hydrogen atoms, $e/m = 9{\cdot}6\times10^3$; for calcium atoms, $e/m = 2{\cdot}4\times10^2$. Hence mv/e and l have the following values for these particles, moving with the various speeds indicated:

TABLE 1

$v =$	10^7		10^8		10^9		10^{10} *cm./sec.*	
	mv/e	l (*cm.*)	mv/e	l (*cm.*)	mv/e	l (*cm.*)	mv/e	l (*cm.*)
Electron	$0{\cdot}56$	$1{\cdot}2\times10^{13}$	$5{\cdot}6$	$3{\cdot}9\times10^{12}$	56	$1{\cdot}2\times10^{12}$	565	$3{\cdot}9\times10^{11}$
H+	$1{\cdot}04\times10^3$	$2{\cdot}9\times10^{11}$	$1{\cdot}04\times10^4$	$9{\cdot}0\times10^{10}$	$1{\cdot}04\times10^5$	$2{\cdot}9\times10^{10}$	$1{\cdot}04\times10^6$	$9{\cdot}0\times10^9$
Ca+	$4{\cdot}2\times10^4$	$4{\cdot}5\times10^{10}$	$4{\cdot}2\times10^5$	$1{\cdot}4\times10^{10}$	$4{\cdot}2\times10^6$	$4{\cdot}5\times10^9$	$4{\cdot}2\times10^7$	$1{\cdot}4\times10^9$

It may be added that 7×10^6 cm. was the value of mv/e attributed by Birkeland to the fast-moving electrons which he supposed to be the cause of aurorae and magnetic storms; the corresponding value of l is $3{\cdot}5\times10^9$.

In all the above cases l is less than the distance ($1{\cdot}5\times10^{13}$) of the sun from the earth. In Plate 36 (for *any* value of γ algebraically less than -1) the outer boundary of the inner unshaded area crosses the plane $z = 0$ at a distance r less than unity on the reduced scale, that is, at a distance less than l. Hence in this figure the sun lies beyond this boundary, so that no trajectory from the sun can reach the origin for $\gamma < -1$: also for $\gamma > 0$ no trajectory reaches the origin. Hence the only trajectories which can reach the origin are for values of γ such that

$$-1 \leqslant \gamma \leqslant 0,$$

as in the two middle sections of Plate 36. For any value of γ between these limits, the part of the earth's surface lying within the unshaded area in Plate 36 consists of two narrow zones; as γ ranges from 0 to -1 these zones traverse the spherical cap extending from the z-axis to the polar angle $\psi = \alpha$, obtained by substituting in (53) $\gamma = -1$ or $\gamma_1 = 1$, and $R = a$ (where a here denotes the earth's radius), or rather $R = a/l$, because in (53) the reduced units are implied. Thus the angular radius α of the polar caps is given by

$$\sin\alpha = (2a/l)^{\frac{1}{2}}. \tag{55}$$

This gives the maximum polar angle at which any such single particle from the sun can impinge upon the earth. Since $a = 6{\cdot}4\times10^8$ cm., the following are the values of α corresponding to the particles and speeds considered in the preceding table.

TABLE 2

v (*cm./sec.*)	10^7	10^8	10^9	10^{10}
Electrons . .	0·6°	1·0°	1·9°	3·3°
H^+ . . .	3·8	6·8	12·1	22·1
Ca^+ . . .	9·7	17·1	32·1	72·8

For the high-speed electrons considered by Birkeland, the value of α is 37·4°.

The observed maximum angular distance of aurorae from the earth's magnetic axis at ordinary times is about 25°, which would correspond to electrons with a speed nearly equal to that of Birkeland's electrons, or to hydrogen ions with a speed 10^{10}, or to calcium atoms with a speed somewhat exceeding 10^8 cm./sec. At times of great magnetic disturbance, the angle increases to 35° or even more.

Störmer showed also that an *inner* limit of the region of auroral precipitation is imposed by the fact that the sun, supposed to be the source of the charged corpuscle, never lies at an altitude of more than 35° above or below the earth's magnetic equatorial plane. By numerical integration for the case of cathode rays (electrons) he showed that the paths which, starting from the sun, pass through the origin, must meet the earth's surface in the zone limited by the inequality

$$-1 < \gamma < -0{\cdot}2.$$

The outer angular radius of this zone is the angle α already found. The inner angular radius, α', is given by

$$\sin\alpha' = (2\gamma_1 a/l)^{\frac{1}{2}} = (0{\cdot}4a/l)^{\frac{1}{2}}. \tag{56}$$

The linear width of the zone is $a(\alpha-\alpha')$ cm.

The inner radius of the zone, or the lower limit of α', does not depend on the properties of the particles in the same simple way as α, the outer radius.

The above considerations refer only to the paths from the sun which, if ideally continued inside the earth, would reach the origin. Other paths exist, not passing through the origin, which can enter the regions marked as forbidden in Plate 36. But since a/l is fairly small for most of the particles considered, Störmer assumed that any paths which enter the atmosphere, where their distance from the origin has the comparatively small value a, will not depart far from the paths that actually pass to the origin; if so, such paths must lie nearly if not wholly within the permitted regions of Plate 36.

If we imagine particles emitted (successively, each singly) in all directions from the sun, with a definite value of mv/e, there will be a certain number of 'distinguished' directions corresponding to paths that pass through the earth's centre. Paths emitted in directions near to these may be expected to come near the earth.

For particles emitted in certain special 'distinguished directions' from neighbouring points on the sun, at slightly different latitudes north or south of the earth's magnetic equator, corresponding to different points of an emitting area on the sun (or different positions of the same point on the sun, as its terrestrial magnetic latitude changes, mainly owing to the earth's daily rotation), Störmer found that the distribution of the points at which the particles meet the earth is very elongated along the auroral zone; for example, for relatively slow electrons ($mv/e = 315$) he found that the region of precipitation might have a length of 275 kilometres in longitude, and a breadth of only 0·072 km.

(72 metres) in latitude. For 'stiffer' particles (greater values of mv/e) the thickness is greater, being several kilometres for α-rays from radium ($mv/e = 8\times 10^5$). In all these cases the corpuscles will enter the atmosphere very nearly along the lines of the earth's magnetic force. These characteristics agree well with those observed in auroral draperies.

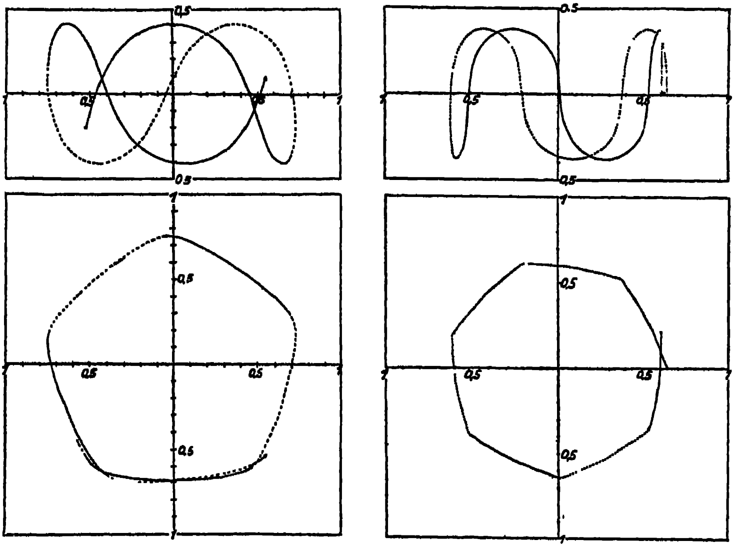


FIG. 8. Four types of periodic orbits of individual charged particles, found by numerical integration by C. Störmer, and demonstrated experimentally by E. Brüche

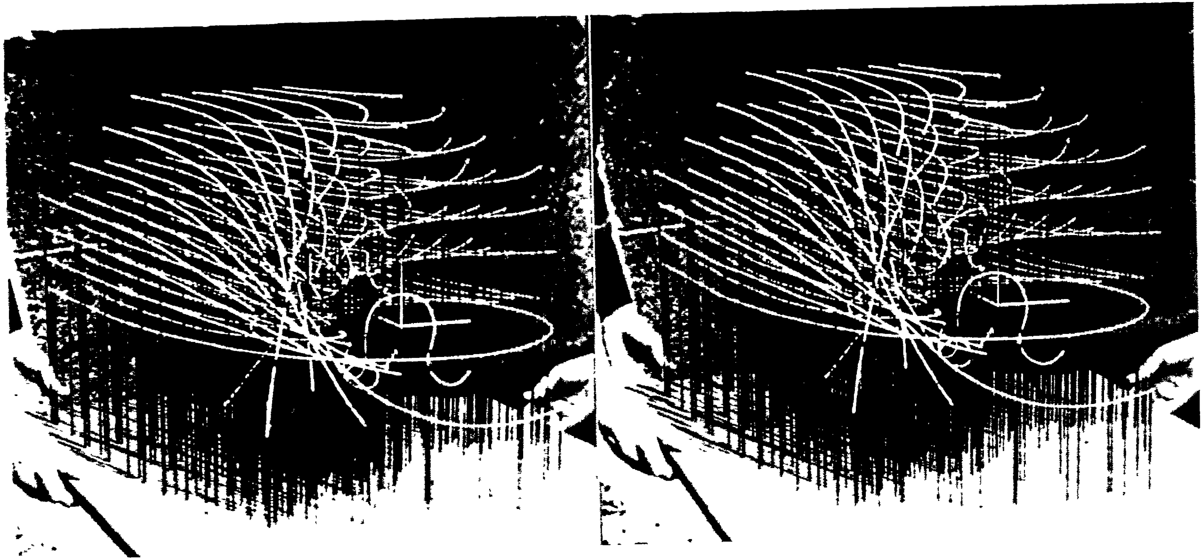
Particles from solar points in neighbouring terrestrial latitudes relative to the earth's magnetic equator may impinge on the earth at widely separated regions along the auroral zone, and their distribution in such regions of the zone may also be in a number of limited parallel bands, corresponding to the parallel auroral draperies that have often been observed.

Fig. 8 shows corpuscular paths of other types, determined theoretically by Störmer and beautifully verified experimentally by Brüche. Its four sections portray four examples of a great variety of *periodic* paths, which consist of successive parts that are congruent, and in some cases form closed periodic orbits. Störmer suggests that particles describing such paths may be the cause of the world-wide magnetic pulsations sometimes stated to exist (10.5); many of the observed pulsations appear, however, to be confined to rather limited regions.

An important application of Störmer's auroral theory, and one which at the outset was quite unexpected, is to the cosmic rays (14.16), whose approach to the earth is modified by the geomagnetic field. This subject is outside the scope of this book, though the distribution of the impact of cosmic rays on the earth imparts additional interest and importance to the study of geomagnetism, and the variations in this distribution, associated with geomagnetic disturbance, may throw valuable light on the mechanism of the latter. Lemaitre, Vallarta [27–31], Epstein [32], and others have made notable contributions to our knowledge of the influence of the earth's (and possibly the sun's) magnetic field on cosmic rays.

24.13. Störmer's equatorial current-ring. Störmer, while expressing no definite conclusion as to the nature of the particles which cause aurorae, appears generally to have considered them to be high-speed electrons, not all moving with the same speed. He seems, however, to have felt the difficulty of supposing the speed to be so high as is necessary to account for the observed auroral zone. In order partly to remove this difficulty, and also to explain why the angular radius of the zone is much enlarged at times of magnetic disturbance, he proposed an additional hypothesis (see pp. 37–104 of [19 *b*]).

If one looks from the sun towards the earth, in such a way that the earth's north pole appears uppermost, then negative particles projected earthwards from the sun will be magnetically deflected to the left, and positive particles will be deflected to the right. Negative particles directed to the right of the earth will therefore move in a path extending partly round the earth, while particles directed to the front of the earth, or to the left of it, will be turned backwards or sideways away from the earth. Störmer imagined a great bundle of negative particles thus flowing eastwards partly round the earth; this is illustrated (by some of the paths) in Plate 37, which with the aid of a stereoscope serves to show the space-distribution of many different types of path. Since the charges in motion round (or partly round) the earth are equivalent to an electric current, they will produce a magnetic field, which near the earth will be similar to the magnetic field of a complete ring of eastward-moving electrons, carrying a westward positive current. Positive particles directed towards the left of the earth would likewise bend partly round it, flowing westwards, and being roughly equivalent to a westward current-ring. Such a ring, in either case, would reduce the magnetic force parallel to the earth's axis, in the middle belt of the earth; this is a main feature of magnetic storms. If the equivalent ring is very large, its radius being many earth-radii, the magnetic field of the ring will be extensive, and over large regions may exceed the earth's



Examples (stereograms) of corpuscular paths calculated by Störmer

own field. It may therefore modify the zones of precipitation of solar corpuscles upon the earth. Störmer showed by a number of detailed calculations that the effect of the ring would be to enlarge the angular radius of the auroral zone. He therefore suggested that even at ordinary times the current-ring exists, and enlarges the radius of the auroral zone, beyond the value indicated by the simple theory which takes account only of the earth's magnetic field; it is then natural to suppose that at times of great magnetic disturbance the current in the ring will increase in strength, and draw the auroral zone equatorwards into unusually low latitudes.

For the purpose of his theory it is necessary to have the current-ring very large, in order to affect the motion of a charged corpuscle over a great part of its path, without introducing an additional magnetic field near the earth, more intense than is actually observed during geomagnetic disturbance.

The particles in the ring are supposed to be circling freely, under the deflecting influence of the earth's magnetic field. At the distances and with the speeds considered, the magnetic deflecting force is much greater than the force of gravity, which can therefore be ignored in calculations concerning the ring. The equation of motion of the particles of the ring, moving with velocity v in a circle of radius r in the magnetic equatorial plane, expresses the balance between the centrifugal and the deflecting forces, namely

$$mv^2/r = evH = evH_0 a^3/r^3, \tag{57}$$

where $H_0 = Ma^3$ is the equatorial value of the magnetic intensity at the earth's surface. Hence

$$r^2 = eH_0 a^3/mv = Me/mv = l^2, \tag{58}$$

by (33); thus the values of r for various values of mv/e are those already given in Table 1.

A ring composed of particles for which mv/e corresponds to any assigned radius l, and carrying a total current of i e.m.u. or A ampères ($A = 10i$), will produce a magnetic field of which the intensity H at the earth is approximately given by 1 (26) (taking $r = a = l$), or

$$H = \pi A/5l. \tag{59}$$

The particles of the ring are not supposed necessarily to have the same value of mv/e as the deflected particles which produce the aurora. Störmer gives several tables showing, for various values of mv/e fo each of the two sets of particles, what is the value of H that woul correspond to various assigned angular radii α for the auroral zone. The corresponding values of A are immediately calculable from H and mv/e.

A few examples may be quoted. If for both the auroral and the ring particles $mv/e = 4\times 10^5$ (as, for example, for Ca^+ ions with speed 10^8 cm./sec.), then for $\alpha = 19°$ and $\alpha = 25°$ the values of H are respectively 10γ and 50γ, which are reasonably small. The value of l is $1{\cdot}4\times 10^{10}$, or about 22 earth-radii (one-third of the radius of the moon's orbit); the values of A corresponding to $H = 10\gamma$ and $H = 50\gamma$ are $2{\cdot}2\times 10^6$ and $1{\cdot}1\times 10^7$ amperes. For the same particles, the respective values $\alpha = 35°, 40°, 45°$ require $H = 315, 660, 1{,}200\gamma$; these are greater than the observations warrant.

If mv/e is 100 for the particles of the ring (corresponding to electrons with speed 2×10^9 cm./sec.), almost the same values of H apply to the cases $\alpha = 19°, 25°$ when mv/e for the auroral particles ranges from 100 to 10^5. Störmer's hypothesis here bears closely on the theory of magnetic storms, and it is important to know whether such a current-ring, composed, as this theory assumes, of charges all of the same sign, could hold together for a sufficiently long period against the mutual repulsions of its particles. It must be remembered that the decrease of horizontal magnetic force produced during magnetic storms often takes days, and sometimes weeks, to die away.

It might seem that the simplest method of explaining the enlargement of the radius of the auroral zone during magnetic storms, if Störmer's general ideas, based on the motion of a single charged particle, are accepted, would be to suppose mv/e to be increased at such times. Störmer rejected this view because it would imply that the particles should penetrate farther into the atmosphere during magnetic storms than at ordinary times; whereas he states that low-latitude aurorae penetrate less far down than normal aurorae. This argument is not conclusive, however, unless it can be shown that the amount of air traversed is less, because low-latitude aurorae, following the lines of magnetic force, come in more obliquely than the ordinary polar aurorae.

The really destructive argument against Störmer's theory of the current-ring is that this would not hold together against the mutual electrostatic repulsion of its particles; this is sufficiently established by the discussion of § 8, unless the corpuscles have improbably great energies, which would not be consistent with the observed levels of auroral penetration. Similar arguments affect the whole of Störmer's auroral theory, in that electrostatic forces in streams of corpuscles must profoundly affect the motions, so that a fundamental revision or extension of his theory is necessary.

Nevertheless, Störmer's theory is very successful in establishing many analogies with auroral features, in the motions of single corpuscles or thin beams of corpuscles. This makes it seem likely that much of the theory will in time be embodied, though in some modified form, in the complete theory of auroral and geomagnetic disturbance [33].

24.14. The action of a magnetic field on a neutral ionized stream. The first detailed attempt to discuss the motion and effects of a neutral ionized stream of solar particles, in the neighbourhood of the earth and under the influence of the geomagnetic field, was made by Chapman [37] in 1923. He sought only to determine the steady state; the stream was supposed from the outset to envelop the earth completely, and to extend to infinity in both directions along the line of motion of the stream, and also laterally. His main aim was to find whether a strictly neutral ionized stream would be deflected by the earth's field so as to impinge mainly in the polar regions; and also to find whether this deflexion could account for the entry of the stream into the atmosphere over the dark hemisphere of the earth. He concluded that the stream would be only slightly deflected, and that it would not explain the aurora polaris.

The problem was taken up afresh some years later by Chapman and Ferraro [38], who concluded that Chapman's earlier investigation [37], by assuming the stream to have been acting on the earth for an indefinite period, had ruled out the features that seemed in actual fact to be of chief importance in the problem. They found that the geomagnetic field might influence the stream, as this advanced towards the earth, in a way that could account for the first phase of a magnetic storm; and that the earth's field would, as it were, carve out a hollow space, of diminishing extent, round the earth, though the matter of the stream could pour into the earth's atmosphere along two 'horns' extending from the surface of this hollow, one towards each polar region. They suggested that the further development of the magnetic storm, and the auroral phenomena, were produced by charged layers induced on, and escaping from, the surface of the hollow. Their work provides, however, only the barest sketch of a theory, in which much is left tentative and uncertain, for further study.

They led up to the actual problem of the solar stream approaching the earth's field, which is a subject of extreme difficulty, by considering a series of idealized simplified illustrative problems. They found that, though Chapman's earlier investigation was defective in some points, one of its main conclusions was correct, namely, that within the solar ionized stream the ions and electrons can move nearly rectilinearly without spiralling, and with only a slight deflexion by the field. The field tends to deflect the ions and electrons in opposite directions, and so to separate them, but this separation is resisted by an opposite tendency due to the electrostatic field set up; all that finally results is a slight polarization of the stream, involving a surface charge-distribution, and also, in general, if the polarization is not uniform, a volume charge-distribution. The electrostatic energy of the polarization is drawn from the kinetic energy of the stream, which is slightly slowed down.

The mere presence of the surface charge, uncomplicated by other effects that occur in general, is exemplified in the case of a stream in the form of an 'infinite plane slab', bounded by two parallel planes ($y = \pm a$), between which the density (N electrons and N positive ions per c.c.) and the velocity are uniform. When such a stream is in the presence of a uniform magnetic field H, in the z direction (normal to the velocity u) and parallel to the planes $y = \pm a$, a uniform surface charge of surface density $\pm uH/4\pi c$ ($= \pm\sigma$) is set up, of opposite sign on the two faces of the slab. These surface charges exert a uniform electric field $4\pi\sigma$ or uH/c, in the y direction, within the stream, and no field outside the stream; the internal electric field just balances the electromagnetic force on the particles, $\pm euH/c$, due to the magnetic field, so that within the stream the particles move rectilinearly as though the field were absent. Chapman and Ferraro considered in detail the state of the charged surface layers of the stream, which are stable; if the particles have no random thermal motions, the two layers can pulsate rhythmically with the spiral frequency of ions or electrons (15 (71)); the ionic layer pulsates with much the greater amplitude, and much more slowly, than the electronic layer, and the two layers have thicknesses in proportion to the masses of the ions or electrons of which they are composed; the weaker the magnetic field, the slower the pulsations of the layers, and the greater the amplitudes. If, however, the particles of the stream, while possessing the uniform mean motion u, form a gas at temperature T, the particles will have random thermal velocities superposed on u; the surface layers will still be stable, since any charge which, by its random motion, is carried out of the layer will be deflected back into the layer by the field; the random motions slightly blur the simple conditions previously described, but do not alter their character in any essential respect. The magnetic field will be affected by the stream only to the extent of a fraction of itself, of the order $(u/c)^2H$; the only electric currents present are those in the two surface layers. If H is sufficiently great, in relation to the density and breadth ($2a$) of the stream, the lateral polarization of the field may separate the positive and negative charges into two distinct layers, there being then no neutral body of the stream; the two layers will pulsate side by side, perhaps overlapping at some stages of the motion. If the stream has a component of velocity parallel to the magnetic field, this component will remain constant, and will not affect the consequences of the motion transverse to H.

The next simplest ideal problem to consider is that of a uniform cylindrical stream (of circular cross-section) moving uniformly through a uniform transverse magnetic field H. As before, the stream is laterally polarized by the field, in the direction transverse both to H and to the velocity u; a surface distribution of charge is set up, which produces a uniform electric field uH/c within the stream, and just neutralizes the deflecting action of the magnetic field upon the ions and electrons in the neutral body of the stream. The surface layers pulsate as before, but in the present case the problem presents a feature which was (exceptionally) absent in the case of the plane-slab stream; the charged surface layers now produce an electric field outside as well as inside the stream, and this field, acting on the particles in the surface layers, tends to disperse them away from the stream. They will be accelerated nearly along the lines of force of the magnetic field; the electrons forming the negatively charged layer will escape (in both the $\pm z$ directions) much more rapidly than the positive ions, because the accelerating electric force is (at the outset) the same for particles of either kind, and the electronic mass is much the smaller. As charges escape from the surface, new charges will flow thither from the interior to maintain the uniform electric field within, just sufficient to balance the electromagnetic deflecting force $\pm euH/c$. This corresponds with the fact that, relative to axes moving with the stream, the electric field inside it must be zero (to a high degree of approximation), since the stream is a good electrical conductor; relative to such axes the magnetic field H (as measured by a 'fixed' observer) produces a transverse electric field uH/c (neglecting quantities of the order $(u/c)^2$), in the direction opposite to that of the field due to the surface charges; it is this 'relativistic' electric component of the magnetic field, relative to the stream regarded as stationary, which produces the electric polarization of the stream and is thereby annulled. The internal electric field is due to the charges over the surface and to those that have escaped from it; the surface field adjusts itself to render the internal field uniform.

Chapman and Ferraro have discussed many of the details of the escape of the surface charges from such a cylindrical stream. It seems likely that the escape and renewal of the charged surface layers will continue until the stream becomes completely dispersed, tending, in fact, to spread in the $\pm z$ directions so as to form a polarized infinite slab stream, in which, besides the forward motion u, there is also a non-uniform distribution of motion in the $\pm z$ directions. The energy of this latter component of motion, originally absent, is gained at the expense of the kinetic energy of the forward motion u, so that this forward motion is retarded.

The uniform motion of a uniform *spherical* neutral ionized cloud in a uniform magnetic field can be discussed in a similar way. From this and the two preceding problems some general inferences can be made as to the uniform motion of a uniform neutral ionized stream or cloud of *any* form, in a uniform magnetic field. Relative to axes moving with the particles, the magnetic field merely produces a uniform transverse polarizing field; this sets up a distribution of surface charge whose field balances the polarizing field, in the interior; the surface charges are unstable and are repelled away. The relative transverse displacement of the interior charges, while the polarization is being set up, is of order $uH/2\pi cNe$, or approximately $uH/90N$. If, for example, $H = 0{\cdot}35\,\Gamma$ (as at the earth's equator) and $u = 10^8$ cm./sec., then the relative displacement of the charges is $4\times 10^5/N$; if $N = 0{\cdot}1$, then the displacement is less than 40 km., which is small compared with the size of the earth. If the motion of the stream is not uniform throughout its interior, then even in a uniform magnetic field the surface charges set up by electro-magnetic deflexion of the particles cannot balance the deflecting forces; electric currents will be set up in the interior, and will be acted on by the magnetic field; the body of the stream will be accelerated in some parts and retarded in others, in a manner tending to make the velocity uniform throughout the stream.

If the stream moves with uniform motion in a non-uniform field, into regions of increasing magnetic intensity, the electric polarization of the stream must continually increase, and this necessitates a continuous transverse flow of charge across the stream; the magnetic field acting on this current retards the forward motion, as indicated by Chapman in his earlier paper. This retardation can be illustrated by considering a neutral ionized stream of uniform mass-density ρ, advancing with initial velocity u into a region of gradually increasing transverse magnetic intensity H, where the velocity is reduced to u'; the retardation is approximately given by

$$(u'/u)^2 = \lambda/(1+\lambda), \tag{60}$$

where

$$\lambda = 4\pi\rho c^2/H^2. \tag{61}$$

Taking $H = 0{\cdot}35\ \Gamma$, Table 3 gives the ratio u'/u for several different values of λ. The table also gives the corresponding values of the mass-density ρ; in this connexion it may be noted that, for a stream containing one hydrogen atomic ion (and one electron) per c.c., $\rho = 1{\cdot}7\times10^{-24}$ gm./c.c.

TABLE 3

λ	*0·01*	*0·1*	*0·2*	*1*	*2*	*10*	*100*
u'/u	0·10	0·30	0·41	0·71	0·82	0·95	1·00
ρ	$1{\cdot}1\times10^{-25}$	$1{\cdot}1\times10^{-24}$	$2{\cdot}2\times10^{-24}$	$1{\cdot}1\times10^{-23}$	$2{\cdot}2\times10^{-23}$	$1{\cdot}1\times10^{-22}$	$1{\cdot}1\times10^{-21}$

The table shows that the stream containing one H^+ ion per c.c. would be considerably retarded in travelling into a field of intensity $0{\cdot}35\,\Gamma$. The retardation would be small, however, for streams of density corresponding to 50 H^+ ions (or about 1 Ca^+ ion) per c.c.

The further discussion of the motion of a neutral ionized stream of dimensions great compared with the earth, up to and beyond the earth (so as to envelop it), is postponed to Chapter XXV, as being properly part of the detailed theory of actual geomagnetic disturbance, according to one of the hypotheses considered there.

XXV

THEORIES OF MAGNETIC STORMS AND AURORAE

25.1. The principal facts to be explained. Aurorae and geomagnetic disturbance are complex phenomena, and any theory that seeks to explain them in detail must likewise be complicated. But in 9.11, 9.12, and 9.14 it has been shown that many systematic properties underlie the complexity and individuality of these D phenomena. The regularities of their distribution in space, development in time, and frequency of occurrence, must be our guide in studying the causes and the mode of production of the D phenomena [24.1]. Sometimes, however, hypotheses have been proposed which pay little regard to the systematic properties—sometimes even apparently with little knowledge of them. Great difficulties have been experienced in all serious attempts to construct a theory of aurorae and magnetic storms, and the difficulties arise quite as much from the regular properties as from the irregularities.

The properties to the explanation of which a theory should chiefly be directed seem to be as follows:

(*a*) The morphology of the average disturbance field as summed up in the current-systems of Figs. 9.21–6, these being regarded as conveniently summarizing the regular features of the D field at the earth's surface, without assuming any special location for the actual current-systems; though these (apart from the associated induced earth currents) must be outside the earth.

(*b*) The development of the D field with respect to storm-time, as indicated by Figs. 9.4, 9.5, including the suddenness of commencement in the case of many storms.

(*c*) The polar incidence and distribution of aurorae, and the general connexions between aurorae and geomagnetic disturbance.

(*d*) The 27-day recurrence-tendency of the D phenomena.

(*e*) The other statistical relationships of these phenomena with the sun.

(*f*) The variety of the irregularities in the D field, some of which are almost world-wide, whereas others are local.

In time the relationships between the D phenomena and the ionospheric data determined by radio methods may make important contributions to the above set of facts (Ch. XV).

25.2. The nature of the solar cause. The cause of the D phenomena may with every confidence be located on the sun; unfortunately it is still a mystery where, when, and how this cause begins to act there. The work of the theorist must therefore at present be largely an imaginative construction; he must explain definite terrestrial phenomena by causes of whose origin on the sun, and of whose mode of transit to the earth, he is mostly ignorant. Speculation is unavoidable, though it must be guided by such poor sign-posts towards the truth of this problem as our present physical and solar-physical knowledge supplies; it must also be controlled by the quantitative development of the hypotheses considered, to the utmost possible extent; unfortunately here also the task is arduous, owing to the inescapably three-dimensional character of many of the problems that demand mathematical investigation, and their complicated electric, magnetic, and also partly hydrodynamic nature (hydrodynamic here referring to problems of gaseous motion, including also some in which kinetic-theory considerations necessarily enter). At present only the fringe of the subject has been touched, and the progress made has been less constructive than destructive by criticism showing that most of the hypotheses yet advanced are not tenable.

In Chapter XXIV the two basic types of theory, as classified according to the nature of the solar agent causing the D phenomena, have been considered. The alternatives are that the solar agent is either ultra-violet light or corpuscles; the conclusion drawn in Chapter XXIV was that corpuscles are the likely cause. The chief factors in favour of this view are

(*a*) the 27-day recurrence tendency shown by the D phenomena;
(*b*) their polar property;
(*c*) the time-lag between the central passage across the sun of the disturbed solar areas thought to be responsible for terrestrial phenomena, and the occurrence of the geomagnetic disturbance; and
(*d*) the dissimilarity of the D phenomena to the magnetic effects of solar flares of ultra-violet light.

Perhaps none of these factors is absolutely conclusive in favour of the corpuscular hypothesis, though especially the first seems very difficult to reconcile, in any unforced way, with the ultra-violet light hypothesis. The weak point in the argument based on the third factor is the uncertainty as to the connexion of the visible disturbed solar areas with the D phenomena considered; this, and the possibility of non-radial emission of corpuscles, or of their deflexion by the sun's magnetic field, is also a weakness in the estimates of the corpuscular speeds based on the time-lags (*c*); these estimates of speed are of the order 1,000 to 2,000 km./sec. The factor (*b*)—the polar property of the D phenomena—seems to tell heavily in favour of a corpuscular hypothesis, although Hulburt [2] and Maris have constructed a hybrid theory which, though based on ultra-violet flares on the sun, has some corpuscular characteristics at the terrestrial 'end' of the problem; the corpuscles of their theory, that is to say, are terrestrial; this theory is considered in § 8. The argument from (*d*) is strong, but not completely proof against objections attributing a less penetrating character to the ultra-violet light which may be supposed responsible for the D phenomena, than to the light of the observed brief solar flares.

Except for an account of the ultra-violet light theory, due to Hulburt [2, 8] and Maris [5], the present discussion of magnetic storm theories will be based on the corpuscular hypothesis. This has been proposed in various forms; again there are two main divisions, according as the stream of corpuscles is supposed electrically charged or neutral. The difficulties encountered by hypotheses based on charged streams or clouds of particles have been discussed in Chapter XXIV; these were first urged by Schuster [24.1] against such hypotheses when proposed by Birkeland [24.2], Lodge, and others, and are based on the electrostatic dispersion of the stream on its way to the earth; subsequently Lindemann [24.15] raised similar and related objections against another form of charged-stream theory, proposed and afterwards abandoned by Chapman [9.2]. It seems impossible to overcome the electrostatic objections to hypotheses based on the direct magnetic action of charged streams from the sun, except by postulating corpuscular speeds so near to that of light (in order to render the particles very energetic and massive) as to project the theory into a pitfall of a quite different but equally serious character (24.10); because the particles then have penetrating power comparable with that of cosmic rays, and the theory comes into conflict with the known levels to which auroral rays penetrate the earth's atmosphere, and with the known very slight variation in the intensity of cosmic rays at times of great magnetic disturbance.

Consequently the only possibly tenable type of solar-corpuscular hypothesis to explain the origin of the D phenomena seems to be that which assumes a stream which is electrically neutral; and in order that the stream may be at all influenced by the earth's magnetic field, as seems to be necessary to account for the polar property of the D phenomena, it must be supposed that the stream, at least when it reaches the earth's neighbourhood, is neutral but ionized. It may have this character from the time of its emission, or, as Lindemann suggested, it may become ionized on its journey from the sun. The distinction between these two possibilities is one for the solar physicist to consider; here the ionized character of the stream, near the earth, will be adopted as a starting-point of the further discussion.

The form and motion of a corpuscular stream continuously emitted by the sun and travelling towards the earth have been discussed in Chapter XXIV, up to the point where the earth's magnetic field is reached. Some ways in which a neutral ionized stream can be influenced by a magnetic field were also considered there, but the phenomena connected with the *approach* of the stream towards the earth, and its passage across it, were deferred to the present chapter, as being essentially the terrestrial part of the theory of the D phenomena.

25.3. The advance of a neutral ionized stream of solar particles into the earth's magnetic field. The attempt to determine what will happen when a neutral ionized stream of solar particles approaches the earth, and in particular how it will be influenced by the earth's magnetic field, represents a purely deductive approach to the theory of the D phenomena, undertaken under the guidance of the considerations described in § 2. The speed and mass of the individual particles, and the density and lateral dimensions of the stream, are not actually known; but if particular (or general symbolic) values for these quantities are adopted, the problem presented is a quite definite one. Unfortunately, owing to its great difficulty, only slight progress has been made towards its solution. The following account is based on the work of Chapman and Ferraro [11] upon this problem. We will suppose that the velocity of the particles is 1,000 km./sec. or more, and that the diameter of cross-section of the stream, near the earth, is 1,000 earth-radii or more, so that its passage across the earth would take at least 8 hours. We suppose that it sweeps across the earth in such a way that the centre-line of the stream passes through or relatively near the earth. At a distance of 50 earth-radii from the earth, the intensity of the geomagnetic field is small—about $0{\cdot}3\,\gamma$; and at that distance the approaching face of the stream will be substantially undistorted, and will appear nearly plane as viewed from the earth, subtending a solid angle of nearly 2π.

This nearly plane face is only slightly inclined to the plane containing the sun's axis and the earth; it approaches the earth by the advance of the particles of the stream, along a direction almost radial from the sun, and therefore at only a small inclination to the stream-surface (24.3). If the lateral forward surface of the stream remained undistorted by the earth's field, it would sweep across the earth and pass beyond it so that after about 25 minutes the earth would be 50 earth-radii *within* the stream; for many hours afterwards the earth would be enveloped in the stream.

The direction of advance of the forward face of the stream towards the earth is such that, near the surface, the positive charges will be deflected in the direction towards the interior of the stream, and the electrons will be deflected away from the surface. That is to say, the stream will be polarized, and a negative surface charge will be induced on its surface. The polarization will decrease inwards into the stream, owing to the decrease of the intensity of the earth's field. When the stream is far distant from the earth, the surface density of charge, calculated approximately as in 24.9, is very small; the motion of the electrons and ions in the surface layer will not be simply periodic and pulsatory, as in 24.9; the field is weak and also non-uniform, and the surface charges may pass beyond the reach of the electric field of the stream, and never return to it. They will escape, but the density of any such dispersing cloud of electrons will be too small to exert any appreciable magnetic field. Many of the electrons will probably find their way along the earth's lines of force towards the polar regions, but their penetrating power will be small, if their velocities are only of the order of 1,000 km./sec., as will in general be supposed in our illustrative numerical descriptions.

When the stream is about 50 earth-radii distant, the spiral radius of the outer layer of electrons, at the point on the surface of the stream that is nearest to the earth, will be only about 20 km., and in the equatorial plane the electron will oscillate to and from the surface over this distance. Those at other points of the surface will be in a weaker field and can range within a greater distance from the neutral body of the stream. But the increasing negative charge on the layer, as the stream approaches the earth, coincides with an increasing 'attachment' of the surface charge to the stream, by reason of the increasing magnetic intensity of the field in which the charged surface layer is situated.

The change in the magnetic field due to the electric current in the surface layer is only of order $(u/c)^2H'$, where u is the velocity of the stream and H' the earth's magnetic intensity in the layer; if $u = 10^8$ cm./sec. = 1,000 km./sec., this is negligible even if the layer were in contact with the earth; if $u = 10^9$ cm./sec., $(u/c)^2H'$ would not then be negligible; but it appears that the surface actually cannot approach to within a few earth-radii from the earth without being greatly distorted; this results in a succession of phenomena of much greater importance than the magnetic effect of the undistorted charged surface layer. No importance is attributed, for the theory of magnetic storms, to the surface charge on the boundary of the solar stream, while this is approaching the earth with only slight distortion.

25.4. The distortion of the advancing stream-surface, and the first phase of a magnetic storm. The distortion of the stream-surface as this advances towards the earth can perhaps most easily be understood by considering the problem with reference to axes moving with the stream. With respect to these the stream is merely a stationary mass of ionized gas of very low density. Its boundary is a great cylinder, the curvature of the surface being so small that over areas whose linear dimensions are many earth-radii it may be considered plane. The earth, for this purpose to be considered as an elementary magnetic dipole, with its axis nearly parallel to the boundary of the gas, is moving towards the surface, with a considerable velocity (u), along a line inclined at only a few degrees to the surface.

The stationary ionized gas is an electrical conductor, so that the situation has some features in common with the motion of a magnet near a conducting mass of metal. It is well known that such motion induces electric currents in the metal, which tend to shield the interior of the metal from the changing magnetic field. Further, there is a mechanical reaction between these electric currents and the magnetic field, which causes a mutual repulsion between the magnet and the currents, or rather between the magnet and the part of the metal in which the currents flow.

If the magnet is moved in the presence of a conducting fluid, like a liquid or an ionized gas, the electromagnetic mechanical force on the part of the fluid which conveys the current can distort the fluid. The part of the surface where the currents are most intense can in fact be pushed inwards by the advancing magnet. Chapman and Ferraro concluded that, under certain conditions, the motion of the terrestrial magnet relative to the ionized gas of the solar stream could 'carve' out a hollow space in the stream; they illustrated this in equatorial section by Fig. 1, which represents successive stages in the advance of the surface of the stream, now viewed relative to axes fixed with respect to the earth. The figure implies a great retardation of the part of the stream advancing nearly directly towards the earth, so that near the 'vertex' of the surface of the hollow space carved in the stream by the earth's magnetic field there must be a great heaping up of gas, as the particles from the rear pour on towards the surface near the vertex.

The distortion of the stream-surface is thus supposed due to the mechanical reaction between the earth's magnetic field, and the current which its motion relative to the surface induces in the ionized gas; these currents tend to shield the interior from the field. If the stream were a perfect conductor, the currents would flow wholly in the surface layer, and no tubes of force of the external field would penetrate behind this layer. Such a change in the magnetic field is a result of the superposition of the field of the currents on the external field; the field of the currents must annul the external field within the conductor; it will strengthen it in the space outside the conductor, at

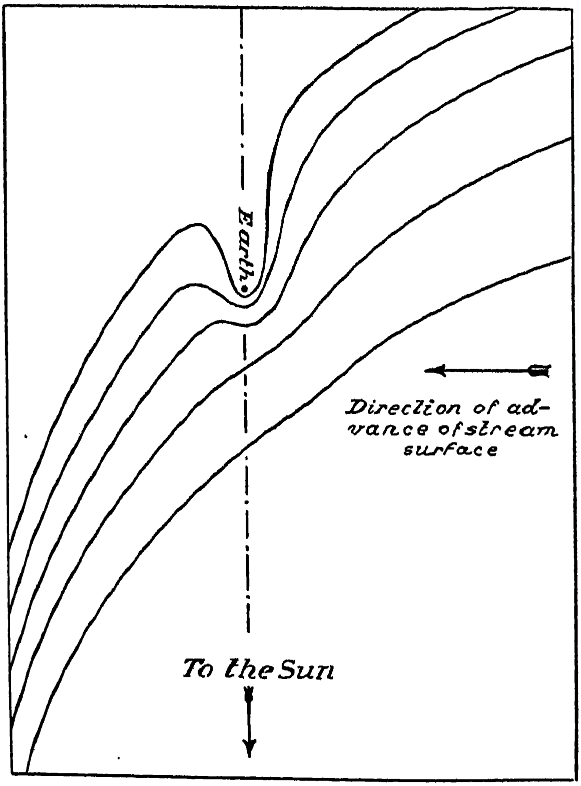


FIG. 1. Sections by the earth's equatorial plane of the advancing front surface of a corpuscular stream, illustrating diagrammatically the formation of a hollow space round the earth by the action of the geomagnetic field on the neutral ionized stream

least in the region near the conductor. This change in the field outside the conductor may be thought of as representing the crowding out from the conductor, by its surface currents, of the closed system of tubes of force associated with the exterior magnet; if we think of the magnet as fixed, and the conductor as moving towards it, the surface (by reason of the shielding property of the induced currents) is able to compress the tubes of force between it and the magnet, progressively crowding them together and strengthening the magnetic field in many regions, though of course not in all.

Another way of regarding the effect of the surface currents is that these are equivalent to a certain magnetic shell of non-uniform strength, lying in the surface, and that this shell may in certain circumstances be equivalent, in its effects *outside* the conductor, to an 'image' magnetic dipole within the metal, of the same sign as the external magnet; the field of the image dipole, near to and in the equatorial plane of the external magnet, will increase the intensity of the field.

This predicted initial increase of the intensity of the field round the external magnet, in the actual case of the solar stream advancing into the earth's magnetic field, is proposed as a possible explanation of the first phase of a magnetic storm, during which, for a period of the order of an hour or two, the horizontal magnetic intensity is in general increased.

Little importance could be attached to such an explanation, so long as it remained a mere qualitative sketch of what *might* happen to the solar stream in the initial stages of its advance upon the earth. A quantitative test of the explanation is imperative. Before describing the rudimentary steps taken by Chapman and Ferraro in this direction, some further qualitative considerations will be mentioned.

The shielding of the interior of the stream by currents induced and flowing in the distorted surface layers of the stream compresses the earth's tubes of magnetic force into a region smaller than that which they would normally occupy, even though they may penetrate to a considerable depth in the stream before a region of practically complete shielding is reached. This compression of the tubes of force increases the magnetic energy of the field, which is distributed with the density $H^2/8\pi$; the gain of magnetic energy must be made at the expense of the kinetic energy of the stream, whose particles, throughout the distorted region, are, as stated, retarded by the field. In the remainder of the stream the motion and kinetic energy of the particles are undisturbed.

The retarding force per unit surface area of this current-layer at any point is proportional to the tangential intensity (H_s) of the field, and to the current-density (i) at the point; and i will be approximately proportional to the normal velocity (v_n) of the surface, and to H_s. Thus, in all, the retarding force will be approximately proportional to $v_n H_s^2$. In the equatorial plane $H_s = H$, the surface being there, by symmetry, perpendicular to this plane; since $H \propto 1/r^3$, where r denotes the distance from the earth's centre, the retarding force varies as v_n/r^6; this rapidly increases as the surface approaches the earth, except in so far as the velocity of approach, v_n, is simultaneously reduced.

The stream will first be retarded, during its approach to the earth, on the advancing side, nearest the earth. As the stream continues to advance, the approaching particles near the surface on the sunward side of the earth will be increasingly retarded, and the surface will become bent round the space occupied by the earth's magnetic field. After the time when, if the field had been absent, the surface of the stream would have swept past the earth, the actual surface, on the forward side, will be entering into a weaker field, where it is less retarded; thus it will be able to advance, along the direction of motion of the particles, beyond the part of the surface immediately between the sun and the earth. Thus the earth will be enveloped on both the forward and rearward sides by the stream, as in Fig. 1.

The actual rate of retardation of the surface, and the effect on the magnetic field, are matters for detailed calculation; they are found to depend essentially on the kinetic energy of the stream per unit volume. If the stream-density is so low that the ions and electrons can be separately deflected as if they were independent particles, the conception of the stream as a conducting medium is inapplicable, and the above description of the phenomena is not valid; the electrons and ions would behave as does the single particle considered in Störmer's auroral theory (24.12); the electrons would be stopped at a far greater distance from the earth than that to which the ions could attain; the excluded regions for each are as described by Störmer for particles with the corresponding values of mv/e. On the other hand, if the stream is very dense, it would be only slightly distorted and retarded; the currents induced during the approach of the stream towards the earth would produce an increase in the field near the earth during the approach of the stream, but the effect would be very fleeting; the 'hollow' would be only a slight depression in the advancing surface, which would sweep up to and beyond the earth; only the region directly eclipsed by the earth would then be free from the stream.

25.5. The quantitative discussion of the first phase. A proper analytical discussion of the distortion of the surface of the stream by the earth's field, and its magnetic consequences, involves difficulties that have not yet been seriously attacked. Chapman and Ferraro only considered some simpler problems that seemed to illustrate the main features of the actual case, to a degree sufficient to yield quantitative conclusions of at least the right order of magnitude.

They first considered the current-system induced in a thin *rigid* infinite plane conducting sheet when a magnetic dipole of moment M approaches it with *normal* velocity w. Maxwell showed that if w/c is small, and if the specific electrical surface resistance of the sheet is expressed as $2\pi w_0$ (the dimensions of w_0 are those of velocity), then the effect of the sheet is to reduce the field of the dipole, throughout the space on the side of the sheet farther from the dipole, in the ratio $w_0/(w+w_0)$, the same at every point; the effect in that region is as if the dipole moment M were reduced in this ratio. In the space on the same side of the sheet as the dipole an additional field is superposed, which is such as would be produced there if the sheet were absent, and replaced by an image dipole, at the (moving) image point. This image dipole would have the same sign as M, and a moment $Mw/(w+w_0)$. If the sheet is perfectly conducting, so that $w_0 = 0$, the space on the far side is completely shielded, and the image dipole has the same moment as the actual one.

The magnetic energy of the field is increased in the region on the side of the sheet on which the dipole approaches, though the intensity is not changed everywhere in the same ratio, nor even everywhere increased.

Fig. 2 shows the current-lines for the induced current-system, due to the relative approach of a dipole to a plane conducting sheet along the *normal*, the dipole being *parallel* to the sheet. A parallel to the dipole axis is indicated by the arrow; the dipole approaches the plane from above. The current circulates round two foci; their distance apart is $\sqrt{2}$ times the distance of the dipole from the sheet at the instant to which the diagram refers. As this distance decreases, the whole current-system shrinks proportionately in linear scale. At the two foci of the current-system, the lines of force of the dipole are perpendicular to the sheet, so that there no e.m.f. is induced.

Fig. 3 gives a rough illustration of the combined magnetic field of the dipole and of the sheet, when this is a perfect conductor, so that the space on the far side of the sheet is completely shielded from the dipole field. In this diagram the points Q represent the foci of the current-system of Fig. 2.

If d denotes the distance of the doublet from the sheet, in Fig. 3, the mechanical force between the dipole and the sheet is the same as that between the dipole and an imaginary equal dipole at a distance $2d$; hence (see 1.10) it is

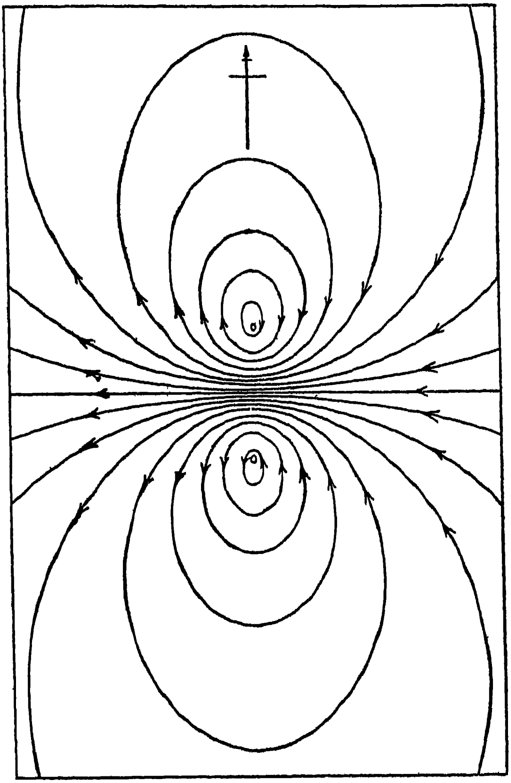


FIG. 2. Lines of flow of the electric currents induced in the plane surface layers of a neutral ionized stream advancing into the geomagnetic field, before the surface is materially distorted by the field

$$3M^2/(2d)^4.$$

The work done in bringing the dipole from infinity to the distance d is the integral of this force with respect to d (*not* $2d$), from ∞ to d; its value is $M^2/16d^3$. If $d = \nu a$, where a is the earth's radius, then, substituting the values $M = 8{\cdot}1 \times 10^{25}$, $a = 6{\cdot}37 \times 10^8$, this work has the magnitude $1{\cdot}6 \times 10^{24}/\nu^3$ ergs. For example, if $\nu = 5$, the energy is $1{\cdot}3 \times 10^{22}$ ergs. The field of the image dipole at the point occupied by the real dipole would be $M/8d^3$ or $M/8\nu^3a^3$ or $H_0/8\nu^3$, where H_0 (about 30,000 γ) is the equatorial surface intensity of the earth's field; if $\nu = 5$, this is about 30 γ; the field of the image dipole would vary from $H_0/(2\nu-1)^3$ to $H_0/(2\nu+1)^3$ at distances $\pm a$ from the real dipole, on the line joining the two; this variation would be from about 40 γ on the near side to 23 γ on the far side.

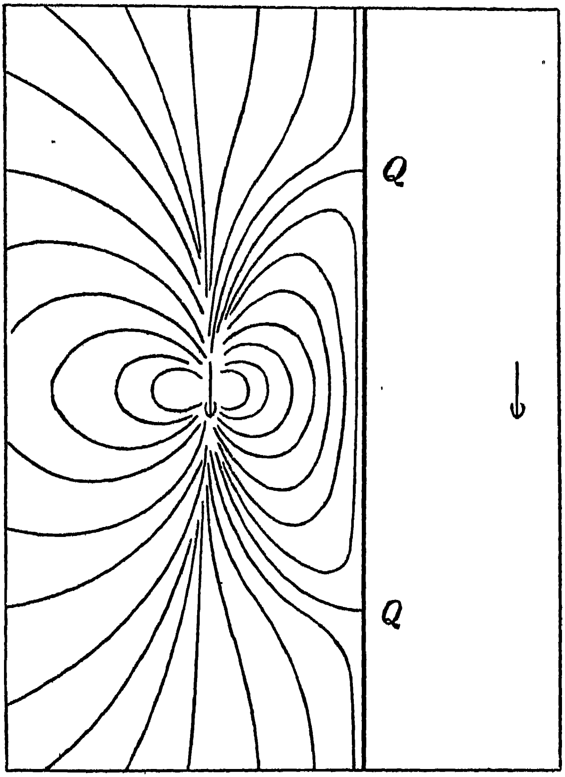


FIG. 3. Diagrammatic sketch of the combined field of the geomagnetic doublet and of the electric currents induced in the plane surface layers of the advancing neutral ionized stream, on the assumption that these currents completely shield the interior of the stream from the geomagnetic field; in this case the magnetic field of the induced current-system is the same as that of the image (shown on the right) of the geomagnetic doublet, relative to the plane face of the stream

The energy $1{\cdot}3 \times 10^{22}$ ergs may be compared with the kinetic energy of a set of particles moving with the velocity 1,000 km./sec. For equality between the two energies, the combined mass of the particles would have to be $2{\cdot}6 \times 10^{6}$ gms.; this is the mass of about 2×10^{30} hydrogen atoms. If these were distributed throughout a square cylinder of side $5a$ and of length such that particles with the speed 10^8 cm./sec. would traverse it in 1 hour (so that the volume of the cylinder would be about $3{\cdot}6\times10^{30}$ c.c.), there would need to be about one H atom in every 1·8 c.c. This illustrates the density of the stream necessary if the above amount of magnetic energy were to be supplied by complete retardation of the stream over this cross-sectional area, and over a length corresponding to that which would pour forward, over a period of time comparable with the duration of the first phase of a storm. Needless to say, the bearing of such a calculation, upon the actual application of the present ideas to the first phase of a magnetic storm, is a matter for careful scrutiny.

In the case of the relative approach of the earth towards the actual solar stream, the motion is at first (while the stream is little distorted) at a very slight angle to the surface. This suggests the need for consideration of what would be the induced currents and magnetic effect if the earth moved parallel to the surface of the stream—a problem more difficult than that of the normal motion. The above calculation of the field of the image doublet, however, shows that at the earth its field will be small until ν is not more than 10, that is, until the surface is not more than 10 earth-radii distant from the earth. It seems likely, however, that by the time the earth is within this distance of the stream, the stream-surface will have become distorted so as to enclose the earth on both the post meridiem and ante meridiem sides, and that then the surface will be nearest to the earth at or near the point where the surface crosses the line from the sun to the earth. This point will be called the *vertex* of the hollow. Thus the approach of the earth to the nearest part of the stream-surface will then be approximately normal.

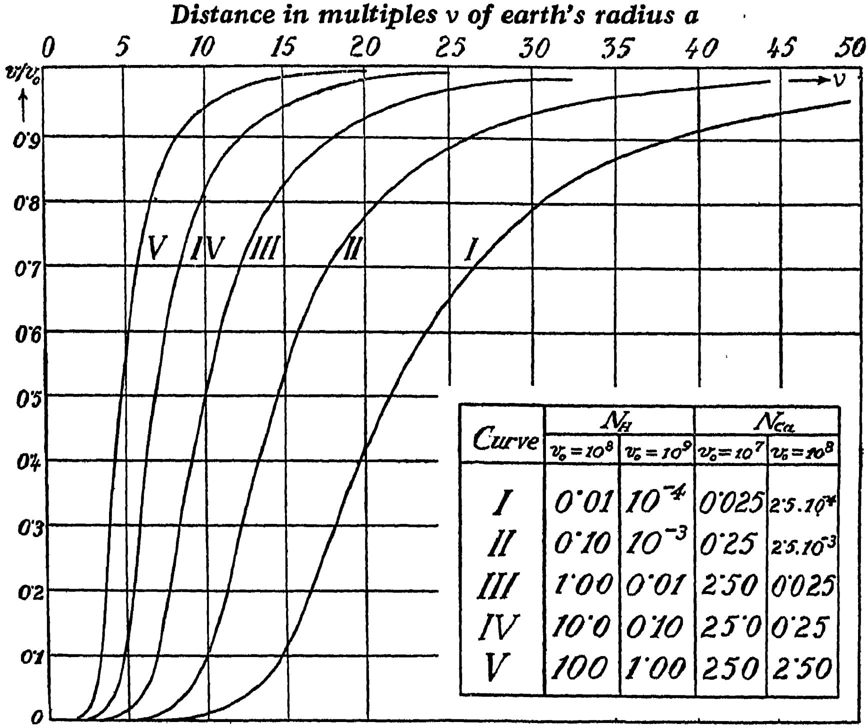
The above calculations for the normal approach of a dipole towards a *rigid plane* current-sheet will not be at all closely illustrative of the current-induction in the retarded ionized gas bounded by the completely *curved* stream-surface which encloses the earth on all sides save that facing away from the sun. But the curvature of the surface is not likely to affect the estimate of the magnetic field of the current-system by a large factor. Other departures of the actual case from the ideal system considered are likely to be more important. One of these is the imperfect conductivity of the ionized gas, as compared with the perfect conductivity assumed in the numerical illustrations based on the problem of the plane current-sheet. This imperfect conductivity has the result that the induced currents will be distributed throughout a layer of some thickness; even if the magnetic field of each thin stratum of this conducting layer can be supposed equivalent to that of an image dipole, the field of the whole current-layer will be represented only by a *distribution* of weak dipoles extending sunwards from a distance $2d$ measured sunwards from the earth's centre O, where d now denotes the shortest distance of O from the stream-surface. The net effect is to blur and weaken the effect of the single image dipole previously considered, by increasing its distance from O, beyond the value $2d$. Consequently the estimate of the magnitude of the 'first phase' increase of H, at or near O, for a given value of d or νa, will be less than that calculated above, for any given value of ν. On the other hand, after allowing for the greater distance of the image dipole (though this conception is only very roughly applicable when the stream-surface is not plane), the fact that the earth's field is compressed within a volume decreased (as compared with the case of the plane current-sheet) by the folding of the surface round the earth, will imply a rather greater increase of magnetic energy, for a given change in H at O, than was calculated above.

Chapman and Ferraro attempted to determine analytically the rate at which the vertex of the stream-surface would approach the earth. Their analysis, which was only approximate, will not be reproduced here. They concluded that the mode of approach would depend on the kinetic-energy density of the (undisturbed) stream, namely on $\frac{1}{2}\rho_0 v_0^2$, where ρ_0 and v_0 are the undisturbed density and velocity. Fig. 4 shows approximately their estimate of how the ratio (v/v_0) of reduction of velocity of the vertex will vary with ν (where νa is the distance of the vertex from the earth's centre) for different values of the parameter γ defined by

$$\gamma = 10\pi\rho_0 v_0^2/H_0^2.$$

In Fig. 4 the value of this parameter is indicated, for the five curves drawn, in terms of the equivalent number of ions per c.c. (both in the case of hydrogen and calcium ions), for two different assumed speeds v_0; note that for H ions these speeds are 10^8 and 10^9 cm./sec., and that for Ca atoms they are 10^7 and 10^8 cm./sec., so that the values in the first H (hydrogen) column are to be compared with those in the second Ca column; if $v = 10^8$ cm./sec., the number of atoms per c.c. even for curve V (which represents the case—among those considered—of most rapid and least retarded approach to the earth) is only 100 (H) or 2·5 (Ca); the velocity in this case is not reduced by half until the vertex of the hollow is within 5 earth-radii from O; nearer the earth the velocity is further reduced, first rapidly and then slowly (when the mass accumulated in the stream-front has become large).

In this calculation of the motion of the vertex Chapman and Ferraro assumed that there was complete magnetic shielding in the rear of the stream, by the currents in the retarded layer, which is continually increased in mass and density by the gas pouring in from behind. It was essential to test whether such shielding is possible in streams of the kind considered. The theory does not make great demands on the (undisturbed) stream-density near the earth, or even on that near the sun, though there the density must be about 40,000 times as great (24.3). The low density, however, implies a long mean free path for the ions and electrons, and the magnetic field in the current-bearing layer will reduce the ordinary transverse conductivity ($\kappa_\perp$) of the gas (15.11); it is therefore necessary to examine whether in such streams the



Curve	N_H $v_0=10^8$	N_H $v_0=10^9$	N_{Ca} $v_0=10^7$	N_{Ca} $v_0=10^8$
I	0·01	10^{-4}	0·025	$2·5.10^{-4}$
II	0·10	10^{-3}	0·25	$2·5.10^{-3}$
III	1·00	0·01	2·50	0·025
IV	10·0	0·10	25·0	0·25
V	100	1·00	250	2·50

FIG. 4. Illustrating the retardation of the central point of the surface of the stream as it advances towards the earth; va is the distance from the earth's centre, and v/v_0 is the ratio of reduction of the speed v from its initial value v_0. The different curves refer to streams of different kinetic-energy density

conductivity is sufficient to permit shielding of the region behind the current-layer, in a layer of reasonable thickness.

In the absence of a magnetic field, the conductivity κ of a rare gas composed of equal numbers of electrons and positive ions is due almost wholly to the electrons, and is approximately $1{\cdot}6\times10^{-9}$ e.m.u. for a temperature of 6,000°, almost independent of the number N of ions or electrons per c.c., over a wide range of N. A conductor having this value of κ could shield its interior from the field of a dipole approaching with a normal velocity of 10^8 cm./sec., by currents in a layer less than a metre thick. In the presence, however, of a magnetic field which reduces the value of $\kappa_\perp$ to that corresponding to the (full) ionic conductivity (10^{-12} for H ions), the electronic contribution to $\kappa_\perp$ being supposed negligible, the corresponding layer-thickness would be about 1·8 km. for a hydrogen-ion stream, and 73 km. for a calcium-ion stream. All these values are very small compared with the dimensions of the stream, or even of the earth; they are small also compared with the mean free paths of the ions and electrons, unless N exceeds 10^6.

However, the magnetic field within a few earth-radii from the earth could reduce $\kappa_\perp$ to values much smaller than 10^{-9} or even 10^{-12}, unless N is greatly increased by the heaping up of matter in the front of the stream, so as to reduce the mean free path. An important question discussed by Chapman and Ferraro (in their § 9) is whether the temperature near the vertex of the hollow in the stream would be greatly raised (as in adiabatic compression) by heaping up of the gas there; if there is such a rise of temperature T, the rise of pressure at the head of the stream will be greater than if T remained at 6,000°; the heaping up of the gas near the vertex of the hollow would therefore be much retarded by the pressure gradient; but unless the density can rise rapidly, as the vertex approaches the earth, and as H increases, $\kappa_\perp$ will be progressively reduced, and with it the current-flow near the stream-surface. In their original paper Chapman and Ferraro were unable to reach a satisfactory solution of these questions, which await further consideration.

But though, at the advancing vertex of the hollow, the electrons will tend to 'hook' themselves on to the lines of magnetic force, the ions will be free to move on, so long as their spiral radius is still large compared with their mean free path. They will thus tend to separate themselves from the electrons; this will set up electrostatic forces, tending to hold them back; the same force, on the electrons, will tend to carry them onward, and prevent them from spiralling; and it will produce a motion of the electrons transverse to H and along the stream-surface, which will cause the electrons to contribute to the shielding current; indeed this current is likely to be due almost entirely to the electrons, on account of their small mass. Later (unpublished) work by Chapman and Ferraro seems to remove the difficulties they originally found in regard to electron-spiralling and the reduction of $\kappa_{\perp}$. It may also be that the front of the stream-surface is continually being renewed by matter from the rear, passing through the retarded shielding current-layer and itself supplying a new contribution to this current-layer. If this be so, the conditions considered by Chapman and Ferraro in their § 9 may not be realized.

As yet the theoretical difficulties of the problem even of the first phase of the advance of the stream-surface towards the earth are, as Chapman and Ferraro remark, exposed rather than overcome. The theory is therefore as yet more qualitative than quantitative.

If the density of the stream is such that the supposed hollow round the earth is really formed, and the current-sheet really does shield the rear of the stream from the earth's field, then despite the distortion of the surface from the plane form to which Fig. 2 relates, there will be two foci of the current-circulations, to north and south of the plane of the magnetic equator, where, as at the points Q in Fig. 3, the lines of force of the field within the hollow are normal to the current-sheet. At these points the mechanical reaction of the field upon the current-sheet vanishes, so that the retarding force on the stream-surface becomes small near these points. Thus the surface will continue to advance in these two regions, while being retarded elsewhere; it would seem that two 'horns' would thus protrude outwards from the surface, along which the stream-gas can advance with little or no retardation. As the retarded hollow surface itself advances, the position of these horns may change; presumably they will become closer together. The matter passing along these horns seems likely to find its way towards the polar regions.

25.6. The main phase of a magnetic storm. The discussion in § 5 suggests how the advance of a neutral ionized solar stream towards the earth may produce the first phase of a magnetic storm, by the formation of a hollow space in the stream, enclosing the earth except on the night side; a current-system near the surface of the hollow might, it is suggested, confine the earth's magnetic field within this space, and so produce the initial rise of the horizontal magnetic force that is characteristic of the first storm-phase. The hollow space will close up round the earth, but the shrinkage will be at a diminishing rate, and may occupy hours if not days. The current-sheet will decay, but Chapman and Ferraro (in their § 10.1) suggest that this decay may be too slow to explain the decline of the initial increase in H, and that this decline is due to the onset of the cause of the large diminution of H that constitutes the main phase of the storm.

The deductive theory of the evolution of the stream phenomena near the earth is far from being elaborated to the stage where it indicates quantitatively how the main phase is produced. All that Chapman and Ferraro attempted in this direction was a tentative explanation of how this *might* come about. On this view it is a phenomenon of the stream-surface in the region beyond the earth, looking from the sun.

Fig. 5 shows an equatorial section of the stream-surface of the hollow, seen from above the north pole. The earth's field will produce a positively charged surface layer on the wall of the hollow opposite the a.m. hemisphere of the earth (lowermost in Fig. 5); opposite the p.m. hemisphere of the earth the surface charges will be mainly electrons. There will be an electric field across the hollow, approximately uniform at the back of the earth, the lines of force being nearly parallel to the equatorial plane. The charges will tend to be repelled from the surface on which they lie, and to cross to the opposite surface; they will be unable, however, to leap the gap unless the spiral radius—corresponding to their velocity and the value of H in their neighbourhood—is comparable with the breadth of the gap. In the earth's field, the spiral radius of an electron, a hydrogen ion, and a calcium ion, having the speed 10^8 cm./sec., is equal to 3 earth-radii at radial distances from the earth equal to about 460, 38, and 11 earth-radii respectively. As it seems likely that the breadth of the cavity near the earth is only a few earth-radii, the electrons in this region will be unable to get across and may drift, under the crossed electric and magnetic fields, towards the vertex. The positive charges, however, as shown in Fig. 5, will describe much larger spiral paths, enabling them to bridge the gap. The positive space charge of the ions crossing the gap may be neutralized by free electrons drawn from the stream 'above and below' the equatorial plane, along the lines of magnetic force; this flow of electrons will be in opposite directions above and below, so that its magnetic effects seem likely to be unimportant. It is supposed, however, that the stream of ions across the hollow will carry a current, which will *reduce* the magnetic field on the side towards the earth; this will enable the surface ions, nearer to the earth, to describe larger spirals, thus contributing to the growth of the 'bridge' stream (so called because it bridges the gap or hollow behind the earth); this bridge stream should steadily increase, perhaps until the main stream has passed onwards, owing to the sun's rotation, beyond the earth. It is supposed that this current across the gap completes a circuit round the earth, perhaps largely by the motion of electrons, especially if the surface layers of the stream still shield the interior from the earth's field. When the stream has passed beyond the earth, the current-ring thus created may be left behind and reduce the earth's field over a period of days, months, or even years (cf. 21.8), until in some way the ring decays.

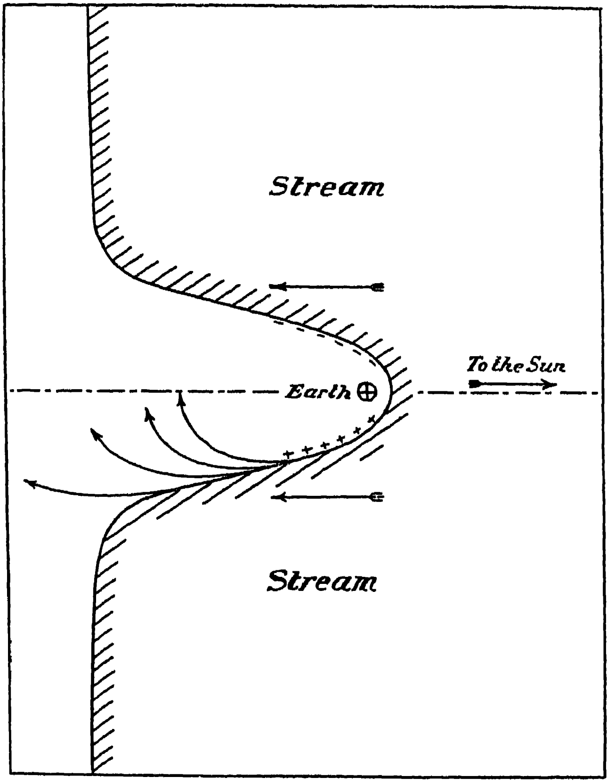


Fig. 5. Illustrating diagrammatically, in an equatorial section, the sign of the electric charges on the surface of the hollow carved in the advancing neutral ionized stream by the geomagnetic field. The curved arrows indicate the curved paths described, under the deflecting influence of the field, by charges which escape from the surface

Chapman and Ferraro discussed the equilibrium and stability of a current-ring, electrically neutral but ionized, and in which the ions travel with a velocity different from that of the electrons. There seems no difficulty in the continued existence of such a ring, over many days, if once formed; the missing link in the theory of the main phase of the storm is the quantitative uncertainty as to the processes which could establish the ring. The processes of decay of the ring, and its current, by recombination or by escape of surface charges towards the polar regions, are surprisingly slow. This part of their discussion is the simplest and most certain, but of course its interest depends on whether the ring is really brought into being by the processes they tentatively suggested or in any other way. Their theory of the main phase of a magnetic storm, based on the neutral ionized stream, is at present only a qualitative sketch.

25.7. Aurorae and other features of magnetic storms. The preceding discussion of the phenomena resulting from the approach of a neutral ionized stream towards the earth deals only with the initial and the main phases of the storm-time variations during a magnetic storm. Much further development of the theory is needed to test whether it can provide explanations of the S_D field, and especially of the intense current-system (mainly of the S_D type) in the polar regions.

It seems likely that much of the matter of the stream that is directed or nearly directed towards the earth will find its way eventually to the earth, and especially towards the polar regions; but the details of this process have not been considered. It would seem that a modified form of Störmer's theory may be valid for particles repelled from the charged surface of the hollow in the stream, the theory applying only to their passage from this surface to the earth, instead of to the whole journey from the sun.

The stream will doubtless have variations of density and speed along it, due to the circumstances of its emission from the sun; these inequalities may suffice to explain irregularities in the two main processes discussed previously, and thus contribute to the irregular features of the storm.

It would seem natural that streams should differ from one another in density, speed, breadth, and length, providing ample reasons for the variety of intensity and detailed development shown by magnetic storms and auroral displays.

Before concluding this account of the present stage of the attempt to develop a corpuscular theory of magnetic storms based on a neutral ionized solar stream, brief mention may be made of an earlier corpuscular theory of magnetic storms proposed by Chapman (in 1918, in a paper devoted mainly to a discussion of the data of magnetic storms), which was later abandoned, on account of criticisms raised by Lindemann. The theory assumed that a solar beam of particles of like charge enters the earth's atmosphere (more on the p.m. than on the a.m. hemisphere), causing a downward motion of the conducting region; by dynamo action, this motion induces an eastward current round the earth, explaining the first phase of the storm. Due to electrostatic self-repulsion of the charged layer thus formed in the atmosphere, the downward motion is followed by an upward expansion, which induces a westward current, explaining the main phase of the storm. Lindemann showed that the stream could not approach the earth after a few seconds, on account of the charge rapidly acquired by the earth; he also renewed the objection made by Schuster against earlier charged stream theories, that the stream would be dispersed on its journey from the sun by its self-repulsion.

25.8. The ultra-violet light theory of aurorae and magnetic storms. In 1928 Hulburt [2] broke away from previous theories of the aurora by suggesting that it is caused by terrestrial instead of solar corpuscles. He supposed that the terrestrial corpuscles are simply ions and electrons produced by solar ultra-violet light ionizing the earth's outer atmosphere; that at high levels above 200 km. or so their frequency of collision is small, and that there will be a rapid migration of the ions and electrons, along the lines of the earth's magnetic field, to the polar regions. There they will concentrate, move to lower levels, and recombine, setting free their energy of recombination in some way which causes the aurora. 'Because of winds in the high atmosphere of the earth and of variation in the solar radiation due to sunspots, etc., the ion layer is probably not uniform but is a thing of shreds and patches. Upon drifting to the poles the patches lengthen into streaks and being touched into luminosity form the auroral streamers.'

With reference to this theory, Chapman [3] pointed out that free ions and electrons in the high atmosphere in middle and low latitudes cannot travel far towards the poles along the earth's lines of magnetic force, because these descend with increasing latitude, and guide the particles to lower levels where their motion is not free. Even a particle starting at a height of 1,000 km. above the equator will descend in latitude 22°; for a particle to travel polewards even as far as latitude 60°, its height at the equator must be 18,000 km. These are simple consequences of the equation for a line of force of a magnetic dipole ($r/\sin^2\theta =$ const., 1 (17)). At such great heights as are reached above the equator by the lines of force descending in the auroral zone, there is no atmosphere, and therefore no supply of particles to travel polewards. Other objections were also raised by Chapman, especially the objection, applicable to all light theories (24.1), based on the 27-day recurrence tendency. Another objection was that the earth's magnetic field does not increase the energy of the particles, so that their power of penetrating the atmosphere will correspond merely to their velocity at their starting-point, increased by their loss of gravitational potential energy; whereas atomic particles that can penetrate to auroral levels must have an amount of kinetic energy corresponding to some thousands of volts. The loss of gravitational energy, from a height of 18,000 km., would supply a velocity of only 9·5 km./sec.

The first of the above objections was boldly faced in a second theory of aurorae published in 1929 by Maris and Hulburt [4–6]. They supposed that terrestrial ions and electrons *do* start polewards from the levels of 30 to 50 thousand km. which are attained over the equator by those lines of geomagnetic force that descend in auroral latitudes. These levels are far above any that could be attained by atmospheric particles in consequence of their thermal velocities, even supposing, as Maris and Hulburt did, that T might be as high as 1,000° K. in the outer hemisphere. The velocity required to reach 18,000 km. is about 9·5 km./sec., and to reach infinity is 11·2 km./sec. Velocities between 10·0 and 10·5 km./sec. suffice to reach heights between 40,000 and 100,000 km. above the earth.

An ingenious explanation was given of the presence of ions and electrons at such high levels. It was supposed that *neutral* atoms of the earth's atmosphere gain velocities of about 10 km./sec. as a result of collisions with other atoms or molecules, that have been either excited by absorption of solar ultra-violet light, or are recombining with other particles. The particles thus acquiring such high speeds must be neutral if they are to attain high levels, because ions and electrons with such speeds would be deflected downwards again by the earth's field, before rising more than a few kilometres above their starting-point.

When the neutral atoms have risen to their maximum height, they are moving slowly; it is supposed that then they are ionized by ultraviolet light, and that the resulting ions and electrons travel along the lines of force to the poles, as in Hulburt's first theory. The time taken for the neutral atom to rise to 40,000 km. is about 3 hours, and another 6 to 14 hours is required before the ions and electrons return to the atmosphere in high latitudes. They are supposed to be carried round with the lines of force so as to have the same angular velocity as the earth. Those ionized near noon will reach high latitudes at 9 p.m. or later, there to produce aurorae.

This interesting theory avoids Chapman's first objection to the original theory, but the second and third objections remain; the particles cannot penetrate beyond the outskirts of the atmosphere, below the high levels from which, as neutral atoms, they started near the equator. Thus the production of aurorae, which seems to require particles able to penetrate to heights of 100 km. or less, is unexplained.

This auroral theory of Maris and Hulburt was also elaborated so as to explain 'practically all observed details' of magnetic storms. These were supposed due to a sudden blast of ultra-violet light, lasting perhaps half an hour. 'For example, if 1/10000 part of the solar surface, which normally is at a temperature of 6,000°, were removed and there were exposed the black body radiations from regions at a temperature of 30,000°, the total solar radiation would be increased by 6·3%'; 'the solar constant would be increased by 1%, and the ultra-violet energy λ 500 to 1,000 A by 10^5'. 'On the whole the observations indicate a higher temperature of the solar volcanoes than 30,000°.' The heating effect of the solar flare expands the earth's atmosphere and produces large numbers of ions at heights between 1,000 and 2,000 km. These are supposed to form a neutral ionized belt round the earth, in which, owing to the action of terrestrial gravity, there is a drift-current (15.10, 23.15–17); this current is eastwards, and to it is attributed the first phase of the storm. It covers a belt extending roughly 2,500 km. north and south of the equator.

This eastward current-ring or 'atmospheric current pulse' is supposed to decay either because the solar flare dies away, or because the ions 'crawl down a magnetic line to a 400 km. level'. While the ring-current is being set up, an opposing current is induced in the earth, which checks the growth of the disturbing field at the earth's surface. 'When the atmospheric pulse dies away, which may require several hours, the earth-current finally reverses the storm field to negative values, which decrease to zero as the earth-current in turn dies away. Thus the theory gives entire agreement with observation.'

The earth-currents induced by the setting up of the first phase of the storm-time field were thus supposed to account for the main phase of the storm. Chapman [7] (1930) showed that this explanation is untenable, because the main phase is due principally to external, not internal, currents. Also Chapman and Price [22.10] later proved that earth-currents would increase, not decrease, the horizontal-force change of the first phase, and could never reverse it (this result also exposed an error in an earlier paper by Chapman and Whitehead [15]—see also [23.9]).

The solar flare was supposed to have the further effect that the number of ion pairs reaching the polar regions would, after an initial decrease, be largely increased. Their great concentration in high latitudes would produce there a diamagnetic ion cloud equivalent to a sheet magnet extending along latitude 67° (the auroral zone), with its south magnetic pole fairly well concentrated at about 200 km. above the earth's surface, and with its north magnetic pole at a much greater height. The field of this magnet, 'readily worked out with a pasteboard globe, plenty of pins and a strong geometrical imagination', was supposed to be in qualitative and quantitative agreement with the facts about the S_D field, described in Chapter IX.

Chapman showed, however, that such a magnet would produce effects differing in the following respects from those which are observed:

(*a*) The daily variation of the horizontal force would change sign on crossing the zone; this agrees with Maris and Hulburt's Fig. 4 (p. 426 of their paper) but not with observation.

(*b*) The daily variation of the vertical force would *not* change sign on crossing the zone; but actually it does change sign, in a very striking way.

Hence he concluded that the ultra-violet light theory failed to fulfil its claims both as regards aurorae and magnetic storms.

In consequence of these cricitisms Hulburt [8] revised his theory, so as to remove the discrepancies 'with no change in the original assumptions'. He recognized that earth-currents are of secondary importance, and no longer attributed the main phase to such currents. The main phase was explained as due to the heating of the high atmosphere by the solar flare; the air expands, and this vertical motion by dynamo action (as in Chapman's first storm-theory—p. 870, and [9.2*a*]) induces a westward current flowing round the earth.

This revised hypothesis of the main phase is, however, open to serious criticism [10]. Since this heating and dynamo action must be confined almost wholly to the day hemisphere of the earth, the westward current and the associated decrease in the horizontal force at the earth's surface would be far greater on the noon than on the midnight meridian; actually the decrease is approximately equal on these meridians. The observed magnetic effects of a solar flare confirm the view that its effects are experienced mainly on the sunlit hemisphere.

The diamagnetic explanation of the (S_D) daily magnetic changes near the auroral zone was also withdrawn, and in its place was substituted a drift-current theory (cf. 15.10 and 23.15–17). The drift currents are supposed intense because of the intense cloud of ions reaching the polar regions from low latitudes; a reconsideration of their passage to the auroral zone led to the conclusion that they would be most numerous at about 5 or 6 p.m. The drift-current along the auroral zone would be eastwards, and a diagram was drawn showing the return currents as flowing partly within the polar cap but mainly between the auroral zone and the equator. But though the magnetic field of this current-system was regarded as being 'in entire agreement' with the observed daily magnetic storm variations, this view seems untenable, because the strongest currents along the auroral zone are *westwards*, not eastwards, and attain their maximum *after*, not before, midnight (Fig. 6, p. 881, or Figs. 9.21–6).

Hulburt also referred to the descent of the electrons and ions reaching the polar regions from low latitudes. 'They fall freely along the lines of the earth's magnetic field into the atmosphere of the auroral zones and continue to descend through the atmosphere by temperature diffusion and electro-magnetic drift.' He concluded that the electrons form a bank with a maximum density at 160 km., below which they become attached rapidly to O_2 molecules to form O_2^- ions. Below 160 km. a bank of positive and negative ions is formed; 10^{11} ion-pairs per sq. cm. per sec. are supposed to move downwards across the 130-km. level with a velocity of 52 cm./sec. This increase of ionization during a magnetic storm is supposed to occur by the descent of ions from the 200-km. to the 110-km. level in 4 or 5 hours (loc. cit., p. 1567). Although it is said that 'altogether the agreement between the theory and the details of the aurora seems very striking', it is not clear how the sudden appearance and motions of luminosity in rayed draperies, down to 100 km. or below, can be explained by such slow entry of particles into the atmosphere. It is true that we cannot say with certainty that these well-known auroral phenomena are due to the sudden penetration of corpuscles to these levels from outside the atmosphere, but this seems likely and is generally assumed. The ultra-violet light theory seems to give no inkling as to how the rapid auroral changes can occur. It has also been criticized by McNish [12] on other grounds.

In a later review of theories of aurorae and magnetic storms (1937) Hulburt [9] contrasts the corpuscular and ultra-violet light theories, and concludes that the latter theory 'appears adequate to explain the complicated storm variations of the terrestrial magnetic field, the auroral zone, the spreading of the aurora into low latitudes during strong storms, the delay of appearance of aurorae after incipience of the storm, and other observed facts'. The range of phenomena which the theory is regarded as explaining is thus wider than is described above, but further details are not given here because the criticisms described above seem to be fatal to the chief merits claimed for the theory. Mention should be made, however, of cometary data regarded by Maris and Hulburt [5] as favouring the view that the solar influence which causes aurorae and magnetic storms spreads outwards in a *wide* angle instead of along a narrow stream as the corpuscular theories suppose; they admit that this wide-angle solar flare 'leaves the recurrence tendency of magnetic storms without an explanation', except 'by the *ad hoc* assumption that the flare puffs out with the recurrence period, due to solar pulsations of some sort'.

Hulburt also remarks that 'all theories of the aurora are geometrical theories in that they merely offer mechanisms whereby energy is supplied to auroral regions. They make no suggestion concerning the manner in which the energy is transformed into auroral light.' Since this was published, a theory of the detailed mechanism of production of the auroral and night-sky light has appeared [14.72]; for the auroral light it requires a source of energy of at least 19·6 volts, sufficient to ionize and excite molecular nitrogen to emit the 'negative' bands that are prominent in the auroral spectrum. This hypothesis of the auroral-light production is consistent with the view that corpuscles enter the earth from outside, with speeds of at least many hundreds of km./sec.; but it is not apparent how the necessary energy quanta could be provided by the slowly descending clouds of ion-pairs considered in the ultra-violet light theory of the aurora.

In concluding this account of the two theories of magnetic storms which may to-day be considered as rivals in the field, it is perhaps appropriate to remind the reader that one author of this book is both the part-author of one theory and a chief critic of the other theory. In these circumstances, some bias in describing the theories and the criticisms of them can hardly be avoided; the interested reader is advised to read the original papers cited before concluding that either theory is right or wrong. He will find that the papers by Hulburt and Maris contain many interesting and stimulating ideas which repay study. Their papers offer scope for criticism partly because of the large number of definite claims made in them, supported by physical arguments and numerical calculations.

A considerable experience of the pitfalls which beset theorists on magnetic storms caused Chapman and Ferraro to try to write in a way that should constantly remind themselves and their readers of the tentative, speculative, and only very partially successful character of their work; oversight, and the need for brevity, may be pleaded in regard to the many cases in which over-confident or over-definite statements appear in such papers. Indications of the same sense of the liability to err may likewise be found in the papers by Hulburt and Maris on this difficult subject.

As regards the corpuscular theory, there is certainly much need for closer scrutiny and criticism, as well as for serious effort towards its further development.

25.9. The location of the inter-zonal D currents. The electric current-system illustrated in Chapter IX, Figs. 21–6, that could account for the main average features of the D field on an idealized earth magnetized along its axis of rotation, was there divided into a storm-time part (Figs. 9.23, 24), symmetrical about the axis, and an S_D part (Figs. 9.21, 22), corresponding to the disturbance-daily variation.

The fact that it is possible to construct such current-systems, supposed to flow in a layer concentric with the earth, and capable of 'explaining' the observed average characteristics of the D field, does not prove that the field is produced by such currents. It is, in principle, impossible to infer uniquely, purely from observations of a magnetic field (of external origin) at the earth's surface, the location of the current-system that is responsible for the field. So long as no further considerations are taken into account, the problem has not only one but an infinity of solutions. This may be illustrated as follows.

Imagine any current-system outside the earth, whether linear or distributed over a surface or throughout a volume of any form. Observation of its field at the earth's surface enables the potential of the field near the surface to be determined and expressed as a series of spherical harmonic terms. Along any radius from the earth's centre O the variation of the term of degree n ($n = 1, 2, \ldots$) is proportional to r^n. This expression for the potential is valid up to the boundary of any sphere centred at O, which does not intersect the external current-system; from it one can readily derive another expression, giving the current-function which describes a current-system flowing in any spherical sheet of radius R ($R > a$, a being the earth's radius), which would give the same *surface* field. But the same surface field could also be produced by an infinite variety of non-spherical current-distributions differing from the actual current-system, but having a potential expressed, relative to O and in the region near the earth's surface, by the same spherical harmonic series.

In the case of the equivalent spherical current-sheet, its radius R may have any value greater than a; the greater the radius, the more prominent will be the spherical harmonic terms of high degree n in the current-distribution, since their magnitudes will increase with R proportionately to $(R/a)^n$.

The current-distribution over a spherical sheet can easily be represented by a diagram using any projection of the sphere upon a plane. This is one method of representing the potential of the field graphically; it is of value in this way even if the field is produced by a non-spherical current-distribution. When used for this purpose, the current-distribution must be drawn for a definite radius, which may conveniently be a itself, or some radius R differing so slightly from a that the relative proportions of the main harmonic terms are not seriously different from those corresponding to a; in that case $(R/a)^n$ will be nearly unity for the values of n which are important. Within the range of R satisfying this condition, the current-system may be considered independent of R; of course this range is fairly small, but if the field is so simple that only the first three or four harmonics in it are important, the range may be considered to include values of R differing from a by not more than 300 or 400 km., which is the thickness of the atmosphere up to and including the main known ionized layers: this is because the ground value of the ratio of the nth to the first harmonic is increased at 400 km. by the factor $(6770/6370)^{n-1}$, and up to $n = 4$ this does not exceed 1·2.

Considerations of continuity indicate that if a field is produced by a current-sheet that departs from the spherical concentric form, but only by a moderate amount, the character of the current-distribution in the sheet can be inferred approximately from that of the spherical current-distribution of the same mean radius, which would give the same surface-field.

In attempting to decide between the infinite number of possible current-systems that could produce a given field, additional 'non-magnetic' considerations of various kinds may be employed.

One of these concerns the space-distribution of electric conductivity; or, what is equivalent in the case of a rare medium such as alone comes into question for the space surrounding the earth, the distribution of ionized gas. Information as to this may indicate some particular region as a likely location for the current-system of a particular field, or may show that it is an improbable or impossible situation for that current-system.

For example, the spherical current-distributions corresponding to the fields (S and L) of the solar (S) and lunar (L) daily magnetic variations are much more intense over the sunlit than over the dark hemisphere, and even over the latter their intensity decreases perceptibly from sunset to dawn (see Figs. 7.15, 16 for the S current-system). Now there are ionized regions in the atmosphere which share these characteristics, and it is natural to suppose, in the absence of evidence to the contrary, that the S and L current-systems flow in one or other of these layers. The conclusion is reasonable (though provisional) despite the dependence of the current-system upon the distribution of electromotive forces as well as upon the conductivity. The conclusion will gain strength if it is found that the S and L fields vary in intensity throughout the year, and from year to year, and also irregularly, in unison with one or other of the ionized layers; in this way it may become possible to associate S with a particular ionized layer, and likewise L with another. There is, in any case, little or no reason to doubt that the S and L currents flow in our atmosphere in layers which are very nearly spherical and concentric with the earth.

One of the outstanding features that distinguishes the D from the S and L current-systems is that the D system is not more intense by day than by night; in the ('interzonal') belt between the two auroral zones the plane that divides the more intense from the less intense parts of the system passes through the sun, the currents being stronger over the p.m. (post meridiem) than over the a.m. hemisphere. In the auroral zones the order of intensity is reversed, but the dividing plane still passes nearly through the sun. We know of no ionized layer of the atmosphere in which the ionization increases from a minimum at dawn to a maximum at sunset, though the general theory of the daily variation of atmospheric ionization suggests that the higher the layer, the later in the daytime should be its hour of maximum ion-content. It may be that above the highest known layer, the F_2 layer, there is another ionized layer with these properties, but of which radio observations can tell us nothing, because its maximum ion-content is always less than that of the F_2 layer: if this be so, any radio waves that could get through the F_2 layer to the supposed upper layer would also penetrate this layer and pass away from the earth, instead of returning with indications of the existence of the upper layer. It may therefore not be possible to exclude the atmosphere as a possible situation for the inter-zonal part of the D current-system.

On the other hand, it is necessary to consider whether these currents may not flow in the practically empty space above the atmosphere. Reasons of two kinds have prompted the suggestion that a current-ring encircling the earth, far outside the atmosphere, is responsible for the decrease in the horizontal magnetic force during and after magnetic storms. The first reason is the high degree of regularity in the geographical distribution of the average disturbing force (both in intensity and in direction at each station), and also the slowness of the decline of this force, over a period of many days. Indeed Schmidt has suggested that the external disturbing field decays so slowly that it would take years to disappear, and that in actual fact it is always present, being reinforced by each new magnetic disturbance, and thereafter declining, at a diminishing rate, though only by a fractional amount before a new storm augments it again. It seems to have been thought that this part of the D field is too regular and perhaps too enduring to be produced by a current-system in our atmosphere. The decline of ionization in the atmosphere is slower, the higher the layer, but it seems likely that if the symmetrical inter-zonal part of the D field is caused by currents in the atmosphere, these must be very high up. Perhaps the S_D part of the field, in the inter-zonal region, is more likely to be of atmospheric origin, because this seems to decay more rapidly than the symmetrical storm-time part.

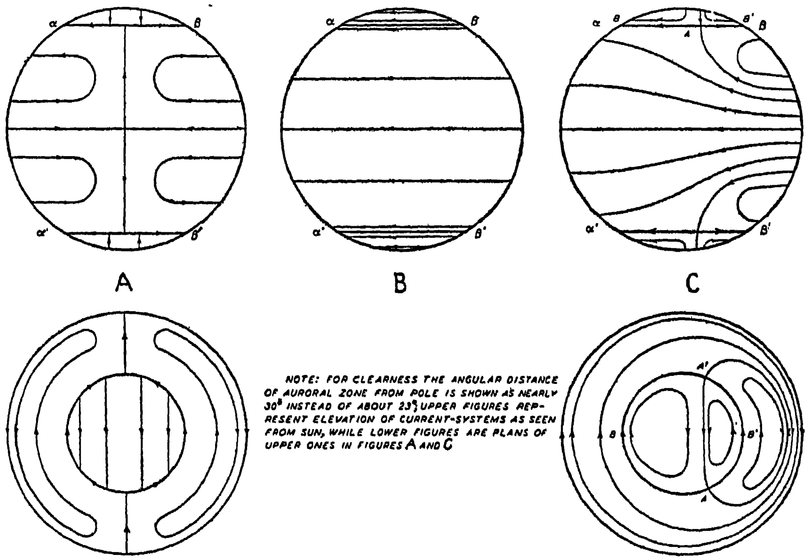
Störmer (24.13) favoured the hypothesis of a ring-current round the earth for a different reason, namely because a difficulty in his auroral theory might be overcome by postulating such a current. He assigned a very large radius to the ring, which he placed beyond the moon's orbit. He thought of the ring as not necessarily closed: it might consist of electrons sweeping along a considerable arc around the earth under the influence of the earth's deflecting field. Later he ascribed an important role to this belt of electrons, in connexion with the production of radio echoes of long delay.

But the hypothesis of a ring- or stream-current at this distance from the earth seems to create more difficulties than it solves. The chief argument against it is that no such current could hold together against the mutual repulsion of its parts, if it is composed of charges all of like sign, as Störmer proposed. If, however, charges of both sign are present, the suggested mode of formation breaks down, because the heavier ions would not follow the path calculated for the more easily deflected electrons.

Chapman and Ferraro have suggested (§ 6) that a ring-current may be formed when a neutral ionized stream sweeps across the earth; while unable to prove that such a ring-current would be formed, they showed that, if it were formed, its radius would probably be much less than Störmer supposed, being only a small multiple of the earth's radius a (e.g. perhaps $5a$); the current in the ring will depend on the relative motion of the ions and electrons in it, and would disappear if their velocities were equal. Collisions between them will tend to equalize their speeds, but a moderate difference of speed could survive, in a stream of the extent and density necessary to provide the required current, for many days. It also appears from later (unpublished) calculations that though the rotation of the earth's magnetic axis would cause a daily 'wobble' of the ring, the orbital motions of the two sets of charges in the ring would adjust themselves to the changing direction of the magnetic axis without the ring being disrupted.

A relatively small ring such as Chapman and Ferraro considered would also require far less energy (§ 14) than a ring of the size suggested by Störmer, and it would be free from the objection against the latter, that it far more than nullifies the earth's field in the region where it is set up, thus destroying and reversing the forces that are supposed to call it into being. Störmer's ring-current would indeed modify the earth's field very greatly throughout an immense region of space beyond it, and would be likely during a magnetic storm to alter the normal paths of cosmic rays, and their distribution over the earth, to an extent far larger than the observations indicate.

Unless it can be shown, more definitely than has yet been possible, that a ring-current could be set up during the impact of a neutral ionized stream upon the earth, or until further supporting evidence for the existence of a ring is obtained (for example, on the basis of the cosmic-ray data during magnetic storms—cf. 14.14), the ring hypothesis must remain in suspense. The alternative view, that the D currents flow in the atmosphere, must be held open as a possibility, unless it can be rendered probable that there is no layer in the upper atmosphere with the necessary properties. In favour of the 'atmospheric' hypothesis of the D field in the inter-zonal region is the consideration that the symmetrical (storm-time) and S_D parts of the D field result only from a convenient analysis of what appears to be a unitary phenomenon; it would be most satisfactory to explain both parts by a single mechanism. On the external-ring hypothesis this would require the ring-current to be very eccentric or elliptical, being much nearer to the earth on the p.m. side (at least during the intense period of the disturbance) than on the a.m. side; it is uncertain whether the ring could be created with, or could preserve, such an ellipticity. On the other hand, there seems no special difficulty in an atmospheric current-flow of the type indicated by Figs. 9.21, 22, if the appropriate conductivity and electromotive forces are present.



NOTE: FOR CLEARNESS THE ANGULAR DISTANCE OF AURORAL ZONE FROM POLE IS SHOWN AS NEARLY 30° INSTEAD OF ABOUT 23°; UPPER FIGURES REPRESENT ELEVATION OF CURRENT-SYSTEMS AS SEEN FROM SUN, WHILE LOWER FIGURES ARE PLANS OF UPPER ONES IN FIGURES A AND C

FIG. 6. Illustrating diagrammatically the idealized overhead electric current-systems that could produce the average field of geomagnetic disturbance. *A* shows, in elevation (*above*, as viewed from the sun) and in plan (*below*, as viewed from above the north pole), the part of the current-system corresponding to S_D, the solar daily disturbance variation field; *B* shows (in elevation) the part of the current-system corresponding to the storm-time variation, symmetrical about the earth's axis; *C* shows the combined current-system

25.10. The polar part of the D current-system. In the polar regions the S_D part of the D field and of the D current-system is more intense than the symmetrical storm-time part; a most prominent feature is the concentration of currents along the auroral zones, as shown in Figs. 9.21–6, which are here reproduced schematically as Fig. 6. The case for locating the polar part of the D current-system in the atmosphere is much stronger than in the case of the inter-zonal part. In the first place, we have evidence from radio measures that during magnetic disturbances the ionization along the auroral zone is enhanced and intense at levels above 90 km. Moreover, the aurorae themselves give visual indication of increased ionization and excitation in the zone, because the 'negative' bands in the auroral spectrum are due to nitrogen ions. This evidence of abnormal ionization, especially along the zone, creates a presumption that the notable magnetic changes near the zone are due to currents situated in this ionized region of the atmosphere.

Another 'non-magnetic' argument in favour of the existence of strong electric currents in the atmosphere, along the auroral zone, is the *a priori* probability on grounds of simplicity. The D field near the zone is highly differentiated locally, and is explicable as due to a current far more concentrated than the currents responsible for the inter-zonal part of the D field; the height and total intensity of the zonal current can be calculated, from the magnetic variations at the ground, as was done by Birkeland and, more recently, by Goldie and by Vestine and Chapman. Such calculations do not *prove* that the local cause of the D field is a current so placed, and of such strength: alternative current-systems remain possible, but the farther their location is supposed to be from that calculated for the concentrated current, the more complicated must be their character. If placed far above the earth, they may involve an improbable distribution of current-bands of opposite sign.

This can be illustrated with reference to the symmetrical part of the auroral-zone current, which flows westwards all round the zone. Such a concentration of current in the system of Fig. 6*B* requires the presence, in the spherical harmonic expression of the field potential near the earth's surface, of a cluster of terms, many of them being of high degree. The surface potential can alternatively be attributed to a current-sheet of any radius R; if R is taken as $a+h$, where a is the earth's radius and h the height of the supposed actual concentrated current along the auroral zone, the current-function must correspond to the supposed concentrated current; the spherical harmonic terms must then be additive at the latitude of this current, and must nullify one another elsewhere. If, however, R is taken to be much greater than $a+h$, the spherical harmonic terms of the higher degrees will gain greatly in relative importance, compared with the lower-order terms in the current-function; hence the latter may then be expected to represent a highly banded distribution of current round the parallels of latitude. This must certainly be regarded as a less probable type of current-system than the simple concentrated current along the auroral zone. McNish [9.23*a*] has confirmed and developed this argument.

The intensity of the current along the auroral zone, according to Fig. 6*C*, is not uniform. It increases from zero at two points A, A′, which in Fig. 6*C* are at about 2 p.m. and 10 p.m., to maxima at two points B, B′, which are there placed at 6 a.m. and 6 p.m. (on the actual earth these points may be displaced by a few hours). Thus, all along the zone, except at the maximum points B, B′, current must be entering or leaving this zone. It is important to know whence comes this current and whither it goes.

Birkeland believed that current came into the zone from outer space, approximately along the earth's lines of magnetic force, and that it left the earth again, along the lines of force, after flowing along the zone for some distance. The inflow, whencesoever it comes, must be continuous over the portion BAB′, while there must be outflow over the remaining part of the zone, B′A′B. Birkeland's view would require that the closed circuit of any elementary tube of this current should be completed by a union, somewhere outside the earth, of the farther ends of the inflowing and outgoing current, possibly greatly diffused and far away: only the nearer parts of the circuit would contribute much to the D field.

Fig. 7 illustrates Birkeland's ideas as to how these zonal currents could be provided by freely moving charges approaching and afterwards receding from the earth. They may approach along a spiral path of diminishing radius of curvature, as in Fig. 7 *A*, later spiralling away again; or they may come and go along paths such as Fig. 7 *C* or *E*. The result of bundles or sheets of rays describing such paths is shown in Fig. 7 *B* or *D*, in which the vertical components of current-flow may be considered as nearly cancelling one another, leaving only a resultant current-flow along a horizontal line, taken by Birkeland to represent the auroral zone. The diagrams *A*–*E* are taken from Birkeland's main memoir, p. 99.

This system of current intake and outflow for the auroral zones may be contrasted with that corresponding to the spherical current-sheet that could also account for the observed D field. Figs. 6 *A* and 6 *C* represent the auroral zone as forming an intense and concentrated part of current-circuits, some of which are inter-zonal, while in others the flow is across the polar caps within the zones. Of course there are other possible current-systems intermediate between the spherical one and that envisaged by Birkeland; for example, part of the zonal currents might be supplied as he suggested, while the remaining part flows in the earth's atmosphere, as in Fig. 6. The details of the current-distributions that correspond to the various theoretically possible situations for the current are not clear without examination; for example, it is not clear that a current-system of the kind suggested by Birkeland, including concentrated currents in the atmosphere along the zone, could explain the observed D field, particularly in the centre of the polar caps, without involving some improbable requirements as to the currents at a distance from the earth. We may imagine the current-sheets of Fig. 6 *C* gradually distorted so that while their concentrated zonal currents remain *in situ*, the two polar sheets, or the inter-zonal sheets, or all three of these current-sheets, are moved outwards, each polar-cap sheet being stretched upwards over the diameter BB′ so as to form two sheets, with downward inflow over the part BB′A, and upward outflow over B′A′B. It seems likely that the current-distribution in these sheets would have to be somewhat singular in order to yield the smooth distribution of the D field that appears to exist near the centre of the polar caps. On the ground of *a priori* probability of simplicity, therefore, the spherical current-sheet distribution over the polar cap seems preferable to the Birkeland theory. Vestine and Chapman [9.20] have enforced this conclusion by actual

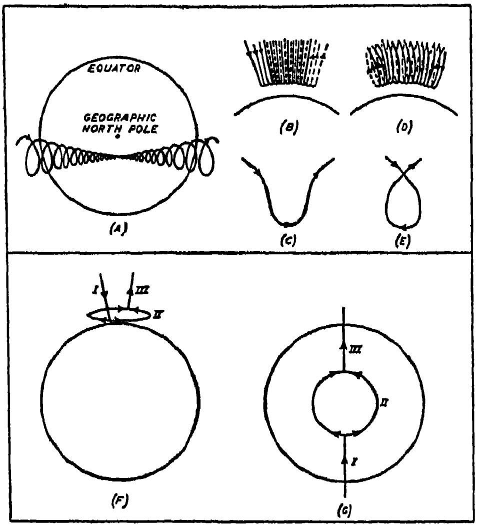


FIG. 7. Illustrating diagrammatically the current-system proposed by K. Birkeland to account for the geomagnetic disturbance field in high latitudes. *A* shows the path of a charged particle spiralling downwards from outer space towards the auroral zone, and subsequently upwards away from the auroral zone into outer space. *B* and *D* show bundles of such paths, of different types, *C* and *E*, simplified by omitting the superposed spiral motions; the bundles of paths are approximately equivalent to a downward current at one end, which turns horizontally along the base of the paths, and then turns upwards again and flows away from the earth. *F* and *G* show, in elevation and plan respectively (as in Fig. 6 *A*), such downward and upward currents, which are connected by horizontal currents along each half of the auroral zone

calculation of the magnetic field of a linear current-system, chosen as giving a rough representation of Birkeland's hypothesis, namely, a linear current descending vertically from an infinite height down to the auroral zone, on the noon meridian, then dividing equally to right and left to flow along the auroral zone (as in Fig. 7, *F*, *G*), and flowing vertically upwards again, to infinity, from the point on the midnight meridian where the two parts of the auroral-zone current reunite. They find that such a system would not give so strong or so uniform a field, near the centre of the auroral zone, as is observed.

Another important argument against the Birkeland hypothesis of the supply and outflow of the zonal currents is the difficulty in accounting for the existence and maintenance of strong free currents in the empty space around the earth. Hence it seems most reasonable to believe in the existence, at some level in the earth's atmosphere, probably between 100 and 300 km. height, of a *polar* current-system of the type represented in Figs. 6 or 9.21–6; it is only doubtful whether the *inter-zonal* part of these current-systems is in or beyond the atmosphere.

As has been indicated in §8, Hulburt and Maris suggested that the D field near the auroral zone was due to diamagnetism of the strongly ionized atmosphere there; later, when it was shown that the field could not thus be accounted for, Hulburt suggested that an easterly electromagnetic drift-current along the auroral zone was the cause of the D field there. This explanation is likewise inappropriate, because the strongest part of the auroral concentrated current is westerly, not easterly (Figs. 9.25, 26). In view of the fact that in different parts of the zone the current has opposite geographical directions, it is clear that no general theory, like those of diamagnetism or drift currents, which predict magnetic effects of a definite sign, can account for the observed field. Chapman and Ferraro offered no explanation of the polar current-system in their theory of magnetic storms; the only bearing of their theory on this question is that it seems able to explain the guidance of a considerable amount of the neutral ionized gas towards the polar regions, and so to account for the abnormal ionization there; but the details of the motion, which must bear closely on a modified theory of the aurora (not improbably incorporating many features of Störmer's theory), have not yet been considered.

Another hypothesis as to the cause of the polar current-system is due to Chapman, who in 1927 [9.2*c*] suggested that the currents are caused by the dynamo action of a horizontal drift of air across the polar regions at high levels. The drift was supposed to be parallel to the plane of the twilight circle, perpendicular to the line joining the centres O_e, O_s of the earth and sun. If the air-flow is from the a.m. to the p.m. hemisphere, as Chapman suggested, the induced electromotive force will be in the direction $O_s O_e$; if the flow is from the p.m. to the a.m. hemisphere, the e.m.f. will be in the opposite direction $O_e O_s$. The currents along the auroral zone, at the points in the twilight plane where the current-strength is greatest, are in the direction $O_s O_e$, in both the a.m. and p.m. hemispheres; the sheet currents across the polar caps are in the opposite direction $O_e O_s$.

If the polar current-system is caused by the proposed dynamo action, it would seem that the current-flow must take place counter to the direction of the induced e.m.f. in one or other of the two parts of the system, that is, either in the zone, or over the polar cap enclosed by the zone. A similar opposition of the current-flow and the e.m.f. occurs in the S_q current-system (23.11, 13) near the equator. It is not easy to see, without mathematical examination, what will be the direction of current-flow over the polar cap and along the auroral zone, due to the distribution of e.m.f. set up by the suggested drift of air across the polar regions. The earth's vertical magnetic force is greatest in the centre of the auroral zone, so that the e.m.f. induced there, by a given speed of air-flow, will be greater than in the zone itself. It is perhaps likely that the induced e.m.f. over the polar cap will be the main factor, and will impel currents in its own direction across the cap, and that these currents will complete their circuit by flow along the much more highly conducting auroral zone, in the direction opposite to that of the e.m.f. induced in the zone itself. If this be so, the air-drift must be from the p.m. to the a.m. hemisphere.

There is need for theoretical treatment of this problem, in which it may be necessary to consider the return path of the air-flow itself; this return flow may take place at an upper or lower level, in the direction opposite to that of the flow which induced the e.m.f., but in a less conducting region (otherwise an opposing system of electric currents would be set up); but in view of the vertical stability of the atmosphere, and the resistance to interchange of air at different levels, it seems more likely that the circuital air-flow is at more or less the same level throughout. In that case the air-flow across the polar cap, say from the p.m. to the a.m. hemisphere, must be completed by flow in lower latitudes, parallel to the equator, easterly over the day hemisphere, and westerly over the night hemisphere. At present this suggested circulation of air across the poles is purely hypothetical.

25.11. The irregular features of magnetic disturbance, D_i. Since progress in explaining the regular features of magnetic disturbance has proved so difficult, it is not surprising that little attempt has been made to account for the irregular features. So far as concerns the polar part of the D field, where the irregular fluctuations are specially intense, it is natural to suppose that many of the rapid changes in the field are due to sudden local alterations in the distribution of the average current-system of Figs. 9.25, 26, itself perhaps due to a fairly regular distribution of e.m.f. induced by a steady air-flow across the polar regions. Such sudden local changes in the current-distribution would be *likely* to occur if the distribution of ionization undergoes rapid irregular variations, which seems quite probable in view of the quick changes in the position and intensity of the aurora. These shifts in the luminosity presumably betoken variations in the incidence of the incoming solar particles, and of the ionization which their entry produces in the atmosphere.

The general polar current-distribution would seem likely to be most unstable where its direction and intensity change most rapidly from point to point; this is specially the case near the points B, B′ (Fig. 6 *C*) on the auroral zone, where the current-intensity is at its maximum, for there, on passing laterally away from the zone, either north or south, the current-intensity is reversed. It is therefore in those regions that temporary shifts in the areas of intense auroral precipitation and ionization would seem most likely to cause sudden large local changes in the current-distribution, without altering the large-scale pattern of the current-system.

These points of maximum current-intensity along the zone are shown in the idealized Fig. 6 as situated on the 6 a.m. and 6 p.m. meridians, but in reality the maximum eastward current seems to occur later than 6 p.m., especially during active disturbance, when the maximum is attained not much before midnight. (This and other departures from the symmetrical idealized current-system proposed by Chapman have been studied by Vestine [9.39], who showed that the actual zonal currents are centred approximately at the magnetic axis pole, and that if time be reckoned relative to the magnetic axis, the daily variation S_D shows no significant change of time-phase with longitude.) It is at this time of maximum, in regions to the south of the zone, that the irregular fluctuations of the field are most frequent; this may be attributed to a tendency for the ionization to be patchy to the south of the auroral zone between 6 p.m. and midnight, due to irregular shifts in the precipitation of the solar corpuscles. Over the polar cap within the auroral zone, on the other hand, the irregular fluctuations, according to Stagg, are most frequent between 6 a.m. and noon; this may perhaps indicate that the solar particles which enter the atmosphere near the auroral zone between these meridians are liable to deviate patchily to the north rather than to the south of their mean position. Except in midwinter the sunlight precludes visible indication of the entry of solar particles between these meridians.

As regards the irregular magnetic changes *between* the auroral zones, the explanation must depend greatly upon whether the inter-zonal part of the average D current-system is situated in or outside the atmosphere. If the currents are atmospheric, irregular changes in their distribution might be attributed to variations in the distribution of ionization, as in the case of the polar part of the current-system. If, however, the currents are external, the irregular magnetic fluctuations must be supposed due to rapid changes in the intensity and position of these currents, perhaps arising from variations in the density or speed of the solar stream, which presumably pours onward towards the earth throughout most of the duration of the storm.

25.12. Magnetic potential energy and field energy. Any permanently magnetized body may be regarded as possessing magnetic potential energy corresponding to the work required to aggregate the small magnetic dipoles of which it is composed from a state of infinite dispersion. This energy is of amount

$$W = -\tfrac{1}{2}\int \boldsymbol{I}\cdot\boldsymbol{H}\,dv_p, \tag{1}$$

where $\boldsymbol{I}$ denotes the intensity of magnetization, $\boldsymbol{H}$ the magnetic field intensity ($= -\text{grad}\,V$), and the suffix p indicates that the integration extends throughout the volume of the permanently magnetized body or bodies; the magnetic field $\boldsymbol{H}$ is supposed due solely to these bodies.

The expression (1) is also valid if the field is partly due to magnetized bodies whose magnetization is not permanent but induced, provided that the induced magnetization ($\boldsymbol{I}_i$, say) is proportional to the inducing field. The volume integration in (1) is not to extend over these regions of induced magnetization.

By means of Green's theorem the expression (1) for W may be transformed to

$$W = \frac{1}{8\pi}\int H^2\,dv + \frac{1}{8\pi}\int BH\,dv_i, \tag{2}$$

where the second integral extends throughout the regions of induced magnetization I_i (indicated by the suffix i), and the first integral extends throughout all the rest of space, including the volume occupied by permanent magnetization. Another alternative form of W (obtained by substituting $H+4\pi I$ for B in (2)) is

$$W = \frac{1}{8\pi}\int_{\infty} H^2\,dv + \tfrac{1}{2}\int I_i H\,dv_i, \tag{3}$$

wherein the first integral extends throughout the whole of space, and the second throughout the regions of induced magnetization. The second term may be regarded as due to a volume-density of $\frac{1}{2}I_i H$ of potential energy of induced magnetization; the first term corresponds to a distribution of *field energy* of density $H^2/8\pi$. If H is measured in c.g.s. units, the energy density $H^2/8\pi$ is expressed in ergs per c.c.

The integral
$$\frac{1}{8\pi}\int H^2\,dv \tag{4}$$
throughout the volume contained within any closed surface S, and including no magnetized matter, can be transformed by Green's theorem into the surface integral
$$-\frac{1}{8\pi}\oint V\mathbf{H}\cdot d\mathbf{S} \tag{5}$$
over the boundary, the surface element $d\mathbf{S}$ having the direction of the normal drawn outwards from the volume of integration. This transformation is valid also if the volume includes a region of *uniform* magnetization, whether permanent or induced. Similarly, for any region enclosing any kind of magnetization, permanent or induced, uniform or not,
$$\int \mathbf{B}\cdot\mathbf{H}\,dv = -\oint V\mathbf{B}\cdot d\mathbf{S}. \tag{6}$$

A sphere of radius a, uniformly and permanently magnetized with intensity I, has a moment $M = \frac{4}{3}\pi I a^3$, and inside the sphere
$$H = \tfrac{4}{3}\pi I = M/a^3.$$
The magnetic potential energy W calculated from (1) is $\frac{1}{2}M^2/a^3$; the total magnetic field energy inside the sphere is $\frac{1}{6}M^2/a^3$, and the field energy outside is $\frac{1}{3}M^2/a^3$; these integrals, which are most readily calculated by means of (5), together account for the whole potential energy $\frac{1}{2}M^2/a^3$.

The magnetic potential energy of the earth, if this is regarded as being permanently and uniformly magnetized throughout its whole volume, is $\frac{1}{2}M^2/a^3$, where a denotes its radius $6{\cdot}37\times10^8$ cm. Taking its moment M to be $8{\cdot}1\times10^{25}$, the energy is found to be $1{\cdot}3\times10^{25}$ ergs. This may be compared with the kinetic energy of rotation, which is of order 10^{36}

ergs, and therefore immensely greater than the magnetic energy calculated on the present hypotheses.

If the earth's magnetic field were due to permanent uniform magnetization within a smaller concentric sphere of radius a', the potential energy would be $\frac{1}{2}M^2/a'^3$, or $(a/a')^3$ times the value just cited. The same would be true if the magnetic field were due to a thin spherical electric current-sheet of radius a', because the field $\boldsymbol{H}$ inside and outside such a sheet is the same as for the uniformly magnetized sphere of the same radius.

If part of the earth's field were due to permanent magnetization or electric currents, and part to induced magnetization set up in magnetically permeable regions by the field of the permanent magnetization or of the currents, these calculations of the magnetic potential energy would need to be modified. It seems hardly worth while, however, to discuss this possibility, since so far as present evidence goes, the magnetic permeability μ of the earth seems to differ little from unity, except in local magnetic deposits of relatively small volume. Hence in later discussions of geomagnetic energy we shall suppose that μ may be taken as everywhere unity, so that $I_i = 0$, and except in permanent magnetic matter $\boldsymbol{B} = \boldsymbol{H}$.

25.13. The energy of superposed magnetic fields. Suppose that there is superposed on the normal or main geomagnetic field $\boldsymbol{F}$ an additional variation field $\Delta\mathsf{F}$ (not necessarily small) due to external electric currents and, maybe, also to associated induced earth-currents. At any point the normal magnetic intensity $\boldsymbol{H}$ will be changed to $\boldsymbol{H}+\Delta\boldsymbol{H}$, and therefore the normal energy density $H^2/8\pi$ of the field will be changed by the amount

$$\frac{1}{8\pi}\{(\boldsymbol{H}+\Delta\boldsymbol{H})\cdot(\boldsymbol{H}+\Delta\boldsymbol{H})-H^2\} \tag{7}$$

or

$$\frac{1}{8\pi}(\Delta\boldsymbol{H})^2+\frac{1}{4\pi}\boldsymbol{H}\cdot\Delta\boldsymbol{H}. \tag{8}$$

The first term here is the energy density that would correspond to the sole presence of the additional field $\Delta\mathsf{F}$, the normal field F being absent; hence $(\Delta\boldsymbol{H})^2/8\pi$ may be called the *self-energy density* of the added field, and its integral throughout all space will be the total *self-energy* S of this field.

The second term in (8) is due to the joint presence of the two fields F and $\Delta\mathsf{F}$, and it may therefore be termed the *joint-energy density*, the integral of which is the *joint energy* J of the two fields. The volume integral of $\boldsymbol{H}\cdot\Delta\boldsymbol{H}/4\pi$ throughout any region can be transformed by Green's theorem to one or other of the surface integrals

$$(1/4\pi)\oint \Delta V\boldsymbol{H}\cdot d\boldsymbol{S} \quad \text{or} \quad (1/4\pi)\oint V\Delta\boldsymbol{H}\cdot d\boldsymbol{S} \tag{9}$$

over the boundary, where ΔV denotes the potential of the field $\Delta\mathsf{F}$. This expression implies that the field $\Delta\mathsf{F}$ has a potential ΔV; this is not the case for a region in which there is a volume-distribution of current-flow. If the current-flow can be supposed confined to thin current-sheets, these may be supposed excluded from the volume of integration by surfaces closely bordering each sheet on both sides; these bordering surfaces must be treated as parts of the boundary of the volume of integration.

If the field $\Delta\mathsf{F}$ is due to currents which may be regarded as flowing in thin spherical sheets concentric with the earth, or along thin tubes lying on such concentric spheres, the potential ΔV, like that of the normal field, can be conveniently expressed in spherical harmonic form. In a region free from currents, between two such concentric current-sheets (the current not necessarily covering the whole of each sphere), the potential ΔV of the field $\Delta\mathsf{F}$ will include terms of both internal and external origin, such as

$$(e_n^m r^n + i_n^m r^{-n-1})S_n^m.$$

The potential V of the earth's field will likewise consist of terms such as

$$(E_n^m r^n + I_n^m r^{-n-1})S_n^m;$$

the coefficients E_n^m are small if the region considered is external to the earth. In calculating the self- or joint energy between two spheres, of radii a_1, a_2, say, the normal to the boundaries is everywhere radial, so that $\boldsymbol{H}\cdot d\boldsymbol{S} = \pm(\partial V/\partial r)\,dS$; and the joint-energy integral becomes

$$(1/4\pi)\left\{-\int_{(a_2)} \Delta V\frac{\partial V}{\partial r}\,dS + \int_{(a_1)} \Delta V\frac{\partial V}{\partial r}\,dS\right\},$$

where the first integral is taken over the outer sphere ($a_2 > a_1$) and the second over the inner one. Similarly, the self-energy integral would in this case be

$$(1/8\pi)\left\{-\int_{(a_2)} \Delta V\frac{\partial \Delta V}{\partial r}\,dS + \int_{(a_1)} \Delta V\frac{\partial \Delta V}{\partial r}\,dS\right\}.$$

In these integrals the radial differentiation of V (or ΔV) does not affect the surface harmonic function S_n^m. Hence, by the orthogonal property of surface harmonic functions, no contribution is made to the values of the surface integrals by the parts of V and ΔV (or of ΔV and ΔV, in the self-energy integral) arising from the product of dissimilar harmonics. Consequently, the self-energy is the sum of the self-energies of the separate harmonic components of ΔV. Similarly, in the joint-energy integral the only terms that require evaluation are those resulting from like surface harmonics existing in the potentials V and ΔV.

25.14. The field-energy changes during magnetic storms [13, 14]. In estimating the field-energy changes during magnetic storms an essential fact is that the permanent field F is everywhere far more intense than the variation field ΔF, so that in general the joint-energy density $\boldsymbol{H}\cdot\Delta\boldsymbol{H}/4\pi$ greatly exceeds the self-energy density $(\Delta\boldsymbol{H})^2/8\pi$. The latter, however, is essentially positive (or zero), whereas the former may be either positive, zero, or negative.

During the main phase of a magnetic storm, the potential of the field ΔF near the earth's surface includes a principal term proportional to the zonal harmonic P_1, corresponding to the storm-time decrease of horizontal magnetic force, and mainly of external origin; there are also terms in S_2^1, S_4^1, representing the S_D field between the auroral zones, and a large number of harmonics of much higher degree, representing the polar disturbance field and the irregular features of the storm field. In calculating the joint energy of the fields F and ΔF, each of these harmonics in ΔF must be associated with the corresponding harmonic in F. The main harmonic in F is P_1, corresponding to uniform magnetization parallel to the earth's geographical axis; in addition there are other less important harmonics, especially P_1^1 (corresponding to the transverse component of magnetization). Clearly the only *like* harmonics which are prominent in both F and ΔF are the terms P_1 occurring in each. Any other correspondence which may chance to exist will be between minor and relatively insignificant components, whereas P_1 is the chief single harmonic in both F and ΔF. It therefore appears that this is the only term that needs to be considered in evaluating the joint energy.

This will be calculated first of all on the assumption that the field F is due to a uniform magnetization throughout the whole earth or, what is equivalent, to an electric current-sheet at the earth's surface; the field ΔF will likewise for the present be attributed to a concentric-spherical current-sheet of radius b $(> a)$, the part of ΔF due to induced earth-currents being ignored. The P_1 or $\cos\theta$ term in V is therefore

$$\left.\begin{array}{ll} -H_0 a^3\cos\theta/r^2 & (r > a), \\ H_0 r\cos\theta & (r < a), \end{array}\right\} \tag{10}$$

where H_0 denotes the surface equatorial value (M/a^3) of the horizontal intensity, of magnitude 0·3 approximately. The P_1 term in ΔV is likewise

$$\left.\begin{array}{ll} \delta H_0 r\cos\theta & (r<b), \\ -\delta H_0 b^3\cos\theta/r^2 & (r>b), \end{array}\right\} \tag{11}$$

where δH_0 denotes the (positive) numerical value of the equatorial horizontal intensity of the $\Delta\mathsf{F}$ field at the radius b *or at any smaller radius*, since the field $\Delta\mathsf{F}$ is uniform inside the sphere b; the $\Delta\mathsf{F}$ field diminishes H at the equator.

In illustrative calculations we may take δH_0 to be 100γ or 0·001 c.g.s. for an active magnetic storm.

The total joint energy of the field outside the sphere b is found, by integrating $V(\partial\Delta V/\partial r)$ over the outer surface of this sphere (cf. (9)), to be

$$\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\left(H_0\frac{a^3}{b^2}\cos\theta\right)\left(2\delta H_0\frac{b^3}{b^3}\cos\theta\right)b^2 d\phi\, d\cos\theta = \tfrac{2}{3}a^3H_0\,\delta H_0. \tag{12}$$

This is positive and, somewhat curiously, is independent of b.

The total joint energy within the sphere a is similarly found to be

$$\tfrac{1}{3}a^3H_0\,\delta H_0. \tag{13}$$

The total joint energy in the space *between* the spheres a and b may be calculated by means of surface integrals over these spheres; these surface integrals are found to be equal in magnitude and opposite in sign, so that there is no net change of field energy in this region. It is curious and interesting that this is so in the region where alone we can directly measure the field. The joint-energy density is positive in high latitudes, and negative in lower latitudes, as illustrated in Fig. 8.

The total joint energy J, therefore, during the main phase of a magnetic storm, is positive and of amount

$$J = a^3H_0\,\delta H_0, \tag{14}$$

according to the above simplifying hypotheses. This may also be written

$$(\delta H_0/H_0)M^2/a^3,$$

which is a fraction $2\delta H_0/H_0$ of the magnetic potential energy of the main field. If $\delta H_0 = 0{\cdot}001$, and $H_0 = 0{\cdot}3$, the change of energy is ⅔ per cent., or about 8×10^{22} ergs.

We next consider how this estimate is affected by modifying the hypotheses on which it is based. Suppose first that the magnetic field F is due to uniform magnetization throughout a sphere of radius a'

instead of a (or to an electric current-sheet of this radius), where $a' < a$. Then if H_0' is the equatorial surface value of the field F at the radius a',

$$H_0' a'^3 = H_0 a^3. \qquad (15)$$

In the present case the calculated joint energy is $a'^3 H_0' \delta H_0$, which by

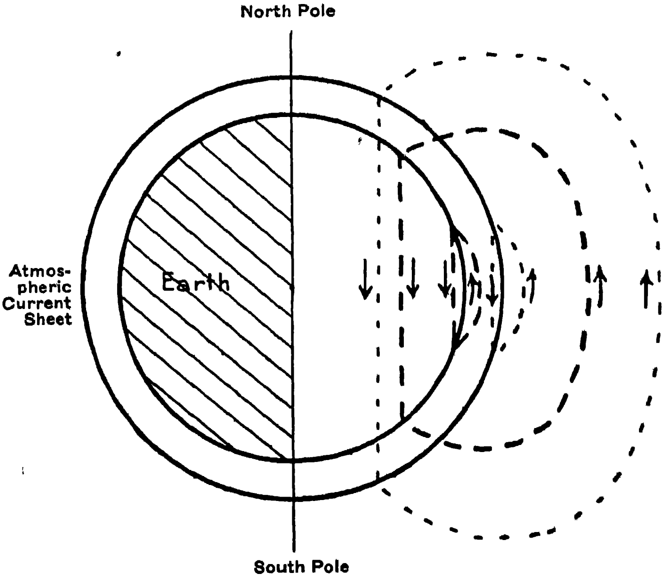


FIG. 8. Illustrating diagrammatically the lines of magnetic force of the earth's main field (the heavy broken lines) and of the storm-time component of the field of geomagnetic disturbance (the thin broken lines). The latter is supposed due to an overhead electric current-system flowing in a layer concentric with the earth; the concentrated currents along the auroral zone (Fig. 6*B*) are here supposed smoothed out. The lines of force within the earth are hypothetical and uncertain. Outside the atmospheric current-sheet and within the earth, the two fields reinforce one another; between the earth and the current-sheet, they reinforce one another near the poles, but near the equator they are opposed

virtue of (15) is identical with (14). Hence (14) is unlikely to depend on the precise region within which the main geomagnetic field F originates.

We next consider how the joint energy is affected by the fact that part of the field ΔF is due to electric currents induced within the earth by the variation of the primary, external part of ΔF. Let $\delta_o H_0$, $\delta_i H_0$ denote the parts of the observed diminution (δH_0) of the equatorial horizontal intensity at the earth's surface, which are respectively of outer and inner origin. The potential ΔV will include terms corresponding to (11), in which, however, δH_0 must be replaced by $\delta_o H_0$; in addition there will be terms

$$\left.\begin{aligned} &-\delta_i H_0 a^3 \cos\theta / r^2 && (r > a''),\\ &\delta_i H_0 (a^3/a''^3) r \cos\theta && (r < a''),\end{aligned}\right\} \qquad (16)$$

due to the induced currents, if these are supposed, for simplicity, to be concentrated in a thin sheet of radius a'' ($< a$).

In the expression (9) for the joint energy ΔV occurs linearly, so that the parts of this energy associated with $\delta_o H_0$ and $\delta_i H_0$ can be calculated separately. The former, found as before, is $a^3 H_0 \delta_o H_0$; the value of the remaining part depends on whether a', the radius of the source of the field F, or a'', the radius of the source of the field $\Delta_i \mathsf{F}$, is the greater. If $a' > a''$, the joint energy depending on $\delta_i H_0$ is $-H_0 \delta_i H_0(a^6/a'^3)$, and if $a'' > a'$, it is $-H_0 \delta_i H_0(a^6/a''^3)$, the value in either case being negative. The magnitude of a'' is approximately known (Ch. XXII), and may be taken to be roughly $0{\cdot}9a$; it seems perhaps most likely that $a'' > a'$, and assuming this, the joint energy depending on $\delta_i H_0$ will be approximately $-1{\cdot}3a^3 H_0 \delta_i H_0$. Hence the total joint energy is

$$a^3 H_0(\delta_o H_0 - 1{\cdot}3\delta_i H_0).$$

The ratio of $\delta_i H_0$ to $\delta_o H_0$ depends on the rate of time-variation of the field $\Delta\mathsf{F}$, but it will not be far wrong to take $\delta_i H_0 = \frac{1}{3}\delta_o H_0$, or

$$\delta_i H_0 = \tfrac{1}{4}\delta H_0, \qquad \delta_o H_0 = \tfrac{3}{4}\delta H_0.$$

Hence the joint energy is estimated as $0{\cdot}4a^3 H_0 \delta H_0$, which is rather less than half the value calculated without allowing for the induced part of the field $\Delta\mathsf{F}$; taking δH_0 as 100γ, as before, the joint energy J is approximately 3×10^{22} ergs.

This estimate is further reduced if the P_1 part of the $\Delta\mathsf{F}$ field is due to a ring-current instead of to a spherical sheet-current. This will be illustrated by supposing that the ring-current is a current of radius R (several times as large as a) in the equatorial plane of the earth, and that its cross-section is circular, of radius c small compared with R.

The potential of a strictly linear circular current i of radius R, expressed in terms of spherical polar coordinates r, θ relative to the earth's centre and axis, is

$$2\pi i \sum_{n=0}^{\infty} (-1)^{n+1} \frac{1.3...(2n-1)}{2.4...2n} \left(\frac{r}{R}\right)^{2n+1} P_{2n+1}(\cos\theta) \tag{17}$$

for $r < R$, and

$$2\pi i \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1.3...(2n-1)}{2.4...2n} \left(\frac{R}{r}\right)^{2n} P_{2n-1}(\cos\theta) \tag{18}$$

for $r > R$. The only term in (17) or (18) which is of importance in association with the term P_1 in V is P_1, or

$$\left.\begin{aligned} &-2\pi i(r/R)P_1, && \text{where } r < R, \\ \text{and} \quad &\pi i(R/r)^2 P_1, && \text{where } r > R. \end{aligned}\right\} \tag{19}$$

The value of i may be expressed in terms of the magnetic field of the ring at its centre (the centre of the earth), namely $2\pi i/R$, so that

$$i = R\delta_o H_0/2\pi, \tag{20}$$

if we ignore the small difference between this central magnetic intensity and $\delta_o H_0$, the intensity at the earth's surface; this is legitimate if R is several times as large as a, as Chapman and Ferraro suggested in their theory of magnetic storms.

Consequently the significant term in $\Delta_o V$ (the part of ΔV of external origin) is

$$\left.\begin{aligned} &-rP_1\,\delta_o H_0, &&\text{where } r < R,\\ \text{and}\quad &\tfrac{1}{2}R^3P_1\,\delta_o H_0, &&\text{where } r > R. \end{aligned}\right\} \tag{21}$$

This agrees with (11), where $r < R$ (since $P_1 = \cos\theta$), but is only half as great where $r > R$. Hence the part of J due to $\Delta_o V$ in space outside a sphere of radius R is now to be halved, being therefore of amount $\frac{1}{3}a^3H_0\,\delta_o H_0$; the whole contribution of $\Delta_o V$ to J is therefore $\frac{2}{3}a^3H_0\,\delta_o H_0$, two-thirds the former value. Hence, since the contribution of $\Delta_i V$ is unaffected,

$$J = a^3H_0(\tfrac{2}{3}\delta_o H_0 - 1{\cdot}3\delta_i H_0). \tag{22}$$

Taking $\delta_o H_0$ and $\delta_i H_0$ as $\frac{3}{4}\delta H_0$ and $\frac{1}{4}\delta H_0$, as before, we find that

$$J = 0{\cdot}2a^3H_0\,\delta H_0 \tag{23}$$

or, if $\delta H_0 = 100\gamma$, $J = 1{\cdot}5\times 10^{22}$ ergs.

The self-energy S of the ΔF field will next be considered. If the field be supposed purely of the P_1 type, and due to a spherical current-sheet of radius b, and if the induced part of the field be neglected, it is readily found that

$$S = \tfrac{1}{2}b^3(\delta H_0)^2. \tag{24}$$

If, however, the outer field is due not to a spherical current-sheet but to a circular ring-current of radius R and cross-sectional radius c, then its self-energy is $\frac{1}{2}Li^2$, where, if c/R is small, L, the coefficient of self-induction of the ring, is approximately

$$4\pi r\{\log_e(8R/c) - 7/4\},$$

and $i = R\,\delta H_0/2\pi$. Hence

$$S = fR^3(\delta H_0)^2, \tag{25}$$

where

$$f = (1/2\pi)\{\log_e(8R/c) - 7/4\}. \tag{26}$$

If the induced part of ΔF is taken into account, δH_0 must be replaced by $\delta_o H_0$ in the above alternative estimates of S, which must also be modified by an additional term, which is either

$$\tfrac{1}{2}(a^6/a''^3)(\delta_i H_0)^2 - a^3(\delta_o H_0)(\delta_i H_0) \tag{27}$$

corresponding to (24), or

$$\tfrac{1}{2}(a^6/a''^3)(\delta_i H_0)^2 - \tfrac{2}{3}a^3(\delta_o H_0)(\delta_i H_0) \tag{28}$$

corresponding to (25). In addition, S will include a contribution from the polar and irregular parts of the disturbance field, but this is insignificant compared with the contributions already considered, because these fields are so much less extended in space than the P_1 component.

The value of S depends very much on the estimate of b or R. If $\Delta\mathrm{F}$ is due to an atmospheric current-sheet, b will be only a few hundred kilometres greater than a, and then S is very much smaller than J; $\frac{1}{2}b^3(\delta_o H_0)^2$ and (27) are together equal to about $\frac{1}{4}a^3(\delta H_0)^2$, or 7×10^{19} ergs if $\delta H_0 = 100\gamma$. This is only about 1/400 of the corresponding estimate of J.

If, however, the main part of the $\Delta\mathrm{F}$ field is due to a ring-current of radius R several times that of the earth, the influence of the induced currents on S, expressed by (28), is unimportant, and approximately $S = fR^3(\delta_o H_0)^2$ or about $\frac{1}{2}fR^3(\delta H_0)^2$. In this case

$$S/J \doteqdot \frac{5}{2}\frac{fR^3}{a^3}\frac{\delta H_0}{H_0}.$$

Taking $\delta H_0 = 100\gamma$, $R = 5a$, and $f = 1$ (f does not vary rapidly as R/c changes, being 0·3 and 1·1 when R/c is 3 and 1,000 respectively) we have $S/J \doteqdot 1$. If R had the much larger value suggested by Störmer, S would be much greater than J. It seems likely, however, in view of the previous discussion, that at most S is of the same order of magnitude as J, and that the whole excess energy during a magnetic storm of the magnitude $\delta H_0 = 100\gamma$ is of order 10^{22} ergs, or about 0·1 per cent. of the magnetic potential energy of the earth as estimated in § 12.

In § 5 an estimate was made, in a quite different way, of the excess magnetic energy during the first phase of a magnetic storm, and this also was found to be of order 10^{22} ergs, though its space distribution differs from that during the main phase. Since the first phase is set up in only a fraction (of order 1/5) of the time taken for the main phase to reach its maximum epoch, the rate of increase in the magnetic energy of the field is probably greater during the first than during the second phase. This rate is of order 10^{19} ergs/sec., which is very small compared with the rate at which energy of solar radiation falls upon the earth, namely 2×10^{24} ergs/sec. According to the theory of Chapman and Ferraro, the magnetic energy of a magnetic storm is derived from the kinetic energy of the particles of the solar stream, though probably only a very small fraction of the kinetic energy is converted into magnetic energy.

XXVI

HISTORICAL NOTES

No comprehensive History of Geomagnetic Discovery has yet been written. In the present treatise on geomagnetism limitations of space forbid the inclusion of even a brief systematic history of the subject, but we have thought that our readers would welcome a few historical notes. Those referring to the earlier periods of discovery are based mainly on two historical articles by A. Crichton Mitchell [1], who consulted the original sources. His initials (A.C.M.) will indicate quotations from these articles, which are provided with ample bibliographical references.

26.1. The directive property of a magnet in the earth's field. [A.C.M.] 'By whom, at what date, and in what form, this was first recognized is now unknown, for the discovery was made in times long before the maintenance of careful scientific record. The investigation of this question has given rise to an extensive literature, but has led to few conclusions of a definite kind. . . . The matter practically resolves itself into an inquiry as to the origin or invention of the mariner's compass, and to the honour of this discovery several claims have been advanced.

'The claim which goes furthest back in history is that made on behalf of the *Chinese*. . . . In general terms it may be stated as follows. The ancient annals of the Chinese Empire contain passages which can only be construed as meaning that the directive property of a magnet in the Earth's field was known in China several thousand years ago; that in later ages there follows a succession of references indicating a constant tradition on the subject; that these gradually merge into others which mention the compass in its earliest form; and thus we reach the seventh or eighth century A.D., when this primitive instrument was employed for surveying and navigation. It is thus contended that the compass is an adaptation in later times of a principle known to the Chinese for the last 4,500 years.

'In examining the evidence in support of this claim, we have to distinguish between that which depends chiefly upon the mythological history of China and that which is gathered from the literature of more recent times.

'With regard to the former, it is stated that in 2634 B.C. the Chinese emperor Hoang-Ti was at war with a tributary prince named Tchi-

Yeou, and that they fought a great battle in the plain of Tcho-luo. In order to escape defeat, or its consequences, Tchi-Yeou raised a dense fog which produced disorder in the imperial army—an anticipation of the smoke-screen of modern times. But Hoang-Ti constructed a chariot upon which stood the diminutive figure of a man with arm outstretched, and this figure being apparently free to revolve on its vertical axis, the arm always pointed to the south. By this means, the emperor was able to locate the direction of his enemy's retreat; Tchi-Yeou was captured and put to death. By the earliest European commentators on the Chinese annals—the Jesuit missionaries who went to China during the latter part of the seventeenth century—it was at once inferred that the movable figure was actuated by a magnet, and this inference forms the basis of the claim now under consideration. . . .

'The Chinese claim to priority, advanced on the story of the south-pointing Chariot, has had its defenders, but is now generally rejected.' Crichton Mitchell gives reasons why this rejection appears to be fully justified. He contends that it is safe to hold that the earliest reliable evidence of Chinese knowledge of the magnet pointing north or south dates late in the eleventh century: the encyclopaedist Shon-Kua (A.D. 1030–93) says that 'fortune tellers rub the point of a needle with the stone of the magnet in order to make it properly indicate the south'.

After a thorough discussion Crichton Mitchell also discards claims ascribing the knowledge of the compass to the ancient Egyptians, to Solomon, King of Israel, and to the Phoenicians.

As to the Greeks and Latins, he writes:

'The literature of classical antiquity contains many references to the attractive quality of the lodestone, but it is now certain that it reveals no knowledge of the polarity of the magnet, or of its directive property in the Earth's field. In every case in which a claim of the kind has been made, it can be shown that the interpretation put upon the passage is a forced one.'

The 'earliest mention in European literature of the polarity of the magnet, of its directive property, and of the application of that property to the art of navigation', which he considers to be 'perfectly definite and well-authenticated', is described by him as follows:

'Alexander Neckam, monk of St. Albans, who was born in 1157 and died in 1217, wrote two treatises, *De Utensilibus* and *De Rerum Naturis*. The dates of their composition are not known exactly, but may with safety be assigned to the last quarter of the twelfth century, possibly not much later than 1187. In the first he describes the use of the magnetic needle to indicate the north, and states that mariners employ this means of finding their course when the sky is clouded so as to obscure sun or stars. In the second, he describes the needle as being placed on a pivot, and it is to be noted that this form of the early compass is generally understood to have been the second stage in its evolution. In neither treatise does Neckam represent the instrument as a novelty, but as one in common use.

'Almost contemporaneous with, but certainly later than, Neckam we have the extensively quoted poem, 'La Bible', of Guyot de Provins. Its date can be fixed as being later than 1204 and not later than 1209, most probably 1206. It describes a floating compass in use by navigators, and hence does not include the developed form described by Neckam. But it must be remembered that Guyot was a poet, and that he only introduced the compass as illustrative to his theme, whereas Neckam wrote for the instruction of others and had, therefore, to be more strictly accurate in technical detail. . . .

'The first attempt, on really scientific lines, at any systematic description of the property of the magnet was that contained in a letter written in 1269 by Petrus Peregrinus, and addressed to Sigerius de Foucancourt. For the full examination of all matters connected with this letter, we are indebted to the researches of Bertelli [2]. The author explained how the position of the poles in a spherical mass of lodestone may be determined and showed how they could be distinguished. He suggested or effected several improvements in the nautical compass, and devised the first form of azimuth-compass. That his letter is, for its time, a wonderful production there can be no manner of doubt. But there has been a tendency among some writers to overrate his influence, and the statement that he was the first to introduce the pivoted needle is erroneous. . . .

'Soon after Guyot wrote his poem, Cardinal de Vitry published, in 1218, a description of Palestine in which he speaks of the compass as a necessary aid to navigation, having seen it in use in 1204. . . .

'Among the many statements made with regard to the origin of the compass, none has attained wider currency than that which attributes its discovery in 1302 to Flavio Gioja, a native of Amalfi, a small seaport in southern Italy. The history of the legend—for such it is, in spite of its being . . . quoted as genuine in a serious history of science of recent date—is interesting, if for no other reason than that it provided the occasion for the important historical researches of Bertelli [2]. . . .

'With regard to this claim of Gioja—apart altogether from what has been stated above—it need only be said that the year to which it is assigned, 1302, is at least a century later than Neckam's description of the compass. . . .

'Voltaire said that the first use of the compass was by the English during the reign of Edward III (1327–77), but he gave no authority for his statement. That it was then in use can, however, be proved. Nicholas cited instances from official records showing that, in 1338 and 1345, certain ships of the British Navy were supplied with "sailing needles and dial", as the compass was then termed. In one case they were purchased in Holland. This is the earliest record of the actual use of the compass in a given ship. Capmany reported that while the compass is not mentioned in the inventory of a ship fitted out at Barcelona in 1331, nor in similar lists for the galleys of Don Pedro IV of Aragon in 1364, it is mentioned in the chronicle of Don Pedro Nino under 1403, and in the galley inventories of Alfonso V of Aragon in 1409.

'The later history of the compass is only concerned with its technical improvement as an instrument, no additional principle in terrestrial magnetism being involved. The matter need not, therefore, be pursued further, except to refer the reader to the work of Schück [3] for information on these later aspects of the question.' (A.C.M.)

The two volumes by Schück on the compass [3] contain a great number of plates illustrating old compass forms.

26.2. The discovery of magnetic declination: rejection of early claims. We quote from Crichton Mitchell's second paper [1]:

'The four centuries from 1200 to 1600 witnessed the completed transition from the age in which medieval scholasticism was dominant into that when the art and logic of observation and experiment had vindicated their entry on the field of scientific investigation. But we have to wait until the publication, in the latter year, of Gilbert's *De Magnete* before the foundations of the science of terrestrial magnetism were well and truly laid. Meanwhile, one notable advance had been made. For at least three hundred years before Gilbert's time, it had been noticed that the suspended magnet did not, always and everywhere, point to the exact geographical north. At first, this was explained as being due to the lodestone, by which the compass-needle was magnetized, having different properties in different parts; later on, it was attributed to imperfections in the method of magnetizing the needle, or to errors in the observation of its direction relative to the geographical meridian. But gradually it came to be recognized that this divergence was a universal phenomenon, and thus was reached the conception of the magnetic declination, the angle at a given place and time between the geographical meridian of the place and the direction of a magnet freely suspended in a horizontal plane.

'The question which immediately presents itself is that relating to the origin of this conception—by whom, when, and in what circumstances, this phenomenon of magnetic declination was first observed. But, as in the analogous question of the nautical compass, we have no extant record of any such observation—the earliest references are found to be imperfect and obscure and thus the question cannot be answered in precise terms of persons, dates, and places. In the strict sense of the term, therefore, we cannot speak of the *discovery* of the declination and it would be more correct to refer to the history of the matter as being that of the gradual evolution of the conception of declination.'

A possible *Chinese* reference to magnetic declination occurs in Shon-Kua's encyclopaedia; another translation by Hirth of the passage already cited in § 1 is quoted by Crichton Mitchell as follows:

'A geomancer rubs the point of a needle with the lodestone to make it point to the south, but it will always deviate a little to the east, and not show the south; that to use the needle, it may be put on water, but it would not be steady; and also it may be put on the nail of a finger or on the lip of a bowl, but it is too apt to drop, because its motion is very brisk; that the best method is to hang it by a thread, and to prepare the contrivance, one has to single out a fine thread from a new skein of floss silk and fix it with a piece of beeswax on the middle of the needle, the latter to be hung up where there is no wind; that the needle would then always point to the south; that, on rubbing a needle with a lodestone, it may happen by chance to point to the north, and he (the author) owned needles of both sorts, and that no one could as yet find out the principle of it.'

But Crichton Mitchell goes on to say:

'This passage, or its substance, also appeared in the supplement to the same cyclopedia, and also in later Chinese works. It is generally on these latter that commentators have based their conclusions, its first sentence being quoted, not invariably with its important context, in support of the claim now under reference. But careful consideration of the whole passage proves that the author's attention is concentrated on the transmission to the needle of the directive property and on the demonstration of that property. Further, the deviation from the meridian is not regarded as a separate physical phenomenon but as an accidental result due to imperfect support or suspension of the magnetized needle. The passage cannot, therefore, be accepted as proof of Chinese knowledge, in the eleventh century, of the magnetic declination. In support of this conclusion, the experience of Father Ricci and other Jesuit missionaries in China, about the beginning of the seventeenth century, may be quoted. They were permitted to take part in the proceedings of the Chinese Tribunal of Mathematicians, and had considerable difficulty in persuading the Tribunal that the geographical and magnetic meridians were not coincident. It was only after actual demonstration of the divergence of 2° between them, that the Tribunal accepted the fact of declination. In 1696, Le Comte reports that the Chinese were still firmly persuaded that the magnetic needle always pointed to the south. In 1870, Amiot found that the Chinese still obtained their first knowledge of the matter from Father Ricci's demonstration. Another piece of evidence is the fact that when the Chinese used the magnetic needle to determine direction, they did not take the declination into account. Thus, the east and west walls of Pekin, built in the time of the second emperor of the Ming dynasty, do not run exactly north and south, but deviate 2½° from that direction. . . .

'From what has been stated above, the conclusion necessarily follows that our knowledge of magnetic declination did not originate in China.'

Crichton Mitchell then deals at length with the claims connected with *Columbus*. We omit his discussion and his extensive quotations from the sources, and quote only his main conclusions (2nd paper, §§ 10, 25):

'Since the beginning of the nineteenth century, the most general opinion regarding the discovery of the magnetic declination has been that it was first observed by Christopher Columbus during his first voyage to the West Indies in 1492. Until 1790, the only record on which this claim could be based was the biography of Columbus, written by his son Fernando. After that date, Navarette, Humboldt, and, especially, Bertelli, founded it on the Las Casa edition of the Journal of Columbus. Later opinion inclined towards the view that the declination was known in western Europe before 1492, and that what Columbus may be credited with is the discovery that its amount diminished, became zero, and then turned westwards, as he sailed westwards across the Atlantic. But even this restricted conclusion has been disputed on the ground that the documents upon which it is based are unreliable.

'There is no intention here of embarking on the wide and angry seas of Columbian controversy. But the main results of that discussion have an important relation to the matter now under consideration and it is impossible to avoid their impact upon it. Two other reasons may be adduced in justification of the detailed examination. . . . The first is that the Columbus claim has been put forward so strongly and in such detail by Bertelli, backed by all the resources of an extensive and profound knowledge of the literature of the subject, that his argument cannot be ignored. The second is that a fresh examination of the original documents, which are reproduced below, throws new light on some aspects of the question. . . .

'We have now to summarize the conclusions reached with regard to the claim advanced on behalf of Columbus, that he was the first to observe the magnetic declination, or at least that he discovered the space-variation of the declination. These claims have been considered on their own merits, without prejudice from the previous history of the matter. With reference to the first of these two questions, the following conclusions appear to be reasonably established by the available evidence:

'(*a*) That the faulty steering of the ships on September 9, 1492, has no bearing on the question. In his transcription of the Las Casa manuscripts, Navarette made one serious mistake, after correction of which the whole matter appears in a simple light and unconnected with questions of declination.

'(*b*) That it is almost entirely certain that an easterly declination had been observed in north-western Europe before Columbus sailed on his first voyage. The proof is meanwhile inferred from the Columbian records themselves. It therefore follows that the magnetic declination was not discovered by Columbus.

'(*c*) With regard to the second question, the conclusion is: That the observations made by Columbus in his first westward voyage in 1492 are, in a rough manner, consistent with his having crossed the agonic line, but that he attributed the results to entirely erroneous causes, and thereby failed to recognize them as evidence of a general space-variation of magnetic declination. There are also slight indications that before this first voyage the fact that declination was not of equal amount at all places was known to several navigators.'

26.3. The actual discovery of magnetic declination. As reliable references to magnetic declination, Crichton Mitchell (2nd paper, § 9) accepts evidence due to Hellmann [4] and Wolkenhauer:

'This is based on the fact that about the middle of the fifteenth century, if not earlier, portable sun-dials were in use, chiefly by travellers, which had attached to them, in the base of each instrument, a small compass which facilitated the correct adjustment of the instrument with its "noon-line" in the geographical meridian. Some of these compasses show a mark indicating the amount of the declination. . . .

'After calling attention to the manufacture of these portable sun-dials, chiefly at Nuremberg and Augsburg, Hellmann begins his argument by giving sound reasons for the belief that the word "*compass*" or "*compassus*" meant nothing more than a horizontal sun-dial provided with a magnetic needle. At what date the manufacture of these instruments began in Nuremberg and Augsburg is unknown. Peuerbach, one of the early authorities on dialling, lectured at Vienna from 1454 to 1460. Among other writings, he left behind two documents bearing on the present subject, namely, a pamphlet entitled *Canones Gnomonis cum nova tabula* and a manuscript entitled *Compositio Compassi cum regula ad omnia climata.* His pupil, Johannes Müller, better known by his latinized name, Regiomontanus, settled in Nuremberg in 1471 and carried on astronomical work until shortly before his death in 1476. He made sun-dials, and either introduced their manufacture to Nuremberg or, more probably, encouraged their manufacture if that had already been begun.

'The next step in Hellmann's argument is to suggest that any maker of sun-dials, with their attached compasses, who was acquainted with the ordinary method of fixing the geographical meridian, must, in course of time, have recognized that, with the noon-line of the instrument in that position, the magnetic needle did not point exactly north but (in Germany at that time) to the east of north. To adjust the sun-dial correctly, it would then be necessary to orient it with the noon-line to westwards of the direction of the magnetic needle. The procedure would then be to mark on the compass-dial a point corresponding to the deviation of the needle and to orient the instrument until the needle stood over that point. The north point of the instrument would then indicate the geographical north. In whatever manner the process was gradually evolved, the fact remains that sun-dials have been found which show quite clearly the difference between the geographical and magnetic meridians, generally by a mark on the compass-dial to the east of the north point. A letter written by Hartmann (of Nuremberg) in 1544 shows that he had observed the declination at Rome in 1510 and found it to be 6° east, whereas it was 10° at Nuremberg. This observation—the earliest of its kind on land—was very probably made with the compass of a portable sun-dial, for it is known that Hartmann was an expert in the construction of these instruments.

'With regard to these sun-dials, Hellmann referred to the instrument found by Le Monnier in Paris. It was constructed by Bellermatus, is stamped with the year 1541, and shows the magnetic meridian making an angle of about 7° east with the north point. Presumably, this relates to Paris. Wolkenhauer was able, however, to go much further back. He called attention to a sun-compass found by E. Meyer in the Spitzer collection of antiques at Paris. It is dated 1453 and shows a deviation of the needle from the noon-line. Other instruments of similar kind found by Wolkenhauer are of dates 1451, 1456, 1470, and 1511, all of them showing an easterly deviation of the magnetic needle. Finally, there are good reasons for believing that they had been constructed at Nuremberg.

'In confirmation of the conclusions towards which the foregoing evidence points, Wolkenhauer referred to certain early road-maps of Germany which were, either directly or indirectly, the work of Erhard Etzlaub of Nuremberg. Of these maps, six copies are still extant. Each has at the side or bottom the figure of a compass, similar to that attached to a portable sun-dial, showing an eastward deviation by about 11¼°. That this deviation was clearly recognized at the time is proved by the instructions, printed at the top of the map, for the use of the compass in travelling by the map. It is generally believed that these maps were issued about 1492.'

Crichton Mitchell states his final conclusion regarding the discovery of the geomagnetic declination as follows (2nd paper, § 27):

'Within a century after the first mention in literature of the nautical compass, we find allusions in the writings of Roger Bacon to the fact that the direction of the magnetic needle does not coincide with the geographical meridian. When, where, and by whom this was first observed is now unknown. At first, this deviation was ascribed to irregularities in the process of magnetization of the compass-needle, or to faulty methods of observation. But gradually it came to be recognized as a general and world-wide phenomenon and it is probable, though there is no extant record to establish the fact, that this stage was reached in the earlier years of the fifteenth century. Very soon thereafter, and certainly not later than 1450, the makers of sun-dials in Germany introduced improvements in their instruments in order to avoid the effects of deviation of the needle. Possibly about the same time, though more probably later, the compass-makers of Holland adjusted their compass needles with the same object. The Mediterranean navigators continued to use the compass without this adjustment. Finally, no original observation, discovery, or instrumental improvement, as made or effected down to the end of the historical period concerned, can be definitely or exclusively associated with the name of any particular person.'

26.4. The earliest observations of declination. Hellmann [4] gives the following values of the magnetic declination in the first half of the sixteenth century:

Year	*Place*	*Decl.*	*Observer or authority*
About 1510	Rome	6° E.	Georg Hartmann
„ 1518	Bay of Guinea	11¼° E.	Piero di Giovanni d'Antonio di Dino
„ 1520	Vienna	4° E.	Johann Georg Tannstetter
„ 1524	Landshut (Bav.)	10° E.	Petrus Apianus (Bienewitz)
1534	Dieppe	10° E.	François or Crignon
1537	Florence	9° E.	Mauro (*Sphera volgare novamente tradotta.* Venetia, 1537)
1538	Lisbon	7½° E.	Pedro Nunes or João de Castro
1539	Dantzic	13° E.	Georg Joachim Rheticus
1541	Paris	7° E.	Hieronymus Bellarmatus
About 1544	Nuremberg	10° E.	Georg Hartmann
„ 1546	Island of Walcheren	9° E.	Gerhard Mercator

Hellmann comments on this table as follows:

'Although, therefore, the deviation of the magnetic needle from the astronomical meridian had been confirmed at different places up to about 1550, it would be very misleading to assume that this knowledge —at any rate, among scholars—soon became common property. On the contrary, up to about the end of the sixteenth century, most of the writers on magnetism and dials made no mention whatever of the declination of the magnetic needle.

'The reason for this should be sought for in the fact that no contemporaneous account of the above-mentioned observations appeared in print which would have contributed to their wider diffusion, as also to the circumstance spoken of above, that the deviation was regarded simply as a peculiarity of the magnet concerned, but not of the station. Again, these first determinations of the magnetic declination were subject to a great uncertainty. This, namely, was the case with the observations made on shipboard, where the imperfect methods, as well as the frequently poor construction of the compass, were to blame. The values of the declination obtained by pilots agreed so poorly with each other—indeed, often directly contradicted each other—that doubts of the correctness of the magnetic declination arose everywhere anew. . . .

'An improvement in the method of determining the declination at sea was, therefore, a first essential, if the hope, cherished with so much love and perseverance, of a solution of the longitude problem by magnetism, was to be realized. An apothecary of Seville, Felipe Guillen, of whom, unfortunately, we know nothing further, was the one who thought out with this aim a new and better method of determining the declination. It is in this connexion interesting to note that the German dials (compasses in the foregoing meaning of the word) furnished to the Spanish observer, not only the suitable magnetic needle, but indirectly also the method itself; for this consisted simply in determining with an arrangement like a sun-dial with magnetic needle, the magnetic azimuth of the sun at equal altitudes before and after noon by means of a centrally placed style or gnomon. The half difference of the azimuths, which were reckoned from N. through E. to S. and from N. through W. to S. as far as 180°, was the desired declination of the magnetic needle from the meridian.

'Felipe Guillen, who presented this instrument (brújula de variación) in 1525 to the king of Portugal, João III, has unfortunately left behind no writing concerning it.

'Soon afterwards Pedro Nunes, who in 1537 likewise pointed out the actual existence of a variation, and emphasized the need of its determination for nautical purposes, improved the Guillen instrument merely by adding to it a contrivance for the determination of sun altitudes, and at the same time devised a new method for determining the latitude at any hour of the day. . . .

'A remarkable opportunity now soon offered itself to prove in a most comprehensive manner both methods, which were first carefully tested in 1533 at Evora. The Infante Dom Luiz, who had received instruction in mathematics and astronomy from Pedro Nunes himself, and had shown great interest in all nautical problems, presented such an instrument to his comrade in study and friend, João de Castro, who commanded one of the eleven ships that sailed to the East Indies in 1538, with the charge to thoroughly test and examine this instrument as well as the new method of determining the longitude. João de Castro, later the fourth viceroy of India, performed his task most brilliantly. He investigated—to consider here only the magnetic side of the matter—not only the variation as often as possible, but he made also all kinds of observations regarding the method itself, concerning the influence of the magnetic needle and its magnetization upon the measured value of the declination, concerning magnetic disturbances, concerning the deviation of the compass, etc. Indeed, he was also the discoverer of the magnetism of rocks, of which with us nothing was said before the seventeenth century. João de Castro carried on his observations also during his voyage along the west coast of India and in the Red Sea, so that we possess a series of 43 determinations of the declination between the years 1538–1541—the first series of this kind that has come down to us. This remarkable mariner kept very copious journals concerning all his nautical, magnetic, meteorological, and hydrographic observations, which contain indisputably the greatest and most valuable treasure of records of that kind from the first half of the sixteenth century, and are worthy the zealous study of all those who intend writing the history of physical geography or of navigation during that period. Since I have read these journals myself, I do not hesitate to pronounce João de Castro to be the most important representative of scientific maritime investigations at the end of the epoch of discoveries.

'The method of determining the declination, first tested by João de Castro, soon was universally introduced on ships and was even at the end of the sixteenth century recommended anew by mariners and scholars in Spain, England, and Holland. They did not already know that this method was of Spanish-Portuguese origin, and was already fifty or perhaps a hundred years old. For neither Rio Riano (1589), nor William Borough (1581), nor Edmund Gunter (1622), nor Henry Gellibrand (1635), nor, finally, Simon Stevin (1599), mention the names of Felipe Guillen, Francisco Falero, or Pedro Nunes.'

26.5. The discovery of magnetic inclination. Georg Hartmann, Vicar of St. Sebald's at Nuremberg, has already been mentioned in § 3. The letter written on March 4, 1544, addressed to the Duke Albrecht of Prussia, in which he describes his observations of declination, contains also a passage which has been interpreted (by L. Moser [5]) as an observation of inclination: 'A non-magnetic needle, which is balanced horizontally on a pivot, becomes inclined when magnetized.' He gives the inclination as generally 9°, instead of more than 70°, as it must then have been. Consequently Gilbert ascribes the discovery of the magnetic dip or inclination to the Englishman Robert Norman, who in 1576 made a dip-circle, and published a work with the title: 'The newe Attractiue, containyng a short discourse of the Magnes or Lodestone, and amongest other his vertues, of a newe discouered secret and subtill propertie, concernyng the Declinyng of the Needle, touched there with onder the plaine of the Horizon. Now first founde out by ROBERT NORMAN Hydrographer. Hereonto are annexed certaine necessarie rules for the art of Nauigation, by the same R. N. Imprinted at London by John Kyngston, for Richard Ballard. 1581.'

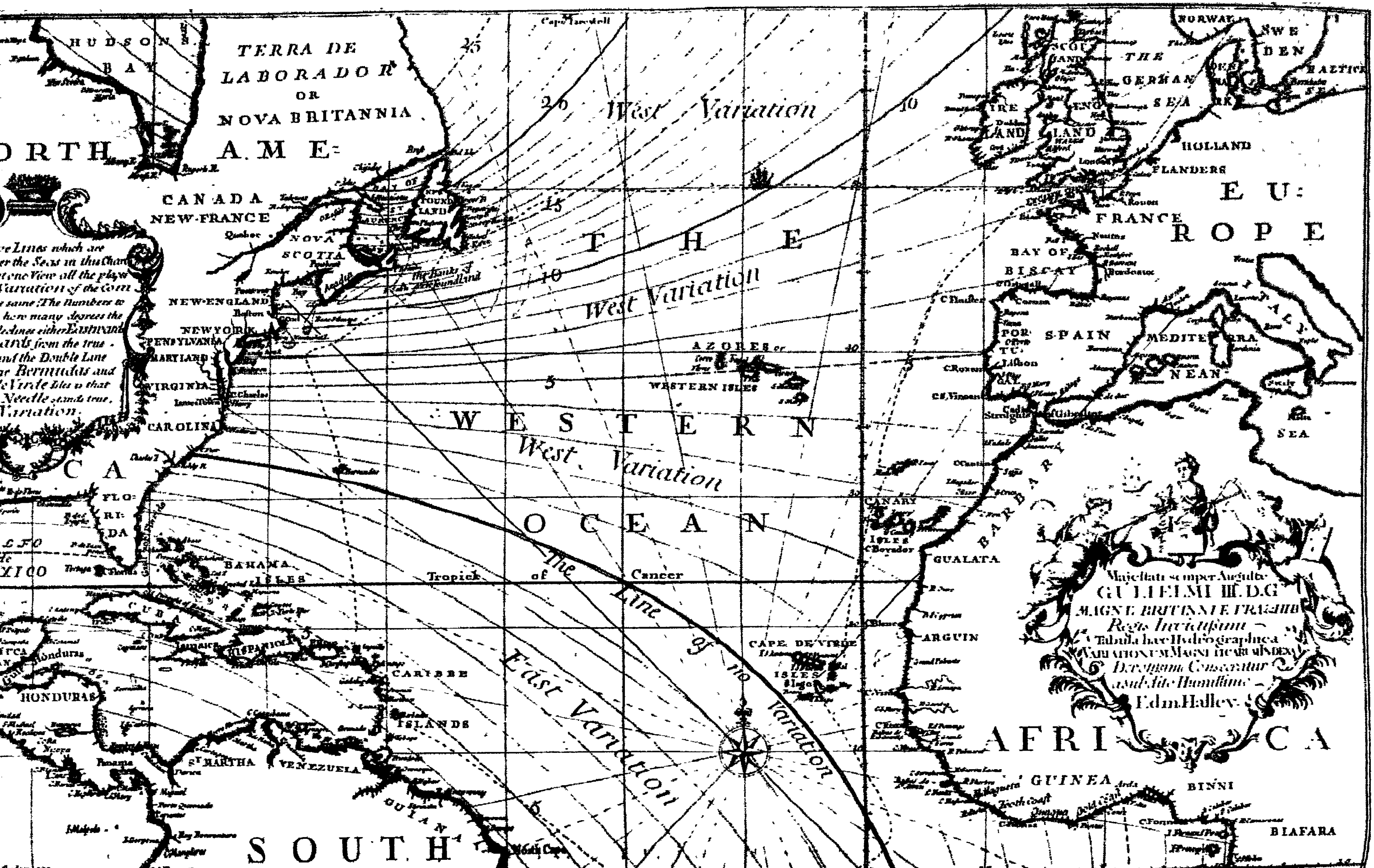
Hellmann reprinted this work in his *Rara Magnetica* [6]; he states that it is the oldest printed work dealing solely with geomagnetism. It was dedicated to Wm. Borough, at that time Comptroller of the Navy; it was favourably received and went through at least four more editions (1585, 1596, 1614, 1720).

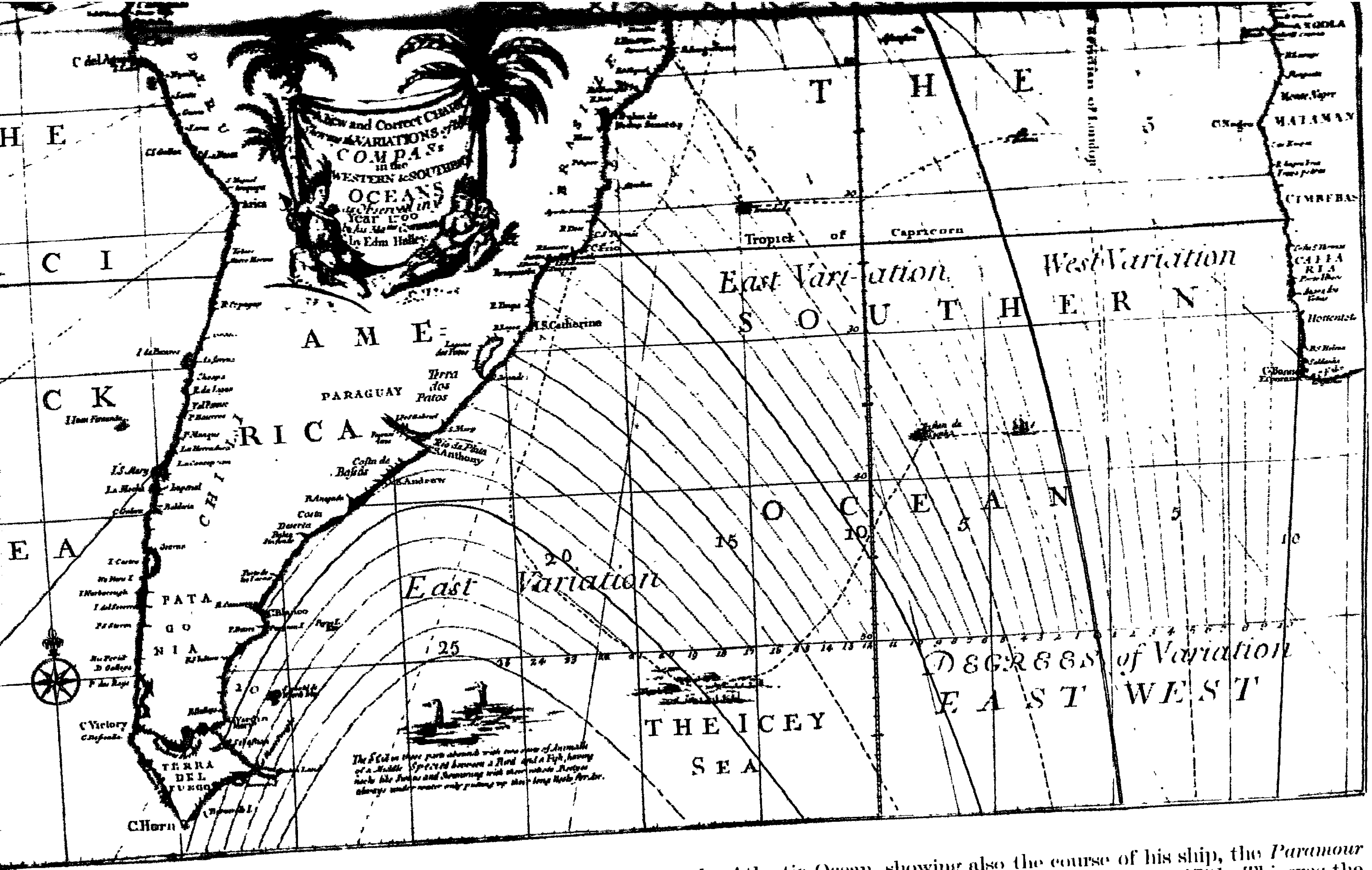
Gilbert made the first practical dip-circle. In his book he refers to the magnetic declination by its nautical name 'variatio', and he called the inclination or dip the 'declinatio'; the present use of the names declination and inclination first became usual in the 18th century. According to Günther [7] this was due mainly to Daniel Bernoulli, who gave much attention to the measurement of I. The Paris Academy proposed the subject as one for a prize-competition; Bernoulli gained the prize for his work *Sur la meilleure manière de construire les boussoles d'inclinaison*, Paris, 1743. Euler measured the inclination at Berlin in 1745 as 71·5°, using an instrument made by the Basel mechanic Dietrich, after Bernoulli's design.

26.6. The discovery of the secular variation. Henry Gellibrand, Professor of Astronomy in Gresham College, discovered that the magnetic declination changed with time. His work, 'A discourse mathematical on the variation of the magneticall needle. Together with its admirable diminution lately discovered. London 1635.', has been republished, in facsimile, by Hellmann [8].

Hellmann says:

> 'At times the credit for this discovery, so important in the science of geomagnetism, has been ascribed to Edmund Gunter, Gellibrand's predecessor as professor of Astronomy at Gresham College, London, but there is nothing in his printed works which could justify this view. Certainly Gunter in 1622 had found the magnetic declination at London to be 5° less than had been found in 1580 by William Borough, but Gunter refrained from drawing any conclusion from this difference. This may have been due either to the great inaccuracy of the older declination observations, or to undue regard for the authority of the statement made in 1600 by William Gilbert: "variatio uniscujusque loci constans est" (the variation at any one place is constant).
>
> 'But though the credit for the discovery of the secular variation of the magnetic declination must be withheld from Gunter, his accurate observations must certainly have helped Gellibrand to recognize with confidence the regular decrease of the declination.'


HUDSONS BAY
TERRA DE LABORADOR OR NOVA BRITANNIA
NORTH AMERICA
CANADA NEW-FRANCE
Quebec
NOVA SCOTIA
NEW-ENGLAND
Boston
NEWYORK
PENSYLVANIA
MARYLAND
VIRGINIA
CAROLINA
FLORIDA
BAHAMA ISLES
CUBA
JAMAICA
HISPANIOLA
HONDURAS
CARIBBE ISLANDS
VENEZUELA
St MARTHA
Panama
GUIANA
SOUTH
The Banks of Newfoundland
THE WESTERN OCEAN
AZORES or WESTERN ISLES
West Variation
West Variation
West Variation
East Variation
The Line of no Variation
Tropick of Cancer
25
20
15
10
10
5
40
30
CANARY ISLES
C. Boyador
CAPE DE VIRDE ISLES
IRELAND
ENGLAND
SCOTLAND
THE GERMAN SEA
NORWAY
SWEDEN
BALTICK
HOLLAND
FLANDERS
FRANCE
EUROPE
BAY OF BISCAY
C. Finisterre
SPAIN
PORTUGAL
Lisbon
C. St Vincent
Streights of Gibraltar
MEDITERRANEAN SEA
ITALY
BARBARY
GUALATA
ARGUIN
AFRICA
GUINEA
BINNI
BIAFARA
Majestati semper Augustæ GULIELMI III. D.G. MAGNÆ BRITANNIÆ FRA: & HIB: Regis Invictissimi Tabula hæc Hydrographica VARIATIONUM MAGNETICARUM INDEX Devotissime Consecratur a sub: dito Humillimo Edm. Halley.
…e Lines which are …er the Seas in this Chart …t one View all the plac… Variation of the Com… same: The Numbers to… how many degrees the… declines either Eastward …ards from the true … and the Double Line …r Bermudas and …e Virde Isles is that …Needle stands true, …Variation.


A New and Correct Chart Shewing the Variations of the Compass in the Western & Southern Oceans as Observed in ye Year 1700 by his Maties Command by Edm Halley
THE SOUTHERN OCEAN
East Variation
West Variation
Tropick of Capricorn
Meridian of London
AMERICA
PARAGUAY
Terra dos Patos
PATAGONIA
TERRA DEL FUEGO
C.Horn
C Victory
Rio da Plata
S.Catherine
THE ICEY SEA
Degrees of Variation
EAST WEST
25
20
15
10
5
MATAMAN
CIMBEBAS
CAFFARIA
Hottentots
ANGOLA
The Sea in these parts abounds with two sorts of Animalls of a Middle Species between a Bird and a Fish, having necks like Swans and Swimming with their whole Bodyes always under water only putting up their long Necks for Air.

s earliest chart of the magnetic 'variation' (or declination—see p. 1) for the Atlantic Ocean, showing also the course of his ship, the *Paramour* n which, between 1697 and 1701, he made the magnetic observations that form the basis of this chart. Probably published in 1701. This was the first isomagnetic (isogonic) chart

The observations in London on which Gellibrand based his conclusion are the following:

Date	*Observer*	*Declination*
1580 Oct. 16	William Burrow (or Borough)	11·3° E.
1622 June 13	Edmund Gunter	6·0° E.
1634 June 16	Gellibrand	4·1° E.

Gellibrand says:

'Hence therefore we may conclude that for the space of 54 Yeares (the difference of time between Mr. Burrowes and of these last observations of ours) there hath beene a sensible diminution of 7 degrees and better.

'If any affected with magneticall Philosophy shall yet desire to see an experiment made for their owne particular satisfaction, where I may prevaile, I would advise them to pitch a faire stone parallell to the Horizon there to rest immoveable, and having a Needle of a convenient length strongly touch't by a vigorous Magnet to draw a Magneticall Meridian thereby, and yearely to examine by the application of the same (well preserved from the ayre and rust, its greatest enemies) whether time will produce the like alteration.'

26.7. The earliest magnetic charts. Hellmann [9] collected and reprinted the earliest geomagnetic charts, the first of which were Edmund Halley's *New and Correct Chart showing the Variations of the Compass in the Western and Southern Oceans, as observed in ye year 1700 by his Majesty's Command by Edm. Halley,* published in 1701, and his *Sea-Chart of the whole World, shewing the Variations of the Compass*, published in London, 1702, in Mercator's projection and in the approximate scale 1:33 millions. Before Hellmann's time, Halley's world chart had already been reproduced by G. B. Airy [10]. His earlier chart (Plate 38) was reproduced by Bauer in 1896 [9*a*].

Hellmann says of Halley's chart:

'Halley in 1683 and 1692 had published two works [10*a*] on the theory of geomagnetism, but further observations seemed necessary to test his theory' (see § 11). 'He wished to obtain the new measurements himself on an extensive ocean voyage, and sought help for this purpose from King William III. The King agreed, and put at his disposal the sloop *Paramour Pink*, on which Halley between October 1698 and September 1700 made two voyages in the North and South Atlantic Ocean (to 52° 24′ S. lat.). These were the first sea voyages made for purely scientific objects, and they led to the production of the first declination chart of the earth. This was not only a masterpiece of the greatest value for practical navigation and the theory of geomagnetism at the time; it was also the pioneer embodiment of a method of graphical representation, which has been of great value in all branches of earth-science.

'Halley's chart, as is indicated by its name (Sea Chart, *Tabula Nautica*), was primarily made for the use of seamen. This accounts for its large scale and for the fact that the lines of equal declination were drawn only over the seas, not over the land. Apparently Halley never published a separate account of his discussion of his own and other observations, on which his isogonic lines in the Atlantic and Indian Oceans were based.'

The oldest charts of inclination, which referred to south-eastern England, are due to WILLIAM WHISTON, of whom Hellmann [9] wrote:

'William Whiston (1667–1752) from his youth upwards was a zealous and fruitful follower of mathematical and astronomical studies, and in 1703 was elected to the Lucasian Professorship of Mathematics at Cambridge (held earlier by Newton); he lost this, however, in 1710, because of religious controversies.

'It was the problem of determining the longitude at sea that gave rise to the preparation of the first inclination-chart. Whiston, like many others before him, believed that this could be solved with the aid of the dip needle.'

Whiston himself writes on this as follows:

'Now as I was musing of this Matter, and when I could think of no other way, Dr. Halley's Map of the Variation, already mentioned, came very seasonably to my mind: And I considered, that by such an inartificial way of describing the Curve Lines of equal Variation, how irregular soever, upon a Map, he had already given some Help for the Discovery of the Longitude, at least near the Cape of Good Hope: tho' the Quickness of the Mutation of those Lines, and their different Position in the rest of the World, rendred that Method of short Use there, and almost wholly useless elsewhere. When therefore I considered, that the Lines of equal Dip could hardly be more irregular than those of the Variation; and well knew that their Mutation was a great deal slower; and that these might probably be useful over all the World; I conceived great hopes, that this way of Application of the Power before us might very probably discover the Longitude.'

The idea that the geographical position of a ship could be found by means of simultaneous observations of D and I was long held; John Churchman's *Magnetic Atlas* [11], 1804, was published with this object in view. The idea is theoretically sound, but its practical application is hindered by the inaccuracy of the measurements on ships, due to their motion (and their iron).

The first chart of inclination for the whole earth was published in 1768 by Johann Carl WILCKE at Stockholm [12]. He called the lines 'parallels of inclination'. He indicated on the chart itself all the observations he had used, so that one could estimate the degree of accuracy of the chart; he stated also that errors up to 1° were to be expected, not only because of the imperfections of dip-circles, but also because of errors in determining the geographical position of the place of observation.

On a sketch of 'isodynamic zones', published in 1804 by Alexander von Humboldt, Hellmann wrote:

'This chart gives no true isodynamic lines, but indicates isodynamic zones on both sides of the magnetic equator, or, in Humboldt's words, the first sketch of an isodynamic system, divided into zones. The chart appeared in the publication "Sur les variations du magnétisme terrestre à différentes latitudes; par MM. Humboldt et Biot. Lu par M. Biot à la classe des sciences mathématiques et physiques de l'Institut National, le 26 frimaire an 13" (24 pp.). The chart is based on measures made by von Humboldt himself during his American journey (1799–1803), which he made through the influence of Borda. He swung a dip-needle in the magnetic meridian plane under the influence of the earth's field, and observed the number of swings during 10 minutes. On the magnetic equator, which in Peru he found had the southern latitude 7° 1′, this number of swings was 211; it decreased to the north and to the south. To smooth out the errors of observation, and also to eliminate local influences to some extent, Humboldt determined the mean number of swings in four northern zones and one southern zone, and entered these on the chart. This indicated a *regular decrease of the (total) magnetic intensity from the poles to the equator*. Even if this fact had already been recognized, it had not previously been published, and unquestionably Humboldt deserves credit for having by his publication of this law stimulated further investigations concerning it. The great importance which Humboldt himself attached to this result of his researches is most clearly evident from a reference in his account (*Relation historique*, iii. 615) of his American journey, where he says: "J'ai regardé la loi du décroissement des forces magnétiques, du pôle à l'équateur, comme le résultat le plus important de mon voyage américain."

'The relative measures of the horizontal intensity, made three years later, in 1807, by Humboldt in conjunction with Gay-Lussac on a journey to Italy, brought into prominence this simpler and more practicable observation of the time of oscillation of a *compass*-needle in the *horizontal* plane, and led to its general adoption on scientific journeys. In this way sufficient observations were gradually accumulated to permit the attempt to draw real isodynamic lines' (Hellmann).

Isodynamic charts for H and F were published by C. Hansteen in 1825 and 1826 [3]; the horizontal intensity H is there indicated by the number of seconds required for Hansteen's magnet to make 300 swings in the horizontal plane; this number was, for example, 753 for Paris, 780 for Oxford, 820 for Edinburgh, 850 for Bergen; these numbers illustrate the northward decrease of H. The chart for F indicates the total intensity expressed in the unit introduced by von Humboldt (see 1.3).

More complete isodynamic charts were presented in 1833 by L. I. Duperrey [14] to the Paris Academy, and E. Sabine published others in 1837 [15].

26.8. William Gilbert. In 1600 William Gilbert published (in Latin) his famous treatise *De Magnete*, the fruit of long years of thought and experiment, on the lines afterwards advocated by Bacon. Gilbert was an amateur in physical science, for his profession was medical; in this also he rose to eminence, being appointed, in 1601, chief physician in personal attendance on Queen Elizabeth. He was born at Colchester in 1540, and died of the plague in 1603. His book has been described as the first modern scientific treatise.

The scope of the book may be seen from Gilbert's Index of Chapters, here reproduced from the revised English translation printed in 1900 at the Chiswick Press, London, for the Gilbert Club:

Book 1.

Chap. 1. Ancient and modern writings on the Loadstone, with certain matters of mention only, various opinions, & vanities.

Chap. 2. Magnet Stone, of what kind it is, and its discovery.

Chap. 3. The loadstone has parts distinct in their natural power, & poles conspicuous for their property.

Chap. 4. Which pole of the stone is the Boreal: and how it is distinguished from the austral.

Chap. 5. Loadstone seems to attract loadstone when in natural position: but repels it when in a contrary one, and brings it back to order.

Chap. 6. Loadstone attracts the ore of iron, as well as iron proper, smelted & wrought.

Chap. 7. What iron is, and of what substance, and its uses.
Chap. 8. In what countries and districts iron originates.
Chap. 9. Iron ore attracts iron ore.
Chap. 10. Iron ore has poles, and acquires them, and settles itself toward the poles of the universe.
Chap. 11. Wrought iron, not excited by a loadstone, draws iron.
Chap. 12. A long piece of Iron (even though not excited by a loadstone) settles itself toward North & South.
Chap. 13. Wrought iron has in itself certain parts Boreal & Austral: a magnetic vigour, verticity, and determinate vertices or poles.
Chap. 14. Concerning other powers of loadstone, & its medicinal properties.
Chap. 15. The medicinal virtue of iron.
Chap. 16. That loadstone & iron ore are the same, but iron an extract from both, as other metals are from their own ores; & that all magnetic virtues, though weaker, exist in the ore itself & in smelted iron.
Chap. 17. That the globe of the earth is magnetic, & a magnet; & how in our hands the magnet stone has all the primary forces of the earth, while the earth by the same powers remains constant in a fixed direction in the universe.

Book 2.

Chap. 1. On Magnetick Motions.
Chap. 2. On the Magnetick Coition, and first on the attraction of Amber, or more truly, on the attaching of bodies to Amber.
Chap. 3. Opinions of others on Magnetick Coition, which they call Attraction.
Chap. 4. On Magnetick Force & Form, what it is; and on the cause of the Coition.
Chap. 5. How the Power dwells in the Loadstone.
Chap. 6. How magnetick pieces of Iron and smaller loadstones conform themselves to a terrella & to the earth itself, and by them are disposed.
Chap. 7. On the Potency of the Magnetick Virtue, and on its nature capable of spreading out into an orbe.
Chap. 8. On the geography of the Earth, and of the Terrella.
Chap. 9. On the Æquinoctial Circle of the Earth and of a Terrella.
Chap. 10. Magnetick Meridians of the Earth.
Chap. 11. Parallels.
Chap. 12. The Magnetick Horizon.
Chap. 13. On the Axis and Magnetick Poles.
Chap. 14. Why at the Pole itself the Coition is stronger than in the other parts intermediate between the æquator and the pole; and on the proportion of forces of the coition in various parts of the earth and of the terrella.
Chap. 15. The Magnetick Virtue which is conceived in Iron is more apparent in an iron rod than in a piece of Iron that is round, square, or of other figure.
Chap. 16. Showing that Movements take place by the Magnetical Vigour though solid bodies lie between; and on the interposition of iron plates.
Chap. 17. On the Iron Cap of a Loadstone, with which it is armed at the pole (for the sake of the virtue), and on the efficacy of the same.
Chap. 18. An armed Loadstone does not indue an excited piece of Iron with greater vigour than an unarmed.
Chap. 19. Union with an armed Loadstone is stronger; hence greater weights are raised; but the coition is not stronger, but generally weaker.
Chap. 20. An armed Loadstone raises an armed Loadstone, which also attracts a third; which likewise happens, though the virtue in the first be somewhat small.

Chap. 21. If Paper or any other Medium be interposed, an armed loadstone raises no more than an unarmed one.

Chap. 22. That an armed Loadstone draws Iron no more than an unarmed one: and that an armed one is more strongly united to iron is shown by means of an armed loadstone and a polished Cylinder of iron.

Chap. 23. The Magnetick Force causes motion toward unity, and binds firmly together bodies which are united.

Chap. 24. A piece of Iron placed within the Orbe of a Loadstone hangs suspended in the air, if on account of some impediment it cannot approach it.

Chap. 25. Exaltation of the power of the magnet.

Chap. 26. Why there should appear to be a greater love between iron & loadstone, than between loadstone & loadstone, or between iron & iron, when close to the loadstone, within its orbe of virtue.

Chap. 27. The Centre of the Magnetick Virtues in the earth is the centre of the earth; and in a terrella is the centre of the stone.

Chap. 28. A Loadstone attracts magneticks not only to a fixed point or pole, but to every part of a terrella save the æquinoctial zone.

Chap. 29. On Variety of Strength due to Quantity or Mass.

Chap. 30. The Shape and Mass of the Iron are of most importance in cases of coition.

Chap. 31. On long and round stones.

Chap. 32. Certain Problems and Magnetick Experiments about the Coition, and Separation, and regular Motion of Bodies magnetical.

Chap. 33. On the Varying Ratio of Strength, and of the Motion of coition, within the orbe of virtue.

Chap. 34. Why a Loadstone should be stronger in its poles in a different ratio; as well in the Northern regions as in the Southern.

Chap. 35. On a Perpetual Motion Machine, mentioned by authors, by means of the attraction of a loadstone.

Chap. 36. How a more robust Loadstone may be recognized.

Chap. 37. Use of a Loadstone as it affects iron.

Chap. 38. On Cases of Attraction in other Bodies.

Chap. 39. On Bodies which mutually repel one another.

Book 3.

Chap. 1. On Direction.

Chap. 2. The Directive or Versorial Virtue (which we call verticity): what it is, how it exists in the loadstone; and in what way it is acquired when innate.

Chap. 3. How Iron acquires Verticity through a loadstone, and how that verticity is lost and changed.

Chap. 4. Why Iron touched by a Loadstone acquires an opposite verticity, and why iron touched by the true northern side of a stone turns to the North of the earth, by the true Southern side to the South; and does not turn to the South when rubbed by the Northern point of the stone, and when by the Southern to the North, as all who have written on the Loadstone have falsely supposed.

Chap. 5. On the touching of pieces of Iron of divers shapes.

Chap. 6. What seems an Opposing Motion in Magneticks is a proper motion toward unity.

Chap. 7. A determined Verticity and a disponent Faculty are what arrange magneticks, not a force, attracting them or pulling them together, nor merely a strongish coition or unition.

Chap. 8. Of Discords between pieces of Iron upon the same pole of a Loadstone, and how they can agree and stand joined together.

Chap. 9. Figures illustrating direction and showing varieties of rotations.

Chap. 10. On Mutation of Verticity and of Magnetick Properties, or on alteration in the power excited by a loadstone.

Chap. 11. On the Rubbing of a piece of Iron on a Loadstone in places midway between the poles, and upon the æquinoctial of a terrella.

Chap. 12. In what way Verticity exists in any Iron that has been smelted though not excited by a loadstone.

Chap. 13. Why no other Body, excepting a magnetick, is imbued with verticity by being rubbed on a loadstone, and why no body is able to instil and excite that virtue, unless it be a magnetick.

Chap. 14. The Placing of a Loadstone above or below a magnetick body suspended in æquilibrio changes neither the power nor the verticity of the magnetick body.

Chap. 15. The Poles, Æquator, Centre in an entire Loadstone remain and continue steady; by diminution and separation of some part they vary and acquire other positions.

Chap. 16. If the Southern Portion of a Stone be lessened, something is also taken away from the power of the Northern Portion.

Chap. 17. On the Use and Excellence of Versoria: and how iron versoria used as pointers in sun-dials, and the fine needles of the mariners' compass, are to be rubbed, that they may acquire stronger verticity.

Book 4.

Chap. 1. On Variation.

Chap. 2. That the variation is caused by the inæquality of the projecting parts of the earth.

Chap. 3. The variation in any one place is constant.

Chap. 4. The arc of variation is not changed equally in proportion to the distance of places.

Chap. 5. An island in Ocean does not change the variation, as neither do mines of loadstone.

Chap. 6. The variation and direction arise from the disponent power of the earth, and from the natural magnetick tendency to rotation, not from attraction, or from coition, or from other occult cause.

Chap. 7. Why the variation from that lateral cause is not greater than has hitherto been observed, having been rarely seen to reach two points of the mariners' compass, except near the pole.

Chap. 8. On the construction of the common mariners' compass, and on the diversity of the compasses of different nations.

Chap. 9. Whether the terrestrial longitude can be found from the variation.

Chap. 10. Why in various places near the pole the variations are much more ample than in a lower latitude.

Chap. 11. Cardan's error when he seeks the distance of the centre of the earth from the centre of the cosmos by the motion of the stone of Hercules; in his book 5, *On Proportions*.

Chap. 12. On the finding of the amount of variation: how great is the arc of the Horizon from its arctick to its antarctick intersection of the meridian, to the point respective of the magnetick needle.

Chap. 13. The observations of variation by seamen vary, for the most part, and are uncertain: partly from error and inexperience, and the imperfections of the instruments: and partly from the sea being seldom so calm that the shadows or lights can remain quite steady on the instruments.

Chap. 14. On the variation under the æquinoctial line, and near it.

Chap. 15. The variation of the magnetick needle in the great Æthiopick and American sea, beyond the æquator.

Chap. 16. On the variation in Nova Zembla.

Chap. 17. Variation in the Pacifick Ocean.

Chap. 18. On the variation in the Mediterranean Sea.

Chap. 19. The variation in the interior of large Continents.

Chap. 20. Variation in the Eastern Ocean.

Chap. 21. How the deviation of the versorium is augmented and diminished by reason of the distance of places.

Book 5.

Chap. 1. On Declination.

Chap. 2. Diagram of declinations of the magnetick needle, when excited, in the various positions of the sphere, and horizons of the earth, in which there is no variation of the declination.

Chap. 3. An indicatory instrument, showing by the virtue of a stone the degrees of declination from the horizon of each several latitude.

Chap. 4. Concerning the length of a versorium convenient for declination on a terrella.

Chap. 5. That declination does not arise from the attraction of the loadstone, but from a disposing and rotating influence.

Chap. 6. On the proportion of declination to latitude, and the cause of it.

Chap. 7. Explanation of the diagram of the rotation of a magnetick needle.

Chap. 8. Diagram of the rotation of a magnetick needle, indicating magnetical declination in all latitudes, and from the rotation and declination, the latitude itself.

Chap. 9. Demonstration of direction, or of variation from the true direction, at the same time with declination, by means of only a single motion in water, due to the disposing and rotating virtue.

Chap. 10. On the variation of the declination.

Chap. 11. On the essential magnetick activity sphærically effused.

Chap. 12. Magnetick force is animate, or imitates life; and in many things surpasses human life, while this is bound up in the organick body.

Book 6.

Chap. 1. On the globe of the earth, the great magnet.

Chap. 2. The Magnetick axis of the Earth persists invariable.

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Chap. 9. On the anomaly of the Præcession of the Æquinoxes, & of the obliquity of the Zodiack.

The central thesis of the book is stated in the title of Chapter 17 of Book 1 (q.v.); the essential arguments for the conclusion there stated are given in Chapter 1, Book 6. We reproduce this chapter here (with its original woodcut page-head) from the 1900 English edition, and also part of p. 191 of this edition, with the woodcut (see p. 921) by which Gilbert indicated to the reader the similarity between the distribution of magnetic dip over the earth, and over a small spherical loadstone, which he called a *terrella*. It should be noted that he used the word declination not in its modern sense, but as meaning dip or inclination; for what we now call declination, he used the word variation, as seamen still do.

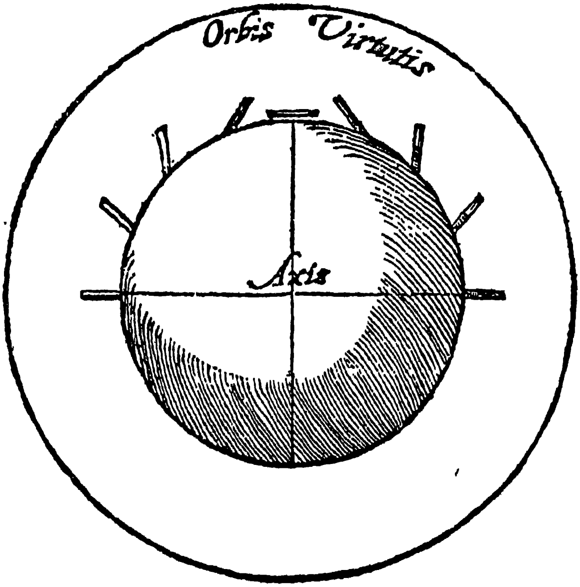
BOOK SIXTH.

CHAP. I.

ON THE GLOBE OF THE EARTH, THE GREAT MAGNET.

Hitherto our subject hath been the loadstone and things magnetical: how they conspire together, and are acted upon, how they conform themselves to the terrella and to the earth. Now must we consider separately the globe itself of the earth. Those experiments which have been proved by means of the terrella, how magnetick things conform themselves to the terrella, are all or at least the principal and most important of them, displayed by means of the earth's Body: And to the earth things magnetical are in all respects associate. First, as in the terrella the æquator, meridians, parallels, axis, poles are natural boundaries, as numerous experiments make plain: So also in the earth these boundaries are natural, not mathematical only (as all before us used to suppose). These boundaries the same experiments display and establish in both cases alike, in the earth no less than in the terrella. Just as on the periphery of a terrella a loadstone or a magnetick piece of iron is directed to its proper pole: so on the earth's surface are there turnings-about, peculiar, manifest, and constant on either side of the æquator. Iron is indued with verticity by being extended toward a pole of the earth, just as toward a pole of the terrella: By its being placed down also, and cooling toward the earth's pole after the pristine verticity has been annulled by fire, it acquires new verticity, conformable to its position earthward. Iron rods also, when placed some considerable time toward the poles, acquire verticity merely by regarding the earth; just as the same rods, if placed toward the pole of a loadstone, even without touching it, receive polar virtue. There is no magnetick body that in any way runs to the terrella which does not also wait upon the earth. As a loadstone is stronger at one end on one side or other of its æquator: so is the same property displayed by a small terrella upon the surface of a larger terrella. According to the variety and artistick skill in the rubbing of the magnetick iron upon the terrella, so do the magnetick things perform their function more efficiently or more feebly. In motions toward the earth's body, as toward the terrella a variation is displayed due to the unlikeness, inequality, and imperfection of its eminences: So every variation of the versorium or mariners' compass, everywhere by land or by sea, which thing has so sorely disturbed men's minds, is discerned and recognized as due to the same causes. The magnetick dip (which is the wonderful turning of magnetick things to the body of the terrella) in systematick course, is seen in clearer light to be the same thing upon the earth. And that single experiment, by a wonderful indication, as with a finger, proclaims the grand magnetick nature of the earth to be innate and diffused through all her inward parts. A magnetick vigour exists then in the earth just as in the terrella, which is a part of the earth, homogenic in nature with it, but rounded by Art, so as to correspond with the earth's globous shape and in order that in the chief experiments it might accord with the globe of the earth.

The declination of a magnetick needle above a terrella is shown by means of several equal iron wires, of the length of a barleycorn, arranged along a meridian. The wires on the equator are directed by the virtue of the stone toward the poles, and lie down upon its body along the plane of its horizon. The nearer they are brought to the poles, the more they are raised up by their versatory nature. At the poles themselves they point perpendicularly toward the very centre. But iron spikes, if they are of more than a due length, are not raised straight up except on a vigorous stone (p. 191).



Variety in the declinations of iron spikes at various latitudes of a terrella.

S. P. Thompson, in the Notes (p. 54) to the 1900 English edition, wrote: 'Gilbert's extraordinary detachment from all metaphysical and ultraphysical explanations of physical facts, and his continual appeal to the test of experimental evidence, enabled him to lift the science of the magnet out of the slough of the dark ages, . . . however, he still gave credence to the *nativities* of judicial astrology, and to the supposed influence of the planets on human destiny.'

It is not surprising that Gilbert's work also contained errors more nearly affecting magnetism, such as the conclusion (Chapter 3, Book 4) that the variation (i.e. D) in any one place is constant: 'Unless there should be a great dissolution of a continent and a subsidence of the land such as there was of the region Atlantis of which Plato and the ancients tell, the variation will continue perpetually immutable.'

Gilbert's book was received with great favour in his time, though some parts of it were considered heretical by many because they upheld the Copernican views, of which he was the first English adherent; Galileo's copy of Gilbert's book was given to him by 'a Peripatetick Philosopher, of great fame, as I believe to free his Library from its contagion'; the Jesuit scientists, however, who followed Gilbert in their magnetic writings, were right in repudiating his idea that the magnetism of the globe gave support to the heretical modern astronomy.

Galileo and Kepler both wrote in praise of Gilbert's book: in one of his *Scientific Dialogues* Galileo makes Salviatus say, regarding the increased lifting power of the loadstone when it is armed by an iron cap: 'I extreamly praise, admire, and envy this Authour, for that a conceit so stupendious should come into his minde. . . . I think him moreover worthy of extraordinary applause for the many new and true Observations that he made, to the disgrace of so many fabulous Authours, that write not only what they do not know, but whatever they hear spoken by the foolish vulgar, never seeking to assure themselves of the same by experience, perhaps because they are unwilling to diminish the bulk of their Books.'

26.9. The discovery of non-secular magnetic time-variations. Hansteen [17] in his Geomagnetic Researches describes how, after the discovery of the secular variation in Europe, it was thought that the magnet needle was turning uniformly westwards; from the London observations Musschenbroek (1719) concluded that the annual motion was 9′, and he calculated how many years would be required for a complete rotation of the needle in the horizontal plane; later it was found that the motion was non-uniform, and the famous instrument-maker Graham [19], with his improved declinatorium, found in 1722, at London, that the compass needle is continually in motion.

Walker [20] gives the following description of Graham's discovery:

'The figure of the three needles, with which the experiments were made, was prismatic, their lengths about 12·2 inches, their ends, which pointed to the divisions, being filed to an edge, forming a fine vertical line.

'The caps of two of the needles were of crystal, and the other of glass, well polished on the inside in that point which touched the pin they moved on.

'They were placed in a brass box, covered with ground glass, and wide enough to admit of 20° oscillation either way. At the ends were placed graduated arcs, raised so as to be in the same horizontal plane with the needle, which could move freely between them. A few of the degrees at the north end were subdivided to 10′, and by means of a convex lens intervals of 2′ could easily be observed. The pin was of hard steel, finely pointed, and, by means of a spring, the needle could be raised off the point at pleasure, so as to prevent its being injured when the box was moved.

'The needle was counterpoised by a small piece of brass.'

Graham says:

'After many trials, I found that all the needles I made use of would not only vary in their direction on different days but at different times on the same day; and this difference would sometimes amount to upwards of 30′ in one day, sometimes in a few hours. And this alteration occurred whether the needles were drawn aside immediately before the observation or suffered to remain undisturbed.'

The fact that the daily oscillation of the compass needle on undisturbed days is greater in summer than in winter was discovered in 1759 by Canton [24] at London. He found that the mean range in December was less than 7′, and in June greater than 13′.

26.10. The connexion between magnetic storms and polar lights. Graham's discovery was confirmed by Andreas Celsius [21] at Upsala in 1740; his observations were continued by Hiorter. From more than 20,000 observations (on the average 20 per day) Hiorter found in 1747 that (to quote Hansteen):

'First: The needle ranges by about 5′, on some days rather more, on others somewhat less, between its most easterly position, at about 8 a.m., and to its most westerly at 2 p.m.

'Secondly, in Hiorter's words: A motion of the magnetic needle has been found which deserves the attention and wonder of every one. Who could have thought that the *northern lights* would have a connexion and a sympathy with the magnet, and that these northern lights, when they draw southwards across our zenith or descend unequally towards the eastern and western horizons could within a few minutes cause considerable oscillations of the magnetic needle through whole degrees? The first time that I saw an aurora to the south and noted simultaneously a great movement of the magnetic needle was on March 1, 1741, in the evening, although at various other times I had noted a displacement of the needle, but on account of overclouded skies had not known whether this was accompanied by northern lights. When I announced this to the professor (Celsius), he said he too had noticed such a disturbance of the needle in similar circumstances, but had not wished to mention it, in order to see (this was his expression) whether I too would light on the same speculation. . . . Subsequently this was always observed, especially on the following April 5, 1741, when the needle at 2 p.m. began to be disturbed, so that at 5 p.m. it was 1° 40′ to the west of its direction at 10 a.m. The most remarkable feature of this motion of the magnetic needle was the following. The professor (Celsius), some weeks earlier, had by letter requested Mr. Graham at London to observe his own needle for some days, so that, should our needles be disturbed, one might know whether this occurred also at many widely separated places, in which case our observations could not be ascribed to any special properties of our observing room and of the iron in it. What happened? The magnetic needle at London had just such an unusual motion at London at the same time as here at Upsala.'

Graham described his observations of Sunday, April 5, 1741, as follows:

'The alterations that day were greater than I have ever met with before. Tho' no alteration of any thing in the Room could occasion it. It was a fine day. I was alone all the time, and observed the Needle with all the care possible, sometimes in 2 or 3 minutes of time a sunder, when I perceived the Needle changing. Whether the cause of the change proceeds from one place, where the Needle was placed, or from any other, I know not; the only thing, in which I am certain, is, that there was no change of position of any thing in the Room, that could cause it, being alone all the day (I mean Sunday the 5th) when the greatest alteration happened. The observations upon the other days were made with the same care, but they are much less, and more regular etc.'

Hiorter remarked that Graham had not mentioned the northern lights, presumably because, to judge by the motion of the needle, they had been strongest during the daylight; for in the evening, after sunset, only a faint diffuse aurora was seen at Upsala.

During a strong aurora on the morning of October 20, 1746 ('terribly red in the south') the declination at Upsala varied by 4° 10′ within four minutes of time.

Among other notable inferences drawn by Hiorter we may mention the following:

(*a*) 'That aurorae must be the highest phenomena of our atmosphere, so high and extensive, that they can simultaneously, here and in England, at Upsala and London, . . ., disturb the magnetic needle.

(*b*) 'That from the magnetic needle one can tell when there is an aurora in the sky, even in broad daylight or on cloudy nights (when, as so often, aurorae are completely hidden from us); and that conversely, from the southerly situation of the aurora one can safely conclude that the magnetic needle will be more or less displaced from its normal direction.

(*c*) 'That the aurora might be partly the cause of the larger of the observed monthly deviations of the magnetic needle from its normal direction; for it was found that after certain aurorae the needle did not return to the direction about which it had previously oscillated, so soon as at other times; hence it would be very difficult, if not impossible, to bring these so-called monthly magnetic motions under rule and order.'

Pehr Wargentin at Stockholm confirmed this discovery. After having seen that for some days the motions of the needle had been very small, he found (we quote from Hansteen's account) that 'on February 28, 1749, in the afternoon at 4 o'clock, the needle had unexpectedly swung westwards through 0·5° in a few minutes. At once I predicted to Mr. Ekström that that evening there would be an aurora; and this actually happened, for as soon as it had become dark we saw one . . .; on that evening . . . the needle made its greatest deviations, between 6° 50′ and 9° 1′.'

WILCKE [22] noted in 1770 that the auroral rays extend upwards in the same direction as the dip-needle: the latter points exactly to the centre of the auroral corona, which is only an optical projection (or perspective) phenomenon, as Mairan [23] had already remarked. Wilcke gave a list of disturbed days, 1772–4; on most of the days in this list aurorae were seen at Stockholm and (by J. A. Euler) also at St. Petersburg.

26.11. The hypothesis of four magnetic poles. In the year 1819 Christopher Hansteen, Professor of Applied Mathematics at the Norwegian University, published his great work *Untersuchungen über den Magnetismus der Erde, erster Teil: Die mechanischen Erscheinungen des Magneten* [17] ('Researches on geomagnetism, first part: the mechanical properties of the magnet'). This book contains many interesting historical quotations, as well as an account of his own original researches. It is noteworthy that Hansteen sought to establish Halley's conception of four magnetic poles, two, B and b, in the north, and two, A and a, in the south. For the year 1800 he extrapolated their positions, from the course of the isogonic lines, as follows: B, 69·9° N., 266·4° E. of Greenwich; b, 85·4° N., 131·7° E.; A, 69·1° S., 134·1° E.; a, 77·8° S., 229·5° E. The points B, A, and b were supposed to move eastwards round the geographical pole, the point a, on the contrary, westwards. The 'orbital times' were given as 1,728, 4,320, 864, and 1,296 years; Hansteen found that 25,920 years (the period of the precession of the equinoxes) was the smallest common multiple of these supposed periods.

Hansteen wrote as follows concerning Halley's poles:

'It is known that Halley regarded his magnetic poles in North America (B) and south of New Holland (A) as stationary (an assumption which the slow motion of these poles might well explain); the other two (b and a) he supposed to move from east to west. That is right as regards b; but his hypothesis also requires that a moves likewise in the same direction. Halley assumed that the earth is hollow, and that the magnetic poles B and A are located in the crust, and must therefore be stationary. He also assumed that there is a smaller sphere or a core (terrella), which is concentric with the earth, and therefore with the inner surface of the crust. The two other poles a and b he located in this core. The impulse which gave the earth (the outer crust) its eastward rotation about its axis, in 24 hours, could not fully impart the same angular velocity to the core, which therefore does not complete a full rotation in 24 hours . . . with the result that the two poles a and b apparently move westwards.'

But since, according to Hansteen, the pole b actually moves in the opposite direction, he concluded that Halley's 'correct' idea (of the existence of four magnetic poles) had been discarded along with the false idea (of the opposite motion of the crust and core). 'Euler saw that the assumption of two magnetic poles, or of one magnetic axis, was sufficient, and his great authority cast a further shadow on Halley's work.'

Gauss, in his *Results of the Observations of the Magnetic Union in the Year 1838*, finally discarded this assumption of four magnetic poles. He defined a pole as a place where the horizontal intensity H is zero. He added: 'If we turn from the actual properties of the earth, and regard the question generally, then clearly more than two magnetic poles can exist; but it appears not to have been noticed that if, for example, there are two north poles, then there must be a third point, between them, which is likewise a magnetic pole, but not truly either a north or a south pole, or, if one prefers, is both at once.'

Hansteen's mistaken advocacy of Halley's four poles should not obscure the memory of his many services to geomagnetism. Only a few years after the appearance of his book the wish was fulfilled (through Gauss) that Hansteen had expressed as follows in his preface: 'The mathematicians of Europe since the times of Kepler and Newton have all turned their eyes to the heavens, to follow the planets in their finest motions and mutual perturbations: it is now to be wished that for a time they would turn their gaze downwards towards the earth's centre, where also there are marvels to be seen; the earth speaks of its internal movements through the silent voice of the magnetic needle. We should also learn much if we could construe the flaming script of the polar lights.'

Halley's idea of an earth-core, which moves within the outer solid crust, and so produces the secular variation, still lives on in our times, as the writings of Carlheim-Gyllensköld and Schmidt testify (see Ch. XVIII). In his Adams Prize Essay for 1865 Walker [20] maintained that Halley's conception of four poles was right, in the sense that there are four poles of maximum total intensity F.

26.12. C. F. Gauss. Carl Friedrich Gauss [18], the son of a day labourer, was born at Brunswick on April 30, 1777. E. Schering [16] states that the first indication of Gauss's intention to turn his thoughts towards geomagnetism is contained in a letter written on March 1, 1803, to the astronomer Olbers of Bremen. In this letter Gauss referred as follows to a newspaper article which stated that the geographical position could be determined by magnetic methods: 'I rather mistrust this idea, although I believe that there may be much still to discover concerning the magnetic force of the earth, and that this offers a greater field for the application of mathematics than has yet been supposed.' On the occasion of the *Naturforscherversammlung* (Conference on the Natural Sciences) at Berlin in 1828, von Humboldt added his influence to lead Gauss in this direction.

Volume 5 of the *Collected Works* of Gauss contains his two famous geomagnetic memoirs: 'Intensitas vis magneticae terrestris ad mensuram absolutam revocata' (1832), and 'Allgemeine Theorie des Erdmagnetismus' (1838), both products of his mature years. The perfection of the form of exposition, characteristic of Gauss's works, veils the course of their development; this may, however, be traced from his letters to Olbers, Schumacher, and Gerling, in the years 1832–3, which Schering has published. We quote some extracts from these.

Writing to Olbers on February 18, 1832, he says:

'I occupy myself at present with geomagnetism, especially with the absolute determination of the magnetic intensity. Friend Weber [Wilhelm] is making the experiments according to my plan. Just as, for example, a clear conception of speed requires reference to a time *and* a length, so, I find, a complete specification of the intensity of geomagnetism requires reference to a weight and a length.'

Writing to Schumacher on March 3, 1832, he says:

'Once the possibility of determining the absolute magnitude of geomagnetism is realized, one must carry this out at a number of places distributed all over the earth; then travelling observers, carrying with them constant magnets, can determine the ratio of the intensity at other places, also testing the constancy of their magnets from time to time, by measurements at the places where absolute determinations have been made; thus all their measures can be expressed in absolute units. This is of special importance for future centuries, in which we may expect that there will be changes in the absolute geomagnetic intensity, as important as we have long known in the magnetic declination and inclination. I have always regarded these great changes as something truly remarkable. Undoubtedly the geomagnetic force is not due to a few great magnets near the earth's centre, which gradually shift their positions through many miles, but rather it is the result of all the polarized particles of iron contained within the earth, owing more to those near the surface than to those near the centre. But what is one to think of the enormous changes which have taken place in a couple of centuries? This phenomenon has always seemed to me to favor the hypothesis, specially advocated by Cordier, that the solid crust of the earth is comparatively thin. Of course the magnetic forces could only have their seat in this crust, the gradual thickening of which, as hitherto fluid layers solidify, would give a quite unforced explanation of the great geomagnetic changes. This also seems to be indicated by the fact that the so-called principal magnetic poles of the earth are situated in the coldest regions, where presumably the earth's crust is thickest.'

On April 2, 1832, he wrote as follows to Gerling:

'I have been very busy with geomagnetism for about a month, attempting not only to carry out (with much help from friend Weber) the ideas I described to you at Christmas, but also to push them much further. Almost every day I light on some new idea, and only regret that the execution lags behind, because it requires now this, now that, auxiliary point first to be rather extensively developed. But even as things now stand, they far surpass my expectations. Already I can almost from minute to minute follow the daily variation, and make changes of a few seconds of arc (e.g. 2 or 3) clearly visible. I hope to be able altogether to transcend the previous accuracy in the measurement, at any moment, of the intensity, the declination, and the inclination, and of their variations. Already now I can determine the time of oscillation with almost incredible accuracy.'

(In this work Gauss was using the mirror-and-scale method of reading, introduced six years earlier, in 1826, by Poggendorff.)

In a letter to Olbers of November 20, 1833, Gauss reported the first installation of the electric telegraph, and also mentioned that the Göttingen magnetic observatory (of non-magnetic construction) was finished, and that for the variometers he was intending to use magnets 2 feet long weighing 4 lb.

26.13. Magnetic observatories and observations. During the late eighteenth and early nineteenth centuries huts of non-magnetic (iron-free) construction for geomagnetic observation began to be built. Walker [20] mentions observations by John Macdonald at Fort Marlborough, Sumatra, in 1794 (published in the *Philosophical Transactions* for 1796 and 1798); Macdonald's description of his observatory is as follows:

'A small building, devoid of iron, was erected at some distance from my house. This building contained the meridian. By repeated observations I found the sphere of attraction of the needle and iron to be very small; a common-sized key presented to it did not affect it at a further distance than five inches. To exclude every influence of iron, I laid aside such as might be about me, previous to taking the observations, and took care to remove the padlock and key at the same time to a certain distance from the constructure. The hasp and staple made use of for the padlock were of wood, and for the nocturnal observations I had wooden candlesticks made.'

Macdonald found that the daily range of the declination is smaller in the tropics than in temperate latitudes. This was confirmed by the French admiral Duperrey, as described by Walker [20] as follows:

'He took his observations, every 15 minutes, with a diurnal variation compass, by the celebrated French maker Gambey, which was placed at his service by the Bureau des Longitudes.

'The instrument consists of a magnetic needle, or bar, of magnetised steel, in the form of a rectangular parallelepiped, suspended horizontally by an assemblage of thread of silk devoid of torsion. Near each end of the needle, and on its upper face, is fixed a small plate of ivory, on which is traced the index line of the needle. From each side of this index proceeds a series of graduations, which, for the instrument in question, are at intervals of 25 of a millimètre.

'The motion of the needle is observed by means of two microscopes, placed with their optical axes vertical, one over each end of the needle; the tubes in which these microscopes slide being capable of a lateral motion by means of adjusting screws, so that the fixed wires of the microscopes can always be made to coincide with the index line of the needle. By the side of each microscope is fixed a scale, graduated to millimètres, and parts of a millimètre, and each scale is furnished with a vernier which moves with the microscope by means of the adjusting screw before mentioned. The reading-off is assisted by means of a lens. To preserve the needle from currents of air, it is inclosed in a wooden box, having at each end, on its upper face, an aperture covered with glass, through which the needle is observed. The suspension thread is also inclosed in glass. The whole rests on a solid block of stone or marble.'

At Greenwich Observatory the magnetic declination was read thrice daily, using a Dollond magnet, from June 1818 to December 1920 (see *Astronomical Observations* by John Pond, Astronomer Royal, and also *Greenwich Observations, 1935*). Magnetic work at Greenwich was then discontinued until 1841.

Arago observed the declination at the Paris (Astronomical) Observatory from 1820 to 1835, using Gambey's instrument. He made over 50,000 individual (eye) observations, mostly at 15-minute intervals near the times of the maxima and minima of the daily variations (see Walker [20]). He noted that often, on days when at Paris there were observed great deviations from the normal course of the daily variation, polar lights (not observable at Paris) were seen at distant places.

Barlow's observations also are noteworthy, because he magnified the range of the swing of the needle by making the latter astatic; he reduced the earth's field near the swinging needle, by placing two magnets close by, in the magnetic meridian, but pointing southwards. Walker [20] remarks that Haüy (1817) had introduced this method.

After his return from his South American journey (1799–1803), Alexander von Humboldt organized simultaneous magnetic observations all over the globe, thus renewing and extending the practice of Celsius and Graham of 1741 (§ 10). Eight periods (per year) for such observations were pre-arranged, each of 44 hours length, when the needle was observed at least hourly. Among these observations, those at Freiberg in Saxony are noteworthy because they were made in a mine; they showed that the daily variation of declination proceeded in the same way at depths where there was no daily variation of temperature, as at the surface. Cassini in 1782 had already shown this by observations made in a cellar (80 feet below ground) at the Paris observatory.

In 1828 von Humboldt constructed his own iron-free magnetic observatory at Berlin.

With reference to Gauss's application of Poggendorff's method of mirror-and-scale readings, von Humboldt, in 1838, wrote the following letter, recently republished in *Nature* [26]:

'On February 12, 1838, at a meeting of the Royal Geographical Society, a letter from Alexander von Humboldt, dated Berlin, January 10, 1838, was read. . . . On the magnetic observations being made at Berlin, he said, "As we make observations here, both with the needle of Gambey, furnished with microscopes, and with the new apparatus of Gauss, furnished with a mirror, we have an opportunity of convincing ourselves more and more of the greater perfection of the latter apparatus, which by degrees will be employed in all our great observatories.

'"As I think that the subject is not without importance to seamen, I beg to invite the influential members of your Society to be good enough to propagate Gauss's manner of observing in all new stations where intelligent people can be found. Points near the magnetic equator and those which are in high latitudes in the southern hemisphere . . . would be most desirable if they would observe at the same epochs indicated by M. Gauss."'

26.14. The Göttingen Magnetic Union. Gauss and Weber [27], at the new Göttingen observatory, began in March 1834 to participate in von Humboldt's scheme of simultaneous observations. Later they considered it necessary to make observations oftener than once an hour, and proposed 5-minute intervals. A number of observatories associated themselves with this proposal in what became known as the *Magnetische Verein* (Göttingen Magnetic Union). Three to six periods of simultaneous observation, each extending over 24 hours, were agreed upon annually.

These observations were limited to the declination, which, Gauss remarked, was the geomagnetic element of most practical interest for seamen and geodesists; but he expressly added that the real reason for limiting the simultaneous observations to the declination was because at that time (1836) there were no instruments suitable for the measurement of the other elements. 'The search for the laws of natural phenomena is an end and has a value in itself for the natural philosopher, and a peculiar charm invests the recognition of order and harmony in happenings that apparently are quite irregular. The apparatus now used for observing the declination leaves nothing to be desired in the certainty, accuracy and ease with which the wonderful and ever-varying changes in this element can be followed; but the same cannot be said for the instruments hitherto used to measure the other elements. At present, therefore, it would be premature to include these in the scope of organized observations. But they will claim the same attention from natural philosophers, as the changes of the declination now receive, as soon as instrumental means are perfected, by which these other elements also, and especially their quickly changing variations, can be recognized with certainty, followed with ease, and measured with accuracy. Let us hope that this time is now not far distant.'

The rich content of the six Annual Reports of the Magnetic Union, 1836–41, may be briefly indicated as follows:

1836: The construction of magnetic observatories; description of instruments; procedure for the simultaneous observations; description of a small apparatus for the absolute measurement of H by travellers.

1837: Description of a bifilar variometer for H; on the influence of temperature upon the magnetization of bars; procedure in determining the time of oscillation (the 'eye and ear' method); the rotational dip inductor; the reduction of magnetometer-readings to absolute declination (introduction of the base-line value and the scale-value).

1838: General theory of geomagnetism; the transportable magnetometer; the inductor as magnetometer; the rotational inductor; magnetism in soft iron.

1839: General theorems relative to the inverse-square law of attraction and repulsion (potential theory); a method of simplifying the observation of magnetic deflexions; unipolar induction; the magnetic apparatus at the Prague astronomical observatory (by C. Kreil).

1840: Various theorems on the bifilar-magnetometer; measurements of local magnetic variations in the garden of the Göttingen astronomical observatory, with a transportable magnetometer; articles by Hansteen, Goldschmidt, Listing.

1841: Various instrumental questions; observations at Göttingen during several aurorae, every 10 or 20 seconds.

The fifty stations which observed more or less regularly during the 28 chosen periods (3 in 1836, 7 in 1837, 6 in 1838, 4 in each year 1839–41) have been listed by Schering [28] as follows, with the number of periods in which they participated:

Europe: Alten (Finnmarken) 3; Altona-Hamburg 1; Augsburg 2; Bellsund (Spitzbergen) 1; Berlin 27; Breda (Holland) 25; Breslau 27; Brussels 8; Katharinenburg 3; Christiania 3; Copenhagen 24; Dublin 6; Freiberg (Saxony) 4; Geneva 1; Göttingen 28; Greenwich 5; Haag 2; Hammerfest 1; Hanover 4; Havösund (Finnmarken) 1; Heidelberg 25; Kierisvara (Lapland) 1; Cracow 8;

Kremsmünster 10; Kuopio (Finland) 1; Leipzig 28; Milan 26; Marburg 26; Makerstoun 2; Munich 18; Petersburg 9; Prague 10; Seeberg near Gotha 4; Stockholm 6; Upsala 28.

Asia: Barnaul (Siberia) 2; Madras 4; Nertschinsk 3; Simla 4; Singapore 3; Trevandrum 3.

Africa: St. Helena 7; Cape of Good Hope 3.

North America: Cambridge 3; Philadelphia 1; Toronto 7.

South Sea: Auckland Island 1; Kerguelen 1; New Zealand 1; Van Diemen's Land (Tasmania) 5.

The 'term-observations' (as they were called) which were made at Washington under this scheme were published *in extenso* [29].

26.15. E. Sabine and the British Colonial observatories. A. von Humboldt, in a letter written in April 1836 to the Duke of Sussex (who in 1786–91 had studied at Göttingen, and who in 1836 was President of the Royal Society of London), urged the institution of magnetic observatories in the British Colonies (this letter, which summarized the then existing state of geomagnetic science, has been reprinted by Schering [28]).

In a circular of the Royal Society dated July 1, 1839, and reprinted in the *Göttinger Resultate* for 1838, we read that Her Majesty's Government had ordered the equipment (then in progress) of a naval expedition of discovery, consisting of two ships under the command of Captain James C. Ross, to proceed to the Antarctic Seas for purposes of magnetic research; and also the establishment of fixed magnetic observatories at St. Helena, Montreal, the Cape of Good Hope, and Van Diemen's Land, having for their object the execution of a series of corresponding magnetic observations during a period of three years, in consonance with the views expressed in that Report (made by the Joint Committee of Physics and Meteorology of the Society to the Council on the subject of an extended system of Magnetic Observation). The Court of Directors of the Honourable East India Company had also, in compliance with the suggestions of the Royal Society, resolved to establish similar observatories at Madras, Bombay, and at a station in the Himalaya Mountains.

The circular stated that observations would be made at the times agreed on, and enumerated the equipment; instructions for the use of the magnetic and meteorological observatories were issued. Edward Sabine was entrusted with the superintendence of the four observatories at Toronto, St. Helena, Cape of Good Hope, and Hobarton. In a report 'On what the Colonial Magnetic Observatories have accomplished' [31], he says:

'Not, as previously, limited to observations of a single element (the Declination),—or combining at the most one only of the components of the magnetic force,—the instructions of the Royal Society, and the instrumental means prepared under its direction, provided for the examination, in every branch of detail, of each of the three elements which, taken in combination, represent, not partially but completely, the whole of the magnetic affections experienced at the surface of the globe, classed under the several heads of absolute values, secular changes, and variations either periodical or occasional,—and proceeding from causes either internal or external.... The Report of the Committee of Physics stated in a very few sentences, remarkable alike for their comprehensiveness and conciseness, the desiderata of magnetical science.'—'The observations will naturally refer themselves to two chief branches, into which the science of terrestrial magnetism in its present state may be divided. The first comprehends the actual distribution of the magnetic influence over the globe, at the present epoch, in its mean or average state, when the effects of temporary fluctuations are either neglected, or eliminated by extending the observations over a sufficient time to neutralize their effects. The other comprises the history of all that is not permanent in the phenomena, whether it appear in the form of momentary, daily, monthly, or annual change and restoration; or in progressive changes not compensated by counter-changes, but going on continually accumulating in one direction, so as in the course of many years to alter the mean amount of the quantities observed.'

Meanwhile Schwabe, from his own sunspot observations during the years 1826–50, had derived the conclusion that the variation of the number of sunspots was periodic, with a period of about 10 years, maxima having occurred in 1828, 1837, and 1848, and minima in 1833 and 1843.

The most notable result of Sabine's magnetic work was his discovery [32] that, during the years 1841–8, the magnetic disturbances varied in intensity in parallel with the sunspot variation. He gave the following comparison between the average disturbance variation in the declination at Toronto (expressed as a ratio relative to the average disturbance intensity), and the number of sunspot groups according to Schwabe:

	1841	*1842*	*1843*	*1844*	*1845*	*1846*	*1847*	*1848*
Disturbance .	1·38	0·91	0·50	0·73	0·63	1·27	1·42	1·45
Spot groups .	102	68	34	52	114	157	257	330

Sabine made the following cogent comment on this comparison [31]: 'The discovery of a connexion of this remarkable description, giving apparently to magnetism a much higher position in the scale of distinct natural forces than was previously assigned to it, may justly be claimed on the part of the Colonial Observatories, as the result of the system of observation enjoined (and so patiently and carefully maintained), and of the investigation for which it has supplied the data.'

Sabine also calculated the lunar daily magnetic variation for Toronto, St. Helena, and Hobarton (Ch. IX); he ascribed the discovery of the moon's influence on geomagnetism to C. Kreil [33], who, in 1850, had calculated it from bi-hourly observations of declination at Prague (taken ten times daily between 10 a.m. and 10 p.m.) in the years 1840–9. Kreil found that the declination in the average over all lunar phases underwent a lunar semi-diurnal wave; at the lunar culmination the magnetic needle, in the annual mean, was about 20″ more to the west of its position 6 hours earlier or later. In the summer months this lunar semi-diurnal range was greater (44″); in winter it seemed to be 'hidden by irregular fluctuations'.

Sabine, in his report [31], clearly expressed his general conception of the duties of a magnetic observatory, in a passage which is still worthy to be read.

26.16. The introduction of photographic recording. Regular magnetic registration by photography was first introduced in 1847, at the Royal Observatory, Greenwich, under the direction of the then Astronomer Royal, G. B. Airy [43], and according to arrangements devised by Charles Brooke, who in 1848 was awarded a prize of £500 offered by the British Admiralty for the invention of satisfactory magnetic self-recording arrangements. In the first of three fundamental memoirs [34] on this subject, Brooke wrote:

'An efficient method of continuously registering the variations of magnetometers having been generally admitted to be a desideratum in science at the Magnetic Conference held during the Meeting of the British Association at Cambridge, it became a matter of philosophical interest to supply the deficiency, and this, it is hoped, has by the following means been satisfactorily accomplished.

'As the mechanical force which a suspended bar magnet is capable of exerting, during its variations of direction, is far too minute to actuate the most delicate mechanism without sensibly affecting its position, the desired object will probably be obtained by photography alone.'

The original arrangement at Greenwich (a camphor lamp with slit, a rotating cylinder carrying the photographic paper, on which a base-line was produced by a separate lamp) was very successful. One of the first published magnetograms, taken at Brooke's house at 29 Keppel Street, London, on April 6, 1846, shows a great disturbance of about 30′ in declination. Brooke clearly distinguished between motions of the needle due to changes in the magnetic field, and those due to local mechanical tremors (pointing out the effects on his magnetogram of a quadrille party in the adjoining house). He introduced automatic temperature compensation of the force magnetometers in 1851.

26.17. Summary of important events in the history of geomagnetism up to about 1850.

A.D. 1030–93.	The Chinese encyclopaedist Shon-Kua described the magnet pointing south.
About 1187.	Alexander Neckam of St. Albans described the magnetic compass.
About 1450.	Sun-dials made in Nuremberg showed marks for magnetic declination.
About 1492.	German road maps containing the figure of a compass indicating the declination.
1538–41.	João de Castro, on a voyage to the East Indies, made forty-three determinations of magnetic declination.
1544.	Letter by Georg Hartmann of Nuremberg, referring to the magnetic inclination.
1576.	Robert Norman, *The Newe Attractiue*.
1600.	William Gilbert, *De Magnete*.
1635.	Henry Gellibrand discovered the secular variation of declination.
1672.	Daniel Tilas died (inventor of the Swedish mining compass for magnetic propecting).
1698–1700.	Halley's voyages in the Atlantic Ocean on the *Paramour Pink*.
1701.	Halley's sea chart of the whole world.
1721.	William Whiston's charts of inclination.
1722.	George Graham discovered non-secular time-variations.
1741 April 5.	Simultaneous magnetic observations by Celsius at Upsala and Graham at London; discovery of the relation between magnetic disturbance and aurora.
1759.	Solar daily variation found to be greater in summer than in winter (by Canton).

1770.	Wilcke observed that auroral rays are parallel to the magnetic inclination.
1782.	Cassini found that the daily variation of declination is independent of the daily variation of air-temperature.
1799–1804.	A. von Humboldt's expedition to America.
1819.	Hansteen's *Untersuchungen über den Magnetismus der Erde.*
1820–35.	Arago's observations of the magnetic declination at Paris.
1826.	Poggendorff introduced readings by means of mirror and scale.
1837.	The earth inductor invented by Weber.
1838.	Gauss's *Allgemeine Theorie des Erdmagnetismus.*
1839.	Lloyd introduced the magnetic balance for recording the variations of the vertical intensity.
1836–41.	The Göttingen Magnetic Union.
1839.	Establishment of the British Colonial Observatories (Sabine).
1846.	Charles Brooke constructed photographic apparatus recording magnetic variations.
1850.	Kreil found the lunar daily variation of declination at Prague.
1851.	Schwabe discovered the sunspot cycle.
1852.	Sabine found the effect of the sunspot cycle in the disturbances of declination at Toronto, 1841–8.

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The General list is itself divided into sections under specific headings, and several of the Chapter lists are also divided (by a slight space) into sections (without headings) comprising groups of papers on kindred topics.

The references in each list or section of the bibliography are not in alphabetical order, but to facilitate the search for references to papers by particular authors the author's index at the end of this volume gives (besides the pages of the text where each author's name is quoted) the list-numbers of each author's papers quoted in the bibliography: as these numbers include the letter G or the chapter number it is easy to look up the bibliographical particulars of any paper included in the bibliography, if the author and the subject are known.

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PART B

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CHAPTER XV. THE EARTH'S ATMOSPHERE

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CHAPTER XXVI. HISTORICAL NOTES

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TABLE A

List of Geomagnetic Observatories

giving (i) their *geographical* coordinates, namely, the north latitude ϕ and the longitude λ east of Greenwich; (ii) their *geomagnetic* coordinates, relative to the geomagnetic north-pole B at $\phi = 78{\cdot}5°$, $\lambda = 291°$ E., and the corresponding geomagnetic equator: namely the north geomagnetic latitude Φ, which is the angular distance from the geomagnetic equator, and the geomagnetic east longitude Λ, which is the angle between the geographical meridian extending southwards from B, and the great circle through B and the station. The table also gives the angle ψ formed by this great circle with the geographical meridian of the station, eastward positive.

The table lists all permanent observatories operating now or in the past, also many stations at which a series of consecutive magnetic observations has been taken for at least a few months, on expeditions, during surveys, etc. New sites of observatories which had to be transferred because of artificial disturbances, etc., are marked *a* or *b*. The letter C before the station-name signifies that the observatory contributed to the International Magnetic Character Figure in the year 1938. The last column gives some approximate information about the year at which observations began and ended.

The table is an extension of that given by A. Schmidt in *Archiv des Erdmagnetismus*, Heft 4 [G 52].

The observatories are listed in decreasing order of north latitude.

No.		*Station*	ϕ	λ	Φ	Λ	ψ	*Years*
			°	°	°	°	°	
1		Fort Conger	81·7	295·2	86·7	169·3	−165·2	1881–3
2	C	Calm Bay (Tikhaya)	80·3	52·8	71·5	153·3	−32·2	1931...
3		Cap Thordsen	78·5	15·7	74·5	131·7	−48·1	1882–3
4		Sveagruvan	77·9	16·8	73·9	130·7	−46·2	1932–3
4*a*		Chelyuskin	77·3	104·3	65·9	177·5	−3·2	1934
5		Axelöen	77·7	14·8	74·2	129·1	−46·5	1902–3
6		Thule	76·5	291·1	88·0	0	0	1932...
7		Bear Island	74·5	19·2	71·1	124·5	−37·9	1932–3
8	C	Dickson	73·5	80·4	63·0	161·5	−12·8	1932–3
9		Ssagastyr	73·4	126·6	62·2	189·5	+6·6	1882–4
10	C	Matotchkin Shar	73·3	56·4	64·8	146·5	−22·4	1882–3, [1932...
11		Point Barrow	71·3	203·3	68·6	241·2	+33·0	1881–3, [1932–3
12		Jan Mayen	71·0	351·5	73·4	96·3	−37·5	1882–3, [1932–3
13		Scoresby Sund	70·5	338·0	75·8	81·8	−36·2	1932–3
14		Bossekop	70·0	23·2	66·6	120·4	−30·2	1882...
14*a*		Kaafjord	69·9	23·0	66·6	120·1	−30·2	1902–3
15	C	Tromsö	69·7	18·9	67·1	116·7	−30·8	1932...
16		Petsamo	69·5	31·2	64·9	125·8	−27·6	1932...
17		Godhavn	69·2	306·5	79·8	32·5	−17·5	1927...
18		King Point	69·1	221·9	70·2	259·5	+33·4	

No.		*Station*	*φ*	*λ*	*Φ*	*Λ*	*ψ*	*Years*
			°	°	°	°	°	
19		Gjöahavn	68·6	264·1	77·8	309	+25·2	1924
20		Kultala	68·5	26·8	64·7	121·3	−27·7	1882–4
21	C	Abisko	68·4	18·8	66·0	116	−29·3	
22	C	Sodankyla	67·4	26·6	63·8	120·0	−26·7	1882...
23		Kandalakcha	67·1	32·4	62·5	124·2	−25·0	1932–3
24		Kingua Fjord	66·6	292·7	78·1	3·2	− 1·6	1882–3
25		Dyra Fjord	66·2	337·5	72·2	73·0	−28·3	1902–3
26	C	Wellen	66·2	190·2	61·8	237	+24·5	1933...
27		Angmagssalik	65·6	322·4	74·2	52·7	−22·5	1932–3
28		College-Fairbanks	64·9	212·2	64·5	255·4	+27·0	1932–3
29		Godthaab	64·2	308·3	74·8	29·6	−13·0	1882–3
30		Chesterfield Inlet	63·3	269·3	73·5	324	+14·9	1932...
31		Fort Rae (new)	62·8	243·9	69·0	290·9	+24·1	1932–3
31*a*		Fort Rae (old)	62·6	244·3	68·9	291·4	+23·8	1882–3
32	C	Srednikan	62·6	152·3	53·2	210·5	+12·7	1935...
33		Dombas	62·1	9·1	62·3	100·0	−23·6	1916...
34	C	Yakutsk	62·0	129·7	51·0	193·8	+ 5·8	1926...
35		Juliannehaab	60·7	314·0	70·8	35·6	−13·8	1932–3
36	C	Lerwick	60·1	358·8	62·5	88·6	−23·6	1923...
37		Petersburg	60·0	30·3	56·3	117·0	−20·8	1829–77
37*a*	C	Pawlowsk (Sloutsk)	59·7	30·5	56·0	117·0	−20·6	1878...
38		Christiania	60·0	10·7	60·0	99·8	−23·1	1841...
39	C	Lovö (Stockholm)	59·4	17·8	58·1	105·8	−22·1	1928...
40	C	Sitka	57·0	224·7	60·0	275·4	+21·4	1842...
41		Sverdlovsk	56·8	60·6	48·6	140·4	−13·4	1834...
41*a*	C	Vyssokaya Doubrawa	56·7	61·1	48·5	140·7	−13·3	1932...
42		Copenhagen	55·7	12·6	55·7	98·6	−20·5	1888...
42*a*	C	Rude Skov	55·8	12·4	55·8	98·5	−20·6	1907...
43		Kasan	55·8	49·1	49·2	130·6	−15·6	
43*a*	C	Zaimische	55·8	48·8	49·3	130·4	−15·7	1925...
44		Makerstoun	55·5	357·5	58·6	83·8	−20·5	1849–50
45	C	Eskdalemuir	55·3	356·8	58·5	82·9	−20·4	1908...
46	C	Königsberg (Gr. Raum)	54·8	20·5	53·3	105·3	−19·5	1929...
47	C	Meanook	54·6	246·7	61·8	301·0	+17·2	1916...
48	C	Hel	54·6	18·8	53·4	103·6	−19·5	
49		Lübeck	53·8	10·7	54·3	95·6	−19·6	1884–94
50	C	Stonyhurst	53·8	357·4	57·0	82·5	−19·6	1858...
51		Wilhelmshaven	53·5	8·2	54·5	92·9	−19·6	1882–1938
51*a*		Wingst	53·7	9·1	54·6	94·0	−19·6	1938...
52		Dublin	53·4	353·7	57·2	78·5	−19·1	1838...
53		Barnaul	53·3	83·8	42·9	158·1	− 7·1	1841...
54		Potsdam	52·4	13·1	52·5	97·1	−18·9	1890–1928
54*a*		Seddin	52·3	13·1	52·4	97·0	−18·9	1908–32
54*b*	C	Niemegk	52·1	12·7	52·2	96·5	−18·8	1932...
55		Irkutsk	52·3	104·3	40·8	174·6	− 1·7	1887...
55*a*	C	Zouy	52·5	104·0	41·0	174·4	− 1·8	1924...
56	C	Swider	52·1	21·2	50·6	104·6	−18·3	1914...
57		Utrecht	52·1	5·1	53·8	89·3	−18·9	1867–98
57*a*		De Bilt	52·1	5·2	53·8	89·6	−18·9	1898–1938
57*b*	C	Witteveen	52·8	6·7	54·2	91·0	−19·3	1938...
58		Valencia	51·9	349·8	56·6	73·5	−18·1	1887...
59		Clausthal	51·7	10·3	52·3	94·1	−18·7	1844...
60		Göttingen	51·5	9·9	52·3	93·7	−18·7	1833...
61	C	Nijnedewitzsk	51·5	38·4	46·9	119·6	−16·2	
62	C	Bochum	51·5	7·2	52·8	88·9	−18·7	1888...

No.		*Station*	ϕ	λ	Φ	Λ	ψ	*Years*
			°	°	°	°	°	
63		Kew	51·5	359·7	54·0	83·3	−18·4	1846–1920
64		Greenwich	51·5	0·0	54·2	83·8	−18·6	1838–1924
64*a*	C	Abinger	51·2	359·6	54·0	83·3	−18·4	1924...
65		Collm (Leipzig)	51·3	13·0	51·5	96·5	−18·5	1935...
66		Nertschinsk	51·3	119·6	39·9	187·0	+ 2·2	1841...
67		Breslau	51·1	17·0	50·5	100·2	−18·2	1870–96
68		Brussels (Uccle)	50·8	4·4	52·7	87·8	−18·4	1841...
68*a*	C	Manhay	50·3	5·7	52·0	88·8	−18·2	
69		Aachen	50·8	6·1	52·3	89·5	−18·4	
70		Hermsdorf	50·8	16·2	50·3	99·3	−18·1	1898...
71		Beuthen	50·3	18·9	49·4	101·6	−17·8	1893...
71*a*		Nikolai	50·1	18·9	49·3	101·5	−17·8	
72		Falmouth	50·2	354·9	54·0	77·9	−17·7	1885...
73		Prague	50·1	14·4	50·0	97·2	−18·0	1839...
74		Janow	49·9	23·7	48·0	105·8	−17·3	
75		Paris (Parc Saint Maur)	48·8	2·5	51·3	86·2	−17·6	1810..., [1882–1901
75*a*		Val Joyeux	48·8	2·0	51·3	84·5	−17·5	1901–36
75*b*	C	Chambon-la-Forêt	48·0	2·3	50·4	83·9	−17·2	after 1936
76		Munich	48·2	11·6	48·7	93·7	−17·4	1840...
76*a*		Maisach	48·2	11·3	48·9	93·4	−17·4	
76*b*		Fürstenfeldbruck	48·2	11·2	48·9	92·4	−17·4	1938...
77	C	Vienna	48·2	16·2	47·9	98·1	−17·2	1852...
78		Kremsmünster	48·0	14·1	48·1	96·0	−17·2	1839...
79		O'Gyalla	47·9	18·2	47·1	99·8	−17·0	1893...
79*a*		Amvrossievka	47·9	38·5	43·4	118·4	−15·2	1939
80	C	Regensberg (Zürich)	47·5	8·5	48·7	88·0	−17·2	
81		Besançon	47·2	6·0	49·0	87·7	−17·1	
82	C	Nantes	47·2	358·4	50·5	80·1	−16·8	1883...
83		Toyohara	47·0	142·8	36·9	203·5	+ 6·7	1932–3
83*a*		Otomari	46·7	142·8	36·6	206·8	+ 7·5	
84		Klagenfurt	46·6	14·3	46·7	95·6	−16·8	before 1893
85		Odessa	46·4	30·8	43·4	110·9	−15·7	1894...
85*a*	C	Stepanovka	46·8	30·9	43·8	110·9	−15·8	
86		Mailand	45·5	9·2	46·6	90·1	−16·5	1836...
87		Pola	44·9	13·8	45·1	94·4	−16·3	1896...
88		Belgrade	44·8	20·2	43·8	100·4	−16·0	
89		Bucharest	44·4	26·1	42·3	105·8	−15·6	1894...
90		Castellacio	44·4	8·9	45·7	90·0	−16·2	1932–3
91		Toronto	43·7	280·5	54·9	346·7	+ 3·6	1840...
91*a*	C	Agincourt	43·8	280·7	55·0	347·0	+ 3·6	1898...
92	C	Mai-Toon	43·2	132·3	32·4	198·3	+ 4·9	
93		Pic-du-Midi	42·9	0·1	46·0	80·1	−15·6	
94		Perpignan	42·7	2·9	45·2	82·8	−15·6	1885...
95		Tiflis	41·7	44·8	36·3	122·1	−13·1	1844...
95*a*		Karsani	41·8	44·7	36·4	122·0	−13·1	1926...
95*b*	C	Dousheti	42·1	44·7	36·7	122·1	−13·2	
96	C	Tashkent (Keles)	41·3	69·3	32·4	143·7	− 9·0	1884...
97		Capodimonte	40·9	14·3	41·2	93·3	−15·3	1881...
98		Tortosa	40·8	0·5	43·9	79·7	−15·0	1905...
99	C	Coimbra	40·2	351·6	45·0	70·3	−14·2	1866...
100		Philadelphia	40·0	284·8	51·4	352·4	+ 2·0	1840–5
101		Pekin	40·0	116·5	28·5	184·8	+ 1·3	1830...
102		Washington	38·9	283·0	50·3	350·2	+ 2·5	1840–2
102*a*	C	Cheltenham	38·7	283·2	50·1	350·5	+ 2·4	1901...

No.		*Station*	*φ*	*λ*	*Φ*	*Λ*	*ψ*	*Years*
			°	°	°	°	°	
103		Baldwin	38·8	264·8	48·9	328·5	+ 7·7	1900–9
104		Lisbon	38·7	350·9	43·7	69·0	−13·8	
105		Athens	38·0	23·7	36·6	101·4	−14·4	1899...
106	C	San Miguel	37·8	334·4	45·6	50·9	−11·3	1901...
107		Zinsen	37·5	126·6	26·4	193·8	+ 3·4	1926...
108	C	San Fernando	36·5	353·8	41·0	71·3	−13·6	1879...
109		Tsingtau	36·1	120·3	24·7	188·3	+ 2·0	1910...
110		Tokio	35·7	139·8	25·4	205·6	+ 6·1	1897–1913
110*a*	C	Kakioka	36·2	140·2	26·0	206·0	+ 6·2	1913...
111		Simoda	34·7	138·9	24·4	205·0	+ 5·9	1932–3
112		Los Angeles (Cal.)	34	242	..	..	..	1882–9
113	C	Ksara	33·8	35·9	30·2	111·7	−12·9	1934...
114		Aso	32·9	131·0	22·0	198·0	+ 4·2	1932–3
115	C	Tucson	32·2	249·2	40·4	312·2	+10·1	1909...
116		Zikawei	31·2	121·4	19·9	189·5	+ 2·2	1874–1907
116*a*		Lukiapang	31·3	121·0	20·0	189·1	+ 2·1	1907–33
116*b*	C	Zo-Sè	31·1	121·2	19·8	189·2	+ 2·2	1934...
117		Simla	31·1	77·2	21·4	149·3	− 6·8	1841...
117*a*	C	Dehra Dun	30·3	78·0	20·5	149·9	− 6·6	1902...
118	C	Helwan	29·9	31·3	27·2	106·4	−12·7	1907...
119		Taihoku	25·0	121·5	13·7	189·8	+ 2·2	
120		Havana	23·1	277·6	34·3	345·1	+ 3·2	
121		Barrackpore	22·8	88·4	12·1	158·7	− 4·5	1903...
121*a*		Tamanrasset	22·8	5·5	25·4	80·6	−12·4	1932
122		Hongkong	22·3	114·2	10·8	183·0	+ 0·6	1884...
122*a*	C	Au Tau	22·4	114·0	11·0	182·9	+ 0·6	
123	C	Honolulu	21·3	201·9	21·1	266·5	+12·3	1901...
124		Cuajimalpa (Mex.)	19·3	260·8	29·1	327·1	+ 6·6	before 1895
124*a*		Teoloyucan	19·8	260·8	29·6	327·0	+ 6·6	1932
125		Toungoo	18·9	96·4	7·8	166·1	− 2·9	1904...
126		Bombay (Colaba)	18·9	72·8	9·5	143·6	− 7·2	1841–1904
126*a*	C	Alibag	18·6	72·9	9·5	143·6	− 7·2	1904...
127		Vieques (Porto Rico)	18·2	294·6	29·6	3·9	− 0·8	1903–25
127*a*	C	San Juan	18·4	293·9	29·9	3·2	− 0·7	1926...
128	C	Manila (Antipolo)	14·6	121·2	3·3	189·8	+ 2·0	1890...
129		Madras	13·1	80·2	3·1	150·1	− 5·9	1841–60
✓130		Kodaikanal	10·2	77·5	0·6	147·1	− 6·3	1902...
131		Trevandrum	8·5	77·0	1·0	146·4	− 6·4	1841–69
132		Palau	7·3	134·5	− 3·2	203·3	+ 4·6	
133		Moka, Fernando Po	3·4	8·7	5·7	78·6	−11·3	1932–3
134		Mogadiscio	2·0	45·4	− 2·7	114·3	−10·5	1932–3
135		Singapore	1·3	103·8	−10·1	172·7	− 1·5	1841–5
136		Tatuoca	− 1·2	311·5	9·6	20·8	− 4·1	
137		Batavia	− 6·2	106·8	−17·6	175·6	− 0·9	1867...
137*a*		Buitenzorg	− 6·6	106·8	−18·0	175·6	− 0·9	1901...
137*b*	C	Kuyper	− 6·0	106·7	−17·5	175·5	− 0·9	1928...
138		Ascension	− 7·9	345·6	− 1·2	53·8	− 9·4	1863–6
139		S. Paul de Loanda	− 8·8	13·2	− 7·1	80·6	−11·5	
140		Elisabethville	−11·7	27·5	−12·7	94·0	+11·7	1932–3
141	C	Huancayo	−12·0	284·7	− 0·6	353·8	+ 1·3	1922
142	C	Samoa (Apia).	−13·8	188·2	−16·0	260·2	+11·7	1902...
143		St. Helena	−16·0	354·3	−10·6	60·9	−10·4	1841–9
144		Tananarivo	−18·9	47·5	−23·7	112·4	−11·2	1902...
145		Mauritius	−20·1	57·6	−26·6	122·4	−10·3	1875...
146		La Quiaca	−22·1	294·4	−10·6	3·2	− 0·7	

No.		*Station*	*φ*	*λ*	*Φ*	*Λ*	*ψ*	*Years*
			°	°	°	°	°	
147		Rio de Janeiro	−22·9	316·4	−12·5	24·3	− 5·1	1882...
147*a*		Vassouras	−22·4	316·4	−11·9	23·9	− 5·0	1915...
148	C	Watheroo	−30·3	115·9	−41·8	185·6	+ 1·3	1919...
149	C	Pilar	−31·7	296·1	−20·2	4·6	− 1·1	1904...
150		Santiago	−33·4	289·3	−22·0	358·5	+ 0·4	
151	C	Cape Town	−33·9	18·5	−32·7	79·9	−13·7	1841...
152		Melbourne	−37·8	145·0	−47·0	220·3	+ 9·4	1858...
152*a*	C	Toolangi	−37·5	145·5	−46·7	220·8	+ 9·5	1910...
153		Hobarton	−42·9	147·5	−51·7	224·6	+11·0	1841–8
154		Christchurch	−43·5	172·6	−48·0	252·6	+15·2	1901–30
154*a*	C	Amberley	−43·5	172·7	−47·7	252·5	+15·1	1929...
155		Kerguelen	−49·4	69·9	−57·4	127·6	−14·1	1902–3
156		Magallanes	−53·2	289·1	−41·6	358·5	+ 0·5	1932–3
157		South Georgia	−54·5	324·0	−44·5	26·3	− 8·8	1882–3
158		New Year's Island	−54·6	295·8	−43·2	3·8	− 1·3	
159		Orange Bay	−55·5	291·9	−44·0	0·7	− 0·2	1882–3
160		Laurie Island	−60·8	315·0	−50·0	18·0	− 7·2	1903...
161		Gauss-station	−66·0	89·6	−76·1	141·9	−17·6	1902–3
162		Cape Denison	−67·0	142·7	..	..	..	1912–13
163		Discovery Bay	−77·8	166·8	−79·0	294·4	+59·6	1902–3
163*a*		Cape Evans	−77·6	166·4	−78·9	293·2	+58·8	1911–12
164		Little America	−78·6	196·2	−74·6	312·1	+48·4	1929

TABLE B

Annual Means of Geomagnetic Elements at Observatories

The number of each station is the same as in Table A. The earliest and latest annual mean known to the authors has been given, also (if possible) one or more annual means a few years earlier than the latest one, to indicate the present trend of the secular variation. Declination *D* positive towards east of north; Inclination *I* and vertical intensity *Z* positive if the north end of the needle points downward. The letter *x* denotes approximate or preliminary values which are not derived from continuous records.

For further information see J. A. Fleming [G 62 or 41 *a*].

No.	*Observatory*	*Year*	*D*		*I*		*H*	*Z*
			°	′	°	′	γ	γ
2	Calm Bay	1934	+21	27·8	+83	8·0	6567	+54529
8	Dickson	1934	+28	29·4	+83	6·8	6909	+57208
10	Matotchkin Shar	1924	+20	37·5	+80	5·4	9491	+54326
		1934	+21	49·3	+80	31·6	9046	+54213
15	Tromsö	1931	− 3	59·6	+77	2·6	11548	+50198
		1937	− 2	53·7			11350	+50308
17	Godhavn	1927	−58	28·4	+81	34·7	8255	+55769
		1935	−56	13·8	+81	34·2	8193	+55285
22	Sodankyla	1914	+ 0	18·3	+75	19·2	12905	
		1923	+ 1	30·6	+75	42·6	12529	+49189
		1933	+ 3	3·9	+76	11·7	12110	+49278
36	Lerwick	1926	−15	2·8	+72	37·1	14618	+46699
		1933	−13	34·0	+72	44·6	14477	+46605
37*a*	Sloutsk	1890	− 0	12·1	+70	44·1	16418	
		1925	+ 3	25·3	+71	27·1	15770	+47000
		1935	+ 4	38·4	+72	2·2	15370	+47404
39	Lovö	1929	− 3	8·3	+71	24·9	15584	+46344
		1933	− 2	30·6	+71	36·5	15459	+46494
40	Sitka	1902	+29	51·1	+74	47·8	15441	+56822
		1925	+30	27·2	+74	22·2	15524	+55488
		1935	+30	2·7	+74	20·1	15450	+55090
41	Sverdlovsk	1890	+ 9	22·3	+70	36·3	17810	
		1931	+10	54·6	+72	26·9	16200	+51220
41*a*	Vyssokaya Doubrawa	1932	+12	49·9	+72	8·6	16312	+50634
		1935	+12	51·5	+72	17·7	16210	+50781
42*a*	Rude Skov	1907	− 9	48·4	+68	44·0	17423	+44765
		1925	− 6	57·7	+69	7·2	17025	+44631
		1935	− 5	8·8	+69	29·6	16804	+44927
43*a*	Zaimische	1925	+ 8	57·0	+70	12·2	17260	+47951
		1935	+ 9	15·4	+70	53·2	16790	+48460

No.	*Observatory*	*Year*	*D*	*I*	*H*	*Z*
			° ′	° ′	γ	γ
45	Eskdalemuir	1909	−18 30·1	+69 38·9	16835	+45385
		1924	−16 1·2	+69 38·7	16673	+44938
		1934	−14 0·6	+69 45·9	16536	+44859
47	Meanook	1916	+27 46·7	+77 55·2	12944	+60481
		1923	+27 23·3	+77 53·2	12881	+60025
		1933	+26 21·9	+77 54·0	12736	+59411
48	Hel	1934	− 2 35·5	+68 25·2	17563	+44384
50	Stonyhurst *x*	1890	−19 16·2	+69 8·4	17083	
		1925	−14 53·4	+68 42·2	17263	+44282
		1935	−12 53·2	+68 50·7	17148	+44311
51	Wilhelmshaven	1900	−12 27·7	+67 44·0	18095	+44193
		1931	− 7 54·5	+67 52·6	17826	+43851
55*a*	Zouy	1916	+ 1 20·7	+71 2·5	19396	+56463
		1929	+ 0 20·2	+71 19·2	19038	+56310
54	Potsdam	1890	−10 48·7	+66 47·8	18616	+43426
		1908	− 9 18·0	+66 19·3	18852	+42990
54*a*	Seddin	1908	− 9 19·2	+66 16·2	18890	+42974
		1931	− 5 28·9	+66 49·8	18450	+43106
54*b*	Niemegk	1931	− 5 36·2	+66 41·6	18526	+42999
		1936	− 4 45·3	+66 52·1	18464	+43220
56	Swider	1921	− 3 30·3	+66 34·4	18712	+43185
		1933	− 1 31·9	+67 9·3	18420	+43724
57*a*	De Bilt	1900	−13 50·6	+66 57·0	18508	+43490
		1926	−10 13·1	+66 55·5	18337	+43040
		1936	− 8 21·4	+67 6·9	18236	+43202
58	Valencia *x*	1899	−21 35·0	+68 33·0	17739	
	(Cahirciveen)	1924	−18 34·9	+68 0·6	17854	+44213
		1934	−16 43·7	+67 57·5	17812	+43993
64	Greenwich	1862	−20 52·6	+68 9·6	17630	+44030
		1925	−13 9·9	+66 51·4	18410	+43080
64*a*	Abinger	1925	−13 22·7	+66 35·1	18597	+42946
		1935	−11 30·3	+66 40·9	18527	+42981
74	Janow	1934	+ 0 12·7	+64 53·9	20081	+42863
75*a*	Val Joyeux	1901	−15 12·0	+64 58·9	19680	+42167
		1926	−11 43·8	+64 39·2	19650	+41482
		1936	− 9 56·7	+64 45·4	19647	+41668
75*b*	Chambon-la-Forêt	1936	− 9 28·9	+64 11·3	20011	+41374
76*a*	Maisach *x*	1927	− 6 52·5	+63 32·5	20314	+40817
		1932	− 5 59·3	+63 39·8	20299	+41005
77	Vienna *x*	1931	− 3 53·3	+63 29·0	20506	+41099
		1934	− 3 25·8	+63 34·4	20501	+41252
82	Nantes	1923	−13 23·5	+63 45·8	20212	+41009
		1934	−11 22·9	+63 43·1	20245	+40995
83	Toyohara	1933	− 8 36·8	+60 41·3	25035	+44591

No.	*Observatory*	*Year*	*D*		*I*		*H*	*Z*
			°	′	°	′	γ	γ
85	Odessa	1899	− 4	36·7	+62	18·2	21869	
		1925	− 1	36·4	+63	18·9	21213	+42206
87	Pola	1900	− 9	25·8	+60	15·9	22192	+38852
		1922	− 6	28·0	+60	12·8	22090	+38591
91*a*	Agincourt	1900	− 5	28·8	+74	31·6	16497	+59594
		1923	− 7	0·9	+74	44·3	15784	+57849
		1933	− 7	37·7	+74	47·4	15453	+56836
95*a*	Karsani	1926	+ 4	12·3	+58	3·0	24694	+39595
		1934	+ 4	26·5	+58	40·9	24574	+40388
98	Tortosa	1905	−13	56·9	+58	7·6	23230	+37359
	(Ebro)	1924	−11	20·2	+57	30·5	23359	+36678
		1934	− 9	45·5	+57	22·7	23456	+36645
99	Coimbra	1900	−17	20·1	+59	24·3	22768	+38506
		1921	−15	13·4	+58	19·2	23110	+37448
		1931	−13	45·5	+57	52·2	23196	+36931
102*a*	Cheltenham	1902	− 5	6·8	+70	21·9	20198	+56612
		1925	− 6	39·2	+71	0·5	18870	+54826
		1934	− 7	6·8	+71	14·2	18384	+54116
106	San Miguel *x*	1913	−19	53·2	+60	49·5	23059	+41283
	(Ponta Delgada)	1923	−19	5·5	+60	11·9	23204	+40514
		1933	−18	12·5	+59	35·3	23374	+39822
107	Zinsen *x*	1926	− 5	57·8	+53	9·0	30001	+40030
		1934	− 6	5·8	+53	12·7	30027	+40155
108	San Fernando	1900	−15	59·3	+55	9·2*x*	24631	+35378
		1924	−13	23·5	+53	46·8*x*	25016	
110*a*	Kakioka	1913	− 5	10·1	+49	30·9	29749	+34851
		1924	− 5	31·6	+49	29·5	29708	+34774
		1934	− 5	47·1	+49	29·5	29720	+34788
109	Tsingtau	1916	− 4	4·7	+52	7·1	30842	+39644
		1925	− 4	22·6	+52	5·9	30831	+39603
		1935	− 4	36·6	+52	5·6	30923	+39714
115	Tucson	1900	+13	25·8	+59	19·6	27379	+46160
		1925	+13	45·3	+59	30·6	26687	+45323
		1935	+13	52·2	+59	40·4	26259	+44889
116*a*	Lukiapang	1909	− 2	58·6	+45	34·8	33199	+33879
		1933	− 3	35·4	+45	23·7	33329	+33791
116*b*	Zo-Sè	1934	− 3	24·6	+45	30·9	33254	+33858
117*a*	Dehra Dun	1915	+ 2	15·5	+44	30·6	33083	+32522
		1924	+ 1	34·6	+45	17·0	32943	+33270
		1934	+ 1	0·0	+45	39·0	33087	+33847
118	Helwan	1907	− 2	55·7	+40	39·4	30033	+25793
		1923	− 1	0·3	+41	20·2	29973	+26366
		1933	− 0	0·6	+41	49·3	30180	+27004
122*a*	Au Tau	1928	− 0	43·1	+30	38·8	37478	+22207
		1934	− 0	42·3	+30	31·5	37557	+22145

No.	*Observatory*	*Year*	*D*		*I*		*H*	*Z*
			°	′	°	′	γ	γ
123	Honolulu	1902	+ 9	19·1	+40	14·5	29255	+24758
		1925	+10	1·8	+39	25·9	28708	+23607
		1935	+10	9·7	+39	10·0	28534	+23244
124*a*	Teoloyucan	1930	+ 9	25·4	+46	54·2	31199	+33343
		1934	+ 9	36·1	+47	7·3	31017	+33403
126*a*	Alibag	1904	+ 1	9·4	+22	54·7	36882	+15588
		1924	+ 0	4·6	+25	12·9	37061	+17453
		1934	− 0	16·2	+25	29·9	37462	+17867
127*a*	San Juan	1926	− 4	21·0	+52	10·6	27743	+35737
		1935	− 5	25·0	+52	46·7	27290	+35926
128	Antipolo	1911	+ 0	41·3	+16	18·6	38072	+11140
		1923	+ 0	31·7	+16	0·0	38174	+10941
		1933	+ 0	30·1	+15	48·6	38285	+10841
137*b*	Batavia (Kuyper)	1928	+ 0	53·0	−32	14·9	36834	−23239
		1935	+ 1	9·2	−32	20·8	37035	−23455
141	Huancayo	1926	+ 7	54·9	+ 1	9·8	29666	+ 601
		1936	+ 7	11·6	+ 2	13·5	29609	+ 1150
142	Apia	1905	+ 9	37·0	−29	11·8	35675	−19935
		1924	+10	19·2	−30	7·5	35249	−20453
		1934	+10	42·2	−30	22·4	35049	−20541
144	Tananarivo	1902	−10	15·0	−54	6·8	23168	−32021
		1919	− 8	4·6	−53	21·2	22218	−29866
		1933	− 8	20·0			21386	−28934
145	Mauritius	1900	− 9	29·0	−54	11·0	23826	−33015
		1928	−11	42·7	−52	44·6	22768	−29934
		1935	−12	59·6	−52	48·6	22529	−29690
146	La Quiaca	1920	+ 6	3·3	−12	39·6	26621	− 5979
		1923	+ 5	40·2	−12	29·5	26505	− 5881
		1933	+ 4	16·7	−12	21·2	26223	− 5743
147*a*	Vassouras	1915	−10	28·1	−14	44·1	24700	− 6496
		1924	−11	53·9	−16	6·0	24371	− 7034
		1932	−12	57·1	−17	20·6	24072	− 7518
148	Watheroo	1919	− 4	22·8	−63	51·4	24925	−50780
		1926	− 4	17·2	−64	10·8	24680	−51007
		1936	− 3	37·1	−64	21·6	24677	−51412
149	Pilar	1905	+ 9	51·7	−26	3·0	25894	−12657
		1925	+ 7	6·2	−25	41·3	25012	−12031
		1935	+ 5	53·5	−26	3·7	24456	−11961
152*a*	Toolangi *x*	1920	+ 8	0·8	−67	55·1	22874	−56384
		1925	+ 8	10·4	−67	44·5	22948	−56071
	„ *x*	1934	+ 8	32·3	−67	49·2	22909	−56193
154*a*	Amberley	1930	+17	51·0	−67	58·4	22351	−55246
		1935	+18	6·3	−67	59·7	22317	−55223

TABLE C

The coefficients in Schuster's series which express $\cos p\theta$ *and* $\sin p\theta$ *in terms of the spherical harmonics* $P_n^m(\theta)$ *of fixed order* m (*odd for* $\cos p\theta$, *even for* $\sin p\theta$).

(See equations 17 (93 and 94).)

$u_{n,p}^m$						
m odd	*p odd*	$p = 1$	$p = 3$	$p = 5$	$p = 7$	$p = 9$
$m = 1$	$n = 2$	+0·8502	−0·8502	..	..	..
	4	+0·3493	+0·8732	−1·2224	..	..
	6	+0·2285	+0·3198	+0·9595	−1·5078	..
	8	+0·1711	+0·2054	+0·3227	+1·0488	−1·7480
$m = 3$	$n = 4$	+0·9241	−1·3861	+0·4620	..	..
	6	+0·6502	+0·2167	−1·6617	+0·7947	..
	8	+0·4985	+0·3324	−0·1108	−1·8002	+1·0801
$m = 5$	$n = 6$	+0·9280	−1·6704	+0·9280	−0·1856	..
	8	+0·7735	−0·3094	−1·5470	+1·4696	−0·3867
$m = 7$	$n = 8$	+0·9152	−1·8304	+1·3074	−0·4576	+0·0654

Example: $\cos 5\theta = -1{\cdot}2224P_4^1+0{\cdot}9595P_6^1+0{\cdot}3227P_8^1+\ldots$

$u_{n,p}^m$						
m odd	*p even*	$p = 0$	$p = 2$	$p = 4$	$p = 6$	$p = 8$
$m = 1$	$n = 1$	+1·1781	−0·5890			
	3	+0·4208	+0·8417	−1·0521	..	..
	5	+0·2614	+0·3268	+0·9149	−1·3724	..
	7	+0·1902	+0·2131	+0·3196	+1·0045	−1·6323
$m = 3$	$n = 3$	+1·6299	−1·0866	+0·2716	..	..
	5	+0·8471	+0·4941	−1·5529	+0·6353	..
	7	+0·5931	+0·4745	+0·0264	−1·7398	+0·9424
$m = 5$	$n = 5$	+1·8941	−1·4206	+0·5682	−0·0947	..
	7	+1·0929	+0·1749	−1·6612	+1·2240	−0·2841
$m = 7$	$n = 7$	+2·0851	−1·6680	+0·8340	−0·2383	+0·0298

Example: $\cos 4\theta = -1{\cdot}0521P_3^1+0{\cdot}9149P_5^1+0{\cdot}3196P_7^1+\ldots$

$v^{m}_{n,p}$						
m even	*p odd*	$p = 1$	$p = 3$	$p = 5$	$p = 7$	$p = 9$
$m = 0$	$n = 0$	+0·7854	..	..	..	..
	2	−0·4909	+1·4726	..	..	..
	4	−0·1104	−0·8284	+1·9328	..	..
	6	−0·0498	−0·2094	−1·0469	+2·3033	..
	8	−0·0285	−0·1027	−0·2689	−1·2236	+2·6220
$m = 2$	$n = 2$	+1·2753	−0·4251	..	..	..
	4	+0·2470	+1·3583	−0·8644	..	..
	6	+0·1084	+0·3974	+1·4088	−1·1921	..
	8	+0·0614	+0·2046	+0·4500	+1·4625	−1·4625
$m = 4$	$n = 4$	+1·6335	−0·8168	+0·1634	..	..
	6	+0·5936	+1·2267	−1·4641	+0·4353	..
	8	+0·3218	+0·8152	+0·7294	−1·8128	+0·6972
$m = 6$	$n = 6$	+1·8753	−1·1252	+0·3750	−0·0536	..
	8	+0·8355	+1·0026	−1·7664	+0·9429	−0·1790
$m = 8$	$n = 8$	+2·0592	−1·3728	+0·5884	−0·1471	+0·0163

Example: $\sin 5\theta = 1{\cdot}9328P^{0}_{4} - 1{\cdot}0469P^{0}_{6} - 0{\cdot}2689P^{0}_{8} - \ldots$

$v^{m}_{n,p}$					
m even	*p even*	$p = 2$	$p = 4$	$p = 6$	$p = 8$
$m = 0$	$n = 1$	+1·1781	..	..	..
	3	−0·6872	+1·7181	..	..
	5	−0·1687	−0·9449	+2·1261	..
	7	−0·0805	−0·2416	−1·1390	+2·4678
$m = 2$	$n = 3$	+1·3308	−0·6654	..	..
	5	+0·3458	+1·3832	−1·0374	..
	7	+0·1678	+0·4287	+1·4353	−1·3328
$m = 4$	$n = 5$	+1·4974	−1·1979	+0·2995	..
	7	+0·7869	+0·9617	−1·6612	+0·5683
$m = 6$	$n = 7$	+1·5603	−1·5603	+0·6687	−0·1114

Example: $\sin 6\theta = 2{\cdot}1261P^{1}_{5} - 1{\cdot}1390P^{1}_{7} + \ldots$

TABLE D

The roots $\theta_1, \ldots, \theta_{n-m}$ *for each associated spherical harmonic function* $P_n^m(\theta) = 0$, *and the numerical factors* p_n^m *in the expression*

$$P_n^m(\theta) = p_n^m \sin^m\theta \sin(\theta+\theta_1)\sin(\theta+\theta_2)\ldots\sin(\theta+\theta_{n-m})$$

(After Adolf Schmidt. See Ch. XVII, formula (27 b).)

$P_1^0 = \cos\theta.$

$P_1^1 = \sin\theta.$

$P_2^0 \ldots p_2^0 = 1{\cdot}5000;\ \log p_2^0 = 0{\cdot}17609.$

$\theta_1 = 54^\circ\ 44{\cdot}1',\ \theta_2 = 125^\circ\ 15{\cdot}9'.$

$P_2^1 = 1{\cdot}7321 \sin\theta\cos\theta.$

$P_2^2 = 0{\cdot}86603 \sin^2\theta.$

$P_3^0 \ldots p_3^0 = 2{\cdot}5000;\ \log p_3^0 = 0{\cdot}39794.$

$\theta_1 = 39^\circ\ 13{\cdot}9',\ \theta_2 = 90^\circ,\ \theta_3 = 140^\circ\ 46{\cdot}1'.$

$P_3^1 \ldots p_3^1 = 3{\cdot}0619;\ \log p_3^1 = 0{\cdot}48599.$

$\theta_1 = 63^\circ\ 26{\cdot}1',\ \theta_2 = 116^\circ\ 33{\cdot}9'.$

$P_3^2 = 1{\cdot}9365 \sin^2\theta\cos\theta.$

$P_3^3 = 0{\cdot}79057 \sin^3\theta.$

$P_4^0 \ldots p_4^0 = 4{\cdot}3750;\ \log p_4^0 = 0{\cdot}64098.$

$\theta_1 = 30^\circ\ 33{\cdot}3',\ \theta_2 = 70^\circ\ 7{\cdot}5',\ \theta_3 = 109^\circ\ 52{\cdot}5',\ \theta_4 = 149^\circ\ 26{\cdot}7'.$

$P_4^1 \ldots p_4^1 = 5{\cdot}5340;\ \log p_4^1 = 0{\cdot}74304.$

$\theta_1 = 49^\circ\ 6{\cdot}4',\ \theta_2 = 90^\circ,\ \theta_3 = 130^\circ\ 53{\cdot}6'.$

$P_4^2 \ldots p_4^2 = 3{\cdot}9131;\ \log p_4^2 = 0{\cdot}59252.$

$\theta_1 = 67^\circ\ 47{\cdot}5',\ \theta_2 = 112^\circ\ 12{\cdot}5'.$

$P_4^3 = 2{\cdot}0917 \sin^3\theta\cos\theta.$

$P_4^4 = 0{\cdot}73951 \sin^4\theta.$

$P_5^0 \ldots p_5^0 = 7{\cdot}8750;\ \log p_5^0 = 0{\cdot}89625.$

$\theta_1 = 25^\circ\ 1{\cdot}0',\ \theta_2 = 57^\circ\ 25{\cdot}2',\ \theta_3 = 90^\circ,\ \theta_4 = 122^\circ\ 34{\cdot}8',\ \theta_5 = 154^\circ\ 59{\cdot}0'.$

$P_5^1 \ldots p_5^1 = 10{\cdot}1666;\ \log p_5^1 = 1{\cdot}00717.$

$\theta_1 = 40^\circ\ 5{\cdot}3',\ \theta_2 = 73^\circ\ 25{\cdot}6',\ \theta_3 = 106^\circ\ 34{\cdot}4',\ \theta_4 = 139^\circ\ 54{\cdot}7'.$

$P_5^2 \ldots p_5^2 = 7{\cdot}6852;\ \log p_5^2 = 0{\cdot}88566.$

$\theta_1 = 54^\circ\ 44{\cdot}1',\ \theta_2 = 90^\circ,\ \theta_3 = 125^\circ\ 15{\cdot}9'.$

$P_5^3 \ldots p_5^3 = 4{\cdot}7062;\ \log p_5^3 = 0{\cdot}67267.$

$\theta_1 = 70^\circ\ 31{\cdot}7',\ \theta_2 = 109^\circ\ 28{\cdot}3'.$

$P_5^4 = 2{\cdot}2185 \sin^4\theta\cos\theta.$

$P_5^5 = 0{\cdot}70156 \sin^5\theta.$

$P_6^0 \ldots p_6^0 = 14{\cdot}4375;\ \log p_6^0 = 1{\cdot}15949.$

$\theta_1 = 21^\circ\ 10{\cdot}6',\quad \theta_2 = 48^\circ\ 36{\cdot}5',\quad \theta_3 = 76^\circ\ 11{\cdot}7',$

$\theta_4 = 103^\circ\ 48{\cdot}3',\ \theta_5 = 131^\circ\ 23{\cdot}5',\ \theta_6 = 158^\circ\ 49{\cdot}4'.$

$P_6^1 \ldots p_6^1 = 18{\cdot}9031;\ \log p_6^1 = 1{\cdot}27653.$

$\theta_1 = 33^\circ\ 52{\cdot}7',\ \theta_2 = 62^\circ\ 2{\cdot}4',\ \theta_3 = 90^\circ,\ \theta_4 = 117^\circ\ 57{\cdot}6',\ \theta_5 = 146^\circ\ 7{\cdot}3'.$

$P_6^2 \ldots p_6^2 = 14{\cdot}9442;\ \log p_6^2 = 1{\cdot}17447.$

$\theta_1 = 45^\circ\ 59{\cdot}6',\ \theta_2 = 75^\circ\ 29{\cdot}4',\ \theta_3 = 104^\circ\ 30{\cdot}6',\ \theta_4 = 134^\circ\ 0{\cdot}4'.$

$P_6^3 \ldots p_6^3 = 9{\cdot}9628$; $\log p_6^3 = 0{\cdot}99838$.

$\theta_1 = 58^\circ\ 31{\cdot}1'$, $\theta_2 = 90^\circ$, $\theta_3 = 121^\circ\ 28{\cdot}9'$.

$P_6^4 \ldots p_6^4 = 5{\cdot}4569$; $\log p_6^4 = 0{\cdot}73694$.

$\theta_1 = 72^\circ\ 27{\cdot}1'$, $\theta_2 = 107^\circ\ 32{\cdot}9'$.

$P_6^5 = 2{\cdot}3268 \sin^5\theta \cos\theta$.

$P_6^6 = 0{\cdot}67169 \sin^6\theta$.

$P_7^0 \ldots p_7^0 = 26{\cdot}8125$; $\log p_7^0 = 1{\cdot}42834$.

$\theta_1 = 18^\circ\ 21{\cdot}5'$, $\theta_2 = 42^\circ\ 8{\cdot}3'$, $\theta_3 = 66^\circ\ 3{\cdot}4'$, $\theta_4 = 90^\circ$,
$\theta_5 = 113^\circ\ 56{\cdot}6'$, $\theta_6 = 137^\circ\ 51{\cdot}7'$, $\theta_7 = 161^\circ\ 38{\cdot}5'$.

$P_7^1 \ldots p_7^1 = 35{\cdot}4696$; $\log p_7^1 = 1{\cdot}54986$.

$\theta_1 = 29^\circ\ 20{\cdot}3'$, $\theta_2 = 53^\circ\ 43{\cdot}3'$, $\theta_3 = 77^\circ\ 55{\cdot}1'$,
$\theta_4 = 102^\circ\ 4{\cdot}9'$, $\theta_5 = 126^\circ\ 16{\cdot}7'$, $\theta_6 = 150^\circ\ 39{\cdot}7'$.

$P_7^2 \ldots p_7^2 = 28{\cdot}9608$; $\log p_7^2 = 1{\cdot}46181$.

$\theta_1 = 39^\circ\ 41{\cdot}7'$, $\theta_2 = 65^\circ\ 6{\cdot}5'$, $\theta_3 = 90^\circ$,
$\theta_4 = 114^\circ\ 53{\cdot}5'$, $\theta_5 = 140^\circ\ 18{\cdot}3'$.

$P_7^3 \ldots p_7^3 = 20{\cdot}4784$; $\log p_7^3 = 1{\cdot}31130$.

$\theta_1 = 50^\circ\ 9{\cdot}6'$, $\theta_2 = 76^\circ\ 56{\cdot}0'$, $\theta_3 = 103^\circ\ 4{\cdot}0'$, $\theta_4 = 129^\circ\ 50{\cdot}4'$.

$P_7^4 \ldots p_7^4 = 12{\cdot}3489$; $\log p_7^4 = 1{\cdot}09163$.

$\theta_1 = 61^\circ\ 17{\cdot}4'$, $\theta_2 = 90^\circ$, $\theta_3 = 118^\circ\ 42{\cdot}6'$.

$P_7^5 \ldots p_7^5 = 6{\cdot}1745$; $\log p_7^5 = 0{\cdot}79060$.

$\theta_1 = 73^\circ\ 53{\cdot}9'$, $\theta_2 = 106^\circ\ 6{\cdot}1'$.

$P_7^6 = 2{\cdot}4218 \sin^6\theta \cos\theta$.

$P_7^7 = 0{\cdot}64726 \sin^7\theta$.

TABLE E

The u-measure of magnetic activity. Monthly and annual means 1872–1938

(The values for 1937 and 1938 are provisional.) u is the average change from day to day, taken without sign, of the horizontal intensity at the earth's magnetic equator, expressed in the unit $10\,\gamma$ ($= 0{\cdot}0001\,\Gamma$).

Approximate annual means for years prior to 1872 may be taken from Fig. 11.5.

Year	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*	*Mean*
1872	1·09	1·90	1·42	1·23	1·13	1·17	1·75	2·14	1·45	1·92	1·32	1·02	1·46
1873	1·41	·88	1·49	1·25	·79	·96	·78	·90	·77	·77	1·04	·79	·99
1874	·99	1·29	1·20	1·28	·71	·72	·82	·80	·82	1·39	·80	·62	·95
1875	·66	1·09	·86	·77	·72	·65	·65	·55	·86	·69	·70	·49	·72
1876	·57	·86	·78	·53	·49	·59	·54	·51	·61	·56	·61	·57	·60
1877	·67	·61	·62	·51	·86	·72	·58	·67	·59	·72	·82	·57	·66
1878	·70	·46	·43	·54	·50	·79	·48	·56	·68	·59	·56	·73	·58
1879	·43	·49	·62	·55	·64	·85	·50	·53	·74	·60	·52	·70	·60
1880	·52	·40	·67	·74	·63	·62	·73	1·38	·97	·82	1·11	·89	·79
1881	·87	1·14	·78	·75	·62	·70	·83	·71	1·20	·87	1·19	·78	·87
1882	·72	·92	·76	2·16	·84	·90	·99	·94	·72	1·82	2·95	·88	1·22
1883	·54	·86	1·07	1·10	·71	·80	·88	·81	1·31	·78	1·41	·79	·92
1884	·92	·86	·79	·94	·82	·94	1·05	·65	1·01	1·14	1·29	·84	·94
1885	·91	·90	1·04	·91	1·15	·94	·91	·79	·96	·70	·83	·75	·90
1886	1·13	·68	·93	·89	·94	·64	·88	·76	·73	·82	·89	·52	·82
1887	·56	·56	·60	·84	·70	·69	·80	·80	·85	·64	·70	·84	·72
1888	1·09	·44	·69	·67	·89	·65	·59	·76	·59	·72	·65	·64	·70
1889	·56	·57	·73	·55	·65	·79	·53	·58	·76	·58	·86	·60	·65
1890	·48	·51	·49	·48	·55	·54	·47	·48	·67	·88	·79	·44	·56
1891	·62	·77	·81	·86	·89	·57	·92	·90	1·14	·72	·84	·92	·83
1892	·98	1·70	1·66	1·13	1·71	1·39	1·61	1·68	·86	·83	1·17	1·25	1·33
1893	·90	1·18	1·00	·90	·73	1·10	·80	1·60	1·07	1·09	1·07	·89	1·03
1894	·83	1·73	1·44	1·41	·92	1·16	1·67	1·85	1·22	·84	1·39	·77	1·27
1895	·88	1·43	·96	·91	·62	·85	·87	·78	·99	1·13	1·00	·76	·93
1896	·81	·84	1·07	·76	1·24	·77	·77	·97	·94	·77	·80	·84	·88
1897	1·10	·79	·75	1·04	·77	·69	·75	·61	·66	·69	·63	1·13	·80
1898	·54	·69	1·18	·71	·69	·67	·57	·69	1·54	·75	·84	·73	·80
1899	·63	·97	·70	·60	·86	·58	·53	·64	·56	·46	·44	·50	·62
1900	·70	·63	1·20	·52	1·06	·42	·43	·53	·48	·56	·40	·42	·61
1901	·46	·45	·64	·41	·44	·56	·54	·50	·44	·44	·46	·54	·49
1902	·42	·52	·45	·61	·48	·46	·51	·56	·42	·56	·60	·40	·50
1903	·44	·57	·44	·79	·53	·50	·59	·77	·61	1·68	1·34	1·09	·78
1904	·89	·46	·47	1·00	·90	·86	·56	·68	·61	·72	·79	·63	·71
1905	·72	·88	·96	·68	·52	·78	·72	·93	·94	·78	1·43	·86	·85
1906	·53	1·08	·70	·68	·88	·70	·78	·69	·70	·50	·75	1·07	·76
1907	·79	1·73	·91	·62	·92	·79	·80	·63	1·04	1·01	1·08	·67	·92
1908	·70	·63	1·04	·82	·91	·61	·81	1·00	2·11	·96	1·14	·90	·97
1909	1·10	·79	1·12	·62	1·31	·67	·71	·68	1·94	1·16	·78	·94	·98
1910	·68	·64	1·10	1·08	·79	·63	·58	1·17	·68	1·02	·62	·69	·81

TABLE E (*continued*)

Year	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*	*Mean*
1911	·53	·54	·70	·78	·89	·60	·66	·59	·61	·69	·53	·85	·66
1912	·46	·46	·55	·67	·53	·45	·57	·63	·64	·55	·75	·47	·56
1913	·54	·48	·51	·56	·45	·46	·38	·45	·54	·58	·39	·41	·48
1914	·46	·49	·45	·73	·54	·47	·68	·50	·77	·67	·66	·60	·58
1915	·59	·70	·80	·78	·57	1·30	·74	·66	·82	1·37	1·19	·87	·87
1916	·76	·59	1·42	1·22	·84	·79	·73	1·37	·95	·94	·71	·54	·90
1917	1·12	·87	·85	1·03	·84	1·08	1·33	2·37	1·04	1·20	·83	1·06	1·14
1918	·74	1·23	1·21	1·49	1·05	·92	·75	1·11	1·06	1·51	1·21	1·54	1·15
1919	1·01	1·04	1·22	·97	1·52	1·00	·91	1·47	1·64	1·55	1·32	1·10	1·23
1920	·84	1·00	2·54	1·20	1·11	·85	·90	1·03	1·32	·99	·92	·94	1·14
1921	·69	·83	·90	1·13	2·70	·80	·64	·76	·77	·92	·88	·75	·98
1922	·74	·85	1·09	·80	·67	·63	·60	·61	·99	·76	·54	·54	·74
1923	·60	·68	·80	·51	·63	·64	·68	·56	·83	·77	·70	·51	·66
1924	·83	·57	·57	·55	·98	1·00	·70	·54	·87	·78	·91	·56	·74
1925	·64	·72	·66	·62	·92	·78	·70	·84	1·14	1·12	·84	1·08	·84
1926	1·41	1·41	1·34	1·43	·89	1·12	·78	·76	1·45	1·73	·84	1·03	1·18
1927	1·08	·84	1·15	1·37	1·00	·63	·88	1·05	·70	1·59	·83	·95	1·01
1928	·58	·58	·83	·64	1·26	·78	1·79	1·01	1·27	1·38	1·02	·72	·99
1929	·84	1·50	1·66	·82	·85	·80	·99	1·01	·89	1·00	1·01	1·21	1·05
1930	·79	·82	·88	·84	1·13	1·01	·70	·77	1·43	1·31	1·01	1·25	1·00
1931	·58	·71	·63	·71	·75	·85	·56	·67	·88	1·06	·80	·51	·73
1932	·54	·59	·88	·68	·94	·65	·63	·67	·71	·81	·59	·73	·70
1933	·68	·63	·68	·48	·96	·61	·56	·81	·88	·61	·51	·64	·67
1934	·61	·67	·81	·58	·86	·68	·61	·59	·76	·53	·56	·75	·67
1935	·73	·97	·69	·65	·75	·79	·79	·60	1·00	·93	·77	·70	·78
1936	·75	·79	·71	·94	·82	1·10	1·16	·73	·79	1·13	1·32	·98	·94
1937	·93	1·05	1·48	1·88	1·69	1·58	1·64	1·52	·99	1·74	·87	1·20	1·38
1938	2·74	1·12	1·17	1·74	1·45	·75	1·31	1·21	1·49	1·46	1·11	1·38	1·41
1939	·64	2·15	·94	1·66									
1940													
1941													
1942													
1943													
1944													
1945													
1946													
1947													
1948													
1949													
1950													

TABLE F

Sunspot data

(*a*) *Zürich Relative Sunspot-numbers for the whole disk. Monthly and annual means* 1872–1938

(After Wolf, Wolfer, and Brunner [5.5]. See also Fig. 11.5.)

Year	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*	*Mean*
1872	80	120	88	102	108	110	106	93	115	104	112	84	102
1873	87	107	98	76	48	45	67	68	48	47	55	49	66
1874	61	64	46	32	45	38	68	61	28	34	29	29	45
1875	15	22	34	29	12	24	12	15	2	13	18	10	17
1876	14	15	31	2	5	2	15	9	10	14	10	8	11
1877	24	9	12	16	21	13	6	6	16	7	14	2	12
1878	3	6	8	0	6	6	0	0	5	1	4	0	3
1879	1	1	0	6	2	5	8	11	6	12	13	7	6
1880	24	28	20	19	24	34	22	48	66	43	31	30	32
1881	36	53	52	52	44	60	77	58	53	64	55	47	54
1882	45	69	68	96	64	45	45	40	58	59	84	42	60
1883	61	47	43	82	32	76	81	46	53	84	84	76	64
1884	92	87	87	76	66	51	53	56	62	48	37	47	64
1885	43	72	50	55	73	84	66	50	40	39	33	22	52
1886	30	26	57	44	31	27	30	17	21	9	0	12	25
1887	10	13	4	7	20	16	23	21	7	7	7	21	13
1888	13	7	8	5	7	7	3	3	9	2	11	7	7
1889	1	8	7	4	2	6	10	21	6	2	0	7	6
1890	5	1	5	2	5	1	12	8	17	11	10	8	7
1891	14	22	10	20	41	48	59	33	54	52	42	32	36
1892	69	76	50	70	80	76	77	101	63	70	65	79	73
1893	75	73	66	88	85	88	89	129	78	80	75	94	85
1894	83	85	52	82	101	99	106	70	66	76	57	60	78
1895	63	67	61	77	68	72	48	69	58	68	47	71	64
1896	29	57	52	44	28	49	45	27	61	28	38	43	42
1897	41	29	29	31	20	11	28	22	48	14	8	33	26
1898	30	36	38	14	26	22	9	31	35	34	31	13	27
1899	20	9	18	14	8	20	14	3	8	13	8	10	12
1900	9	14	9	16	15	12	8	4	8	13	4	0	10
1901	0	2	4	0	10	6	1	1	1	4	4	0	3
1902	5	0	12	0	3	1	1	2	8	16	10	1	5
1903	8	17	14	26	15	16	28	29	11	39	44	46	24
1904	32	24	37	43	40	42	51	58	30	54	38	55	42
1905	55	86	56	39	48	49	73	59	55	79	107	56	64
1906	46	31	64	55	58	63	104	48	56	18	39	65	54
1907	76	108	61	53	43	40	50	54	85	65	62	47	62
1908	39	34	29	58	41	48	40	90	87	32	46	40	48
1909	57	47	66	32	36	23	36	23	39	58	56	54	44
1910	26	32	21	8	22	12	14	12	26	38	5	6	19

TABLE F (*continued*)

Year	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*	*Mean*
1911	3	9	8	16	9	2	4	4	4	3	4	2	6
1912	0	0	5	4	4	4	3	0	10	5	1	6	4
1913	2	3	0	1	0	0	2	0	1	3	1	4	1
1914	3	3	3	17	5	11	5	8	13	8	16	22	10
1915	23	42	39	41	33	69	72	70	50	54	42	34	47
1916	45	55	67	72	74	68	54	35	45	51	66	53	57
1917	75	72	95	75	114	115	120	154	129	72	96	129	104
1918	96	65	72	80	77	59	108	102	80	85	83	59	81
1919	48	80	66	52	88	111	65	69	55	53	42	35	64
1920	51	54	70	15	33	39	28	19	36	50	27	30	38
1921	32	28	27	32	22	34	42	23	18	18	18	20	26
1922	12	26	55	11	8	6	11	6	5	6	7	18	14
1923	4	2	3	6	3	9	4	0	13	12	10	3	6
1924	0	5	2	11	21	24	28	19	25	26	22	16	17
1925	6	23	18	32	43	48	38	38	60	69	59	99	44
1926	72	70	62	38	64	74	52	62	61	72	60	79	64
1927	82	93	70	94	79	59	55	54	68	63	67	45	69
1928	84	74	85	81	77	91	98	84	90	61	50	59	78
1929	69	64	50	53	58	72	70	66	34	54	81	108	65
1930	65	49	35	38	37	29	22	25	32	34	36	26	36
1931	15	43	30	31	25	15	17	13	19	10	19	18	21
1932	12	11	11	11	18	22	10	7	4	9	8	11	11
1933	12	22	10	3	3	5	3	0	5	3	1	0	6
1934	3	8	4	11	20	7	9	8	4	6	9	15	9
1935	19	20	23	12	27	46	34	30	42	53	64	62	36
1936	63	74	77	75	55	70	52	87	76	89	115	123	80
1937	132	128	84	109	117	130	145	138	101	125	74	89	114
1938	98	119	86	101	127	98	165	116	90	99	122	93	110
1939													
1940													
1941													
1942													
1943													
1944													
1945													
1946													
1947													
1948													
1949													
1950													

TABLE F (*continued*)

(*b*) *Epochs of sunspot maxima and minima, given to tenths of a year, and the relative sunspot-number for the interval of twelve months centred at that epoch*

(After Wolfer and Brunner.)

1784·7	10	1843·5	10	1901·7	3
1788·1	141	1848·1	132	1907·0	64
1798·3	3	1856·0	3	1913·6	1
1805·2	49	1860·1	98	1917·6	105
1810·6	0	1867·2	5	1923·6	6
1816·4	49	1870·6	140	1928·4	78
1823·3	0	1878·9	2	1933·8	3
1829·9	72	1883·9	75	1937·4	119
1833·9	7	1889·6	5		
1837·2	147	1894·1	88		

(*c*) *The annual number (F) of days free from sunspots, and the annual mean distance of spots from the solar equator, according to the Greenwich observations*

	F	*dist.*		*F*	*dist.*		*F*	*dist.*
		°			°			°
1890	...	22·0	1910	65	10·5	1930	4	9·9
91	...	20·3	11	183	6·5	31	42	8·3
92	...	18·4	12	246	8·1	32	92	8·3
93	...	14·5	13	311	23·2	33	234	10·6
94	0	14·2	14	143	21·8	34	145	4·1
95	0	13·5	15	6	18·8	35	21	23·3
96	8	14·3	16	3	15·8	36	0	20·4
97	32	8·0	17	0	14·6	37	0	17·02
98	48	10·5	18	0	12·8	38	0	
99	123	9·5	19	0	10·8			
1900	191	7·7	1920	0	10·4			
01	289	10·4	21	4	7·9			
02	248	17·6	22	31	8·0			
03	56	19·9	23	109	15·3			
04	0	16·6	24	171	22·7			
05	2	13·1	25	97	20·2			
06	5	14·0	26	19	18·7			
07	1	12·1	27	2	15·0			
08	6	10·4	28	0	13·5			
09	5	9·7	29	0	10·5			

For 1934 the Greenwich record gives the mean distance of the spots separately for those of the old cycle, 4·1°, and for those of the new cycle, 27·6°.

CORRIGENDA, pp. T 19–T 77

Page T 19. Line 11, *insert* 407; lines 13, 14, *delete from* A *to* month; line 15, *for* J *read* I; lines 17–19, *replace last sentence by* The decimal point of each international magnetic character figure (p. 196) is omitted, thus multiplying them by 10.

Pages T 31–77, Table J. The heading 'Sunspots/5' printed at the lower left-hand corner of these pages (odd numbers) should be deleted.

Additional Note to Tables I, J. Some idea of the time-pattern of solar and geomagnetic disturbance is given by mere inspection of these tables, since the print is more dense where the numbers are largest, particularly where they have two digits instead of only one.

Table G. International quiet days, 1884–1937 (*left pages*).

Table H. International disturbed days, 1884–1937 (*right pages*).

These are the dates of the Greenwich days (midnight to midnight) selected internationally as being the five quietest and the five most disturbed days in each calendar month.

Table I. The daily relative sunspot numbers for the whole disk, divided by 5; 1890–1937 (*left pages*).

Table J. The daily international magnetic character-figures, multiplied by 10; 1890–1937 (*right pages*).

These tables give the values for each day, 1890–1937, arranged in solar 'rotations' (see Ch. XII, p.   ), i.e. rows of 27 days each, with the number of the rotation, and the date of the first day, given at the left of each row. A small mark separates the value for the last day of a calendar month from the first day of the next month. The Zürich sunspot-numbers (see p. 176) are divided by 5; more exactly, a figure 0 in our table J means sunspot-numbers 0, a figure 1 may signify sunspot-numbers 1, 2, 3, 4 or 5, a figure 2 may signify sunspot-numbers 6, 7, 8, 9, 10, etc. The international magnetic character-figures (see p. 195) are unchanged, only the decimal point has been omitted.

Table G. Quiet Days, 1884 - 1895

	1884	1885	1886	1887
Jan.	1 6 14 24 31	1 5 14 15 26	6 12 13 17 18	1 2 9 10 21
Febr.	10 12 13 14 15	1 20 24 25 26	9 13 14 26 28	6 8 17 18 26
March	10 11 12 13 18	9 18 19 26 30	1 2 5 9 14	3 18 29 30 31
April	5 8 9 23 30	5 6 22 23 30	3 8 9 10 24	1 13 19 26 27
May	4 5 17 21 29	1 9 21 22 23	5 7 25 30 31	9 10 20 21 29
June	8 16 21 26 27	3 6 7 29 30	1 2 3 11 20	3 14 15 16 28
July	10 17 18 21 22	2 3 20 23 24	7 8 13 25 26	3 22 26 28 29
Aug.	4 5 17 28 30	13 14 23 24 25	9 10 22 29 30	11 12 19 20 23
Sept.	9 16 24 25 27	11 13 20 21 29	2 25 26 27 28	5 6 7 8 20
Oct.	11 12 22 23 31	4 5 6 21 26	4 5 16 24 25	3 4 20 21 29
Nov.	15 16 21 22 26	4 16 17 21 30	1 10 22 27 28	7 15 16 25 26
Dec.	4 5 6 10 13	4 11 12 24 25	10 12 20 25 31	4 10 11 15 24

	1888	1889	1890	1891
Jan.	3 5 10 20 30	3 16 17 18 27	2 8 9 27 31	4 7 8 26 31
Febr.	1 2 6 15 27	2 9 10 11 12	7 10 23 25 28	3 4 20 21 28
March	3 5 6 25 26	3 9 10 11 19	2 27 28 29 30	1 10 11 28 29
April	1 10 21 22 23	5 6 11 15 16	3 4 10 18 26	6 19 26 27 30
May	5 6 14 15 25	2 8 11 15 16	2 27 28 29 30	1 2 23 24 25
June	1 9 11 14 17	5 7 12 18 27	8 14 16 26 27	10 12 13 28 29
July	6 12 13 26 27	9 10 12 16 22	2 14 16 27 28	9 12 19 22 31
Aug.	9 15 23 28 29	4 5 6 18 19	3 11 12 13 30	6 7 23 24 25
Sept.	4 5 6 11 23	5 14 15 20 28	1 8 9 23 27	6 7 18 20 25
Oct.	3 15 16 28 29	11 12 24 29 30	2 4 7 23 29	6 15 16 17 22
Nov.	13 14 22 23 24	8 13 14 20 23	3 5 6 28 29	2 6 7 9 30
Dec.	11 20 21 22 28	5 10 11 12 18	4 16 18 19 27	1 17 18 24 27

	1892	1893	1894	1895
Jan.	2 9 14 25 27	15 16 17 25 27	8 15 16 17 20	13 14 27 28 31
Febr.	10 11 17 22 23	1 13 23 24 26	1 8 10 11 13	4 22 25 26 27
March	17 19 20 22 23	18 19 20 21 22	5 7 13 27 28	11 12 21 24 29
April	16 17 19 20 21	4 9 21 22 23	3 4 9 11 23	8 22 28 29 30
May	12 13 15 23 26	1 2 22 28 29	10 11 12 25 26	4 12 16 25 26
June	11 12 13 14 15	8 13 17 23 24	5 6 7 24 26	8 13 14 20 25
July	5 6 7 20 23	5 6 9 30 31	7 10 11 14 26	7 10 18 19 25
Aug.	2 14 15 19 28	1 17 24 27 28	1 2 17 18 31	3 4 7 22 27
Sept.	5 9 18 19 25	4 7 22 23 24	3 4 6 13 26	2 7 8 11 28
Oct.	9 25 26 28 29	19 20 21 22 23	10 11 12 15 23	3 10 22 24 25
Nov.	8 11 12 13 20	11 15 16 20 21	4 5 6 21 22	6 7 18 19 21
Dec.	2 3 9 10 20	7 17 18 21 22	3 4 11 25 26	6 16 28 29 31

Table H. Disturbed Days, 1884 - 1895

	1884	1885	1886	1887
Jan.	11 12 19 25 26	2 9 22 23 30	4 9 10 19 30	14 15 16 23 25
Febr.	2 23 24 25 29	5 10 12 27 28	5 11 16 19 22	12 13 14 20 21
March	1 2 3 20 29	1 13 14 15 16	19 20 23 30 31	6 9 15 20 21
April	10 11 18 24 25	3 13 15 27 28	12 13 14 15 18	5 6 7 11 28
May	1 10 11 12 22	13 25 26 27 28	8 9 10 11 18	2 3 12 13 24
June	2 19 22 23 24	4 20 22 25 26	5 12 22 29 30	5 10 19 21 22
July	2 3 4 14 26	1 6 17 18 25	1 20 21 27 28	7 8 11 19 20
Aug.	8 9 10 21 22	1 2 7 28 29	12 14 16 17 24	1 2 3 5 29
Sept.	11 14 17 18 19	4 5 15 16 23	9 10 11 12 21	10 25 26 27 28
Oct.	2 3 7 14 29	13 15 16 22 23	6 7 8 9 10	22 23 26 27 30
Nov.	2 3 4 23 28	7 8 10 11 18	3 4 5 6 30	10 20 21 22 29
Dec.	14 15 20 23 28	1 6 7 8 28	1 2 26 27 29	16 17 18 21 22

	1888	1889	1890	1891
Jan.	8 13 14 23 24	7 11 20 21 22	3 11 18 20 21	16 17 18 19 24
Febr.	9 10 11 19 21	3 8 17 18 27	3 15 18 19 20	11 12 13 14 15
March	9 15 16 17 18	1 6 17 28 29	11 12 16 17 18	2 3 5 24 31
April	4 5 11 12 13	3 8 9 25 28	1 7 22 23 30	7 8 9 12 17
May	7 8 10 20 21	4 5 22 26 31	4 5 6 17 25	4 13 14 15 16
June	3 4 5 22 23	9 14 21 22 28	2 4 5 21 22	5 14 15 20 27
July	1 20 22 28 29	1 6 17 18 19	6 17 18 19 20	4 6 7 17 24
Aug.	3 4 16 17 18	13 20 26 27 28	14 15 16 17 18	2 3 19 29 30
Sept.	13 14 15 27 28	9 10 11 22 23	11 12 15 16 20	9 10 11 12 28
Oct.	5 19 20 21 31	5 6 7 20 21	6 10 15 18 19	8 23 24 25 26
Nov.	11 16 17 18 28	1 2 26 27 28	8 9 13 14 15	15 16 17 20 21
Dec.	8 14 15 24 26	6 7 20 22 27	5 21 22 23 28	7 8 9 22 30

	1892	1893	1894	1895
Jan.	4 5 6 17 29	5 9 21 22 29	3 4 11 12 13	1 6 17 18 19
Febr.	13 14 20 21 27	4 5 14 16 17	21 23 24 25 28	9 10 15 16 24
March	1 6 7 11 12	14 15 25 26 28	1 21 22 30 31	8 9 13 14 22
April	9 24 25 26 27	12 13 26 27 28	6 13 17 18 19	5 6 11 12 23
May	1 2 8 18 19	7 8 9 18 30	1 14 15 28 31	2 10 28 29 30
June	2 3 16 27 28	9 18 19 28 29	9 10 11 17 21	2 3 4 17 30
July	12 14 16 17 26	14 15 16 21 22	2 18 19 20 21	1 13 14 15 27
Aug.	3 4 12 25 26	6 7 12 13 18	13 14 15 20 21	9 10 11 18 23
Sept.	11 13 16 21 22	5 8 9 26 30	14 15 19 20 21	4 5 15 19 20
Oct.	12 15 16 18 19	2 10 25 26 29	5 16 17 25 31	12 13 14 17 27
Nov.	4 5 17 18 24	1 2 3 27 28	13 14 17 18 23	9 10 11 23 24
Dec.	4 5 13 23 24	5 6 24 25 30	5 13 15 21 22	7 8 9 21 24

Table G. Quiet Days, 1896 - 1907

	1896	1897	1898	1899
Jan.	1 2 23 26 28	6 9 20 21 22	3 4 7 9 23	8 9 10 25 27
Febr.	7 20 22 23 24	2 16 17 18 19	1 7 23 26 27	4 7 8 10 18
March	17 18 19 21 24	16 18 20 21 26	3 4 7 25 29	5 19 20 27 30
April	7 14 16 20 30	3 11 12 15 22	19 21 22 26 29	13 15 16 26 27
May	5 26 27 28 31	8 9 26 27 28	19 21 23 24 25	13 14 25 28 29
June	1 7 20 23 24	6 9 10 24 30	5 13 18 20 21	7 8 17 20 25
July	9 17 19 28 31	5 13 18 26 29	2 14 15 16 18	19 22 23 30 31
Aug.	5 13 16 27 28	4 5 6 7 24	9 10 14 24 25	11 12 16 19 26
Sept.	1 9 11 27 28	2 8 19 26 28	6 7 19 21 26	5 6 7 8 20
Oct.	6 7 26 27 28	8 13 14 20 21	4 11 12 17 18	2 10 11 29 30
Nov.	2 3 22 24 25	7 8 12 23 30	10 11 13 14 28	8 10 16 18 20
Dec.	8 12 18 19 20	8 13 26 27 28	10 12 23 25 26	6 11 14 15 24

	1900	1901	1902	1903
Jan.	3 7 8 9 30	12 13 16 19 20	1 12 14 21 22	1 7 15 17 29
Febr.	6 7 13 18 19	11 15 16 27 28	15 18 19 27 28	1 2 18 27 28
March	5 11 21 27 28	5 10 15 16 28	4 14 19 20 28	16 17 25 26 27
April	2 3 19 22 28	9 12 20 21 25	7 15 25 26 27	14 16 20 22 23
May	9 10 15 16 27	5 13 16 17 22	3 11 12 16 22	1 11 18 19 20
June	11 15 16 22 26	5 25 26 27 28	2 3 8 20 23	6 12 13 26 27
July	1 3 9 13 15	3 21 28 29 30	2 6 14 21 31	4 8 22 23 24
Aug.	5 6 10 23 24	12 13 19 24 28	5 12 13 14 30	3 7 19 29 31
Sept.	7 11 12 21 25	7 8 19 20 25	8 9 14 24 26	3 16 17 18 26
Oct.	1 2 15 19 31	2 3 23 30 31	7 10 14 16 22	9 10 20 21 24
Nov.	5 7 10 11 30	13 15 23 29 30	4 5 11 28 29	14 15 25 27 28
Dec.	5 6 20 21 23	14 15 16 17 23	5 8 14 18 31	12 17 18 24 26

	1904	1905	1906	1907
Jan.	13 14 19 20 23	2 3 8 9 16	1 7 17 24 30	3 18 19 20 21
Febr.	18 20 21 27 28	11 18 19 20 28	12 13 20 21 22	4 5 6 18 27
March	6 15 16 17 22	18 21 25 26 28	19 20 21 22 23	15 16 17 30 31
April	14 16 23 27 28	11 17 18 23 24	5 6 7 15 16	1 2 9 21 30
May	5 7 9 10 25	5 6 15 16 21	2 3 4 5 23	1 8 9 26 27
June	3 13 24 29 30	13 18 19 27 28	19 20 21 22 30	5 8 15 16 17
July	4 11 12 21 25	4 11 15 20 31	9 15 16 17 22	4 15 16 21 30
Aug.	24 25 26 27 28	1 9 10 15 18	4 5 6 18 23	6 12 13 28 29
Sept.	18 19 20 21 27	6 13 14 16 17	8 9 10 11 19	8 9 22 23 24
Oct.	4 12 17 18 24	2 3 23 29 31	6 7 8 9 18	6 7 18 19 31
Nov.	9 12 13 19 20	1 2 11 20 29	2 3 14 28 29	1 19 20 29 30
Dec.	8 12 13 23 24	1 9 22 23 31	4 5 25 30 31	1 2 3 9 24

Table H. Disturbed Days, 1896 - 1907

	1896					1897					1898					1899				
Jan.	3	4	19	30	31	1	2	3	29	30	15	16	17	18	19	15	18	23	28	29
Febr.	3	4	14	28	29	3	4	25	26	27	11	12	14	15	16	12	14	23	24	28
March	4	12	26	27	28	3	4	8	10	29	11	15	16	18	20	10	11	21	22	23
April	21	22	23	24	25	2	6	20	23	24	4	7	12	13	14	5	10	18	19	29
May	2	3	17	18	20	2	14	17	21	30	4	11	12	29	30	3	4	5	15	31
June	9	14	16	17	29	2	3	16	17	18	7	8	15	26	27	1	27	28	29	30
July	4	5	12	23	27	14	15	22	30	31	20	21	22	23	27	2	3	4	6	25
Aug.	1	2	7	21	29	1	9	15	20	30	3	16	17	22	28	3	20	21	29	31
Sept.	3	17	18	20	24	4	5	11	14	23	3	9	10	28	29	1	15	18	26	27
Oct.	1	9	10	11	12	1	2	10	18	28	22	25	28	29	30	5	6	15	23	24
Nov.	5	6	7	8	9	17	18	20	24	25	17	18	20	21	22	4	19	22	23	30
Dec.	3	4	5	13	14	11	12	20	21	31	5	14	15	16	19	2	18	19	28	29

	1900					1901					1902					1903				
Jan.	5	15	19	20	26	2	5	22	23	24	15	16	17	24	26	4	5	23	26	27
Febr.	4	5	9	11	24	2	19	20	22	23	7	8	20	21	25	8	12	13	15	22
March	1	8	9	13	14	18	19	23	24	29	8	11	12	24	25	5	7	12	13	31
April	5	9	10	11	30	1	2	14	15	27	8	10	11	20	21	5	6	9	18	26
May	1	3	5	6	30	10	11	23	24	31	8	9	10	18	31	5	6	16	17	23
June	2	5	27	28	29	1	7	13	14	21	6	15	21	22	29	1	2	28	29	30
July	17	20	24	25	28	11	12	17	18	20	8	12	24	25	26	6	19	26	27	28
Aug.	1	7	8	20	27	3	14	15	16	31	9	16	21	22	25	11	12	21	22	26
Sept.	5	15	16	22	23	10	11	12	16	17	2	12	18	19	20	5	19	20	23	29
Oct.	4	20	25	26	27	8	9	13	25	28	11	24	27	30	31	12	13	14	26	31
Nov.	1	2	13	18	24	4	5	9	11	19	14	22	23	24	25	1	10	18	19	22
Dec.	2	7	10	27	28	1	2	9	27	28	9	12	13	23	24	4	13	20	30	31

	1904					1905					1906					1907				
Jan.	1	10	16	28	29	5	14	17	22	31	9	13	14	21	31	5	8	11	14	15
Febr.	5	6	7	9	16	3	4	5	14	23	19	24	25	26	28	7	8	9	10	11
March	3	4	5	11	26	2	3	5	7	16	1	4	7	8	25	10	11	12	21	22
April	1	2	4	18	19	1	2	3	5	29	9	10	23	28	29	5	6	14	16	18
May	12	13	14	27	28	1	12	19	27	29	8	14	15	20	21	13	15	18	19	29
June	6	15	16	17	27	5	9	10	22	23	1	2	3	4	8	10	11	19	20	22
July	1	6	7	10	14	6	7	8	23	24	11	12	27	29	30	2	10	11	25	28
Aug.	2	3	4	21	30	2	3	7	29	30	7	8	12	13	27	1	2	21	24	30
Sept.	8	11	16	24	25	3	4	18	19	27	3	4	22	23	24	10	11	12	17	30
Oct.	7	8	13	21	22	6	13	18	26	28	1	12	21	22	27	13	14	15	22	27
Nov.	2	4	5	16	25	4	12	13	15	16	8	10	11	21	22	3	10	11	21	22
Dec.	3	9	14	15	16	4	12	13	20	29	8	9	16	22	23	7	8	11	12	13

Table G. Quiet Days, 1908 - 1919

1908	1909	1910	1911
3 14 19 20 23	6 7 12 16 22	8 11 15 16 31	7 12 17 20 21
2 14 18 20 21	11 12 15 19 25	6 7 8 12 19	11 12 15 19 20
3 14 15 18 24	9 11 15 16 24	8 12 24 25 26	10 11 12 17 18
0 11 12 13 20	7 8 15 21 22	7 10 11 15 21	5 13 14 15 26
7 9 15 16 19	7 8 24 25 30	6 8 12 21 22	1 4 13 22 24
5 10 11 22 23	4 9 12 20 27	2 15 16 18 30	3 17 18 19 25
2 3 10 19 31	5 6 10 18 19	1 12 13 14 27	13 14 15 16 26
4 5 16 27 28	6 14 15 16 21	6 7 8 16 26	7 8 10 11 29
1 2 20 21 26	11 12 13 18 19	3 4 12 18 19	2 3 14 25 26
9 10 22 23 28	5 13 14 16 28	9 16 17 18 31	1 5 15 23 28
4 5 6 22 23	2 4 5 12 25	6 12 13 14 24	1 7 22 23 24
2 11 15 20 21	7 8 9 11 24	10 11 12 17 26	2 9 21 22 23

1912	1913	1914	1915
2 15 16 26 27	1 7 12 16 24	9 10 24 25 26	2 3 18 19 31
5 6 15 20 21	3 4 23 24 28	1 10 11 21 25	7 11 14 16 17
4 17 18 19 24	1 2 10 26 27	5 22 28 29 30	2 3 14 15 28
1 8 11 21 28	6 20 21 22 26	12 15 26 29 30	10 11 12 13 28
1 16 22 23 26	14 20 21 22 23	9 10 14 19 20	7 8 11 28 29
5 6 15 19 20	7 8 11 12 27	12 13 16 17 23	3 4 10 20 30
10 11 12 15 24	4 9 17 19 28	2 8 13 17 19	4 15 16 17 24
4 8 12 13 26	1 5 20 29 30	9 10 14 16 22	5 13 14 15 24
2 15 16 27 28	4 14 20 27 29	2 7 14 21 26	7 8 18 19 20
2 5 18 19 31	2 3 23 24 28	12 14 24 25 26	2 5 9 18 29
3 12 21 29 30	4 14 15 16 25	9 20 21 22 23	3 4 14 29 30
4 5 20 21 28	10 11 13 17 23	2 13 15 21 26	1 5 18 21 22

1916	1917	1918	1919
2 8 15 17 19	3 15 18 28 29	8 11 17 18 19	2 11 25 26 27
1 6 16 21 25	1 9 12 13 27	8 19 22 25 26	7 10 11 12 25
1 13 15 23 27	2 3 28 29 30	5 6 19 24 25	8 9 10 18 24
4 5 10 13 24	10 11 14 20 27	13 14 15 20 24	3 5 14 26 28
13 14 15 18 27	6 8 13 19 20	7 8 9 26 27	7 8 11 28 29
2 3 10 15 16	1 2 19 20 30	2 3 4 29 30	7 8 19 20 30
7 15 27 28 29	6 16 17 18 20	7 19 20 21 22	5 6 15 16 28
1 15 16 17 25	5 6 19 28 29	1 18 19 21 30	6 9 13 14 31
1 19 20 21 29	1 11 23 25 26	11 12 15 25 26	1 5 12 28 30
4 16 17 18 28	16 19 20 21 22	11 12 13 14 27	12 13 14 21 25
1 14 20 21 24	9 10 15 16 23	5 6 7 26 27	7 9 14 19 20
6 10 21 22 23	10 13 22 23 31	5 6 28 29 30	1 17 26 29 31

Table H. Disturbed Days, 1908 – 1919

	1908	1909	1910	1911
Jan.	8 9 27 28 29	1 3 29 30 31	17 22 24 25 26	8 24 25 28 31
Febr.	5 6 7 11 22	2 6 22 23 24	14 17 20 21 25	2 13 21 22 23
March	1 2 26 27 28	18 19 27 28 29	27 28 29 30 31	5 20 21 23 26
April	6 7 15 22 23	12 16 25 26 27	1 2 18 27 28	8 9 16 17 30
May	2 24 25 26 27	14 15 18 19 26	13 18 19 24 25	7 14 15 16 31
June	3 4 18 19 24	21 22 23 29 30	8 9 20 21 22	4 5 9 10 11
July	15 16 17 24 25	12 13 23 24 28	5 6 7 24 29	1 17 19 28 29
Aug.	8 9 12 19 21	2 7 9 29 30	10 19 22 28 29	19 23 24 25 26
Sept.	5 11 12 29 30	3 21 25 26 30	22 25 27 29 30	16 19 20 21 22
Oct.	5 6 12 13 31	2 18 19 23 24	4 6 13 19 27	9 10 11 17 18
Nov.	1 8 9 10 17	15 16 20 27 30	8 17 18 19 29	3 9 13 14 15
Dec.	4 5 6 13 26	1 2 13 14 21	2 15 22 28 29	6 11 17 26 31

	1912	1913	1914	1915
Jan.	11 12 13 17 22	3 10 18 19 30	5 7 13 16 22	1 5 25 26 27
Febr.	12 13 16 17 26	12 14 15 25 26	3 6 18 22 28	8 19 20 23 24
March	7 8 9 21 29	14 15 16 17 23	2 6 7 15 17	7 8 20 21 22
April	5 6 10 15 16	1 9 10 12 16	1 6 7 8 16	7 8 15 22 26
May	5 6 12 13 14	4 5 6 7 27	6 15 17 26 31	1 2 16 17 27
June	1 8 9 10 28	1 2 19 28 29	1 25 26 27 28	12 13 17 18 22
July	3 4 5 27 31	12 13 14 20 25	5 25 29 30 31	2 6 9 11 27
Aug.	1 6 18 19 22	9 10 11 12 23	3 6 23 29 30	2 7 26 27 29
Sept.	4 17 18 23 24	7 8 9 10 22	23 27 28 29 30	22 23 24 28 29
Oct.	1 11 14 15 16	5 6 7 8 18	1 17 27 28 29	15 19 23 24 25
Nov.	10 11 14 16 26	2 3 7 8 28	1 3 4 11 27	1 5 6 16 17
Dec.	2 6 7 22 23	4 7 25 26 27	7 9 16 27 28	6 7 15 16 26

	1916	1917	1918	1919
Jan.	10 11 12 20 23	4 5 12 22 23	5 12 29 30 31	3 4 5 18 31
Febr.	8 12 17 18 27	15 16 18 19 20	5 6 12 13 15	1 14 21 22 28
March	8 9 10 29 30	4 5 8 21 25	8 11 12 15 16	1 2 20 21 22
April	25 26 27 28 29	5 6 9 16 26	5 6 11 19 26	7 8 16 17 18
May	21 22 23 30 31	1 2 3 16 28	1 16 17 18 19	2 3 13 21 24
June	8 18 19 22 23	7 13 23 24 25	9 10 11 12 15	9 10 11 23 26
July	1 8 9 17 23	2 13 22 29 31	8 11 25 28 29	8 17 18 22 23
Aug.	6 22 23 26 27	9 10 13 14 21	15 16 25 26 27	11 12 19 20 28
Sept.	3 4 12 27 30	2 5 9 19 30	1 18 19 20 21	2 6 19 20 24
Oct.	1 6 7 8 13	3 13 14 28 29	2 8 16 17 31	1 3 4 5 28
Nov.	4 5 6 12 27	12 14 25 26 27	11 12 15 23 29	4 11 16 17 22
Dec.	1 2 3 15 27	8 16 17 18 26	1 8 9 25 26	3 14 15 22 24

Table G. Quiet Days, 1920 - 1931

	1920					1921					1922					1923				
Jan.	4	5	13	19	27	2	8	13	14	22	3	4	15	21	22	7	8	18	19	27
Febr.	2	3	9	23	29	8	9	12	16	23	7	11	23	24	25	9	12	13	21	23
March	2	3	29	30	31	5	8	17	19	20	7	8	9	23	24	8	9	10	11	23
April	1	11	13	14	28	4	5	7	26	27	4	5	6	7	19	2	5	17	26	27
May	6	7	11	22	23	5	6	7	25	30	2	3	15	30	31	1	12	13	22	24
June	2	8	14	18	22	5	15	18	19	25	10	11	15	24	25	9	10	17	18	24
July	2	3	21	28	29	2	11	18	21	25	8	11	12	21	22	5	14	15	21	24
Aug.	2	6	17	27	28	1	9	23	25	29	2	3	17	18	28	1	18	21	28	29
Sept.	6	12	20	21	26	11	12	13	24	25	2	22	23	24	26	7	8	16	21	22
Oct.	3	14	20	21	30	3	16	17	18	19	1	16	18	19	22	2	3	23	24	30
Nov.	8	10	14	23	24	2	3	4	27	29	6	7	13	18	26	5	10	15	20	25
Dec.	1	11	12	22	30	6	7	19	20	21	3	8	16	22	23	2	7	17	21	22

	1924					1925					1926					1927				
Jan.	1	12	13	14	20	4	10	11	25	28	5	20	21	25	30	9	10	21	22	23
Febr.	14	15	18	28	29	3	4	21	22	23	6	7	8	9	27	6	7	21	22	23
March	1	14	15	17	28	3	8	18	28	31	8	15	25	26	31	2	22	23	24	25
April	4	5	11	13	30	4	17	19	24	25	2	20	28	29	30	17	18	20	21	22
May	6	7	14	18	31	2	12	14	15	16	1	2	15	26	31	11	12	17	18	26
June	3	6	7	8	14	8	9	19	20	21	4	12	20	25	26	8	19	20	21	24
July	3	4	23	30	31	13	16	17	18	20	11	14	21	22	23	3	10	15	16	29
Aug.	11	12	20	21	25	11	12	13	15	28	7	8	21	22	23	6	7	13	25	26
Sept.	2	11	16	17	20	10	11	12	29	30	1	4	5	28	29	16	17	22	23	24
Oct.	11	12	14	29	30	2	3	18	29	30	1	10	21	22	23	1	4	17	21	31
Nov.	5	8	17	18	30	5	21	22	27	28	7	8	14	15	16	2	7	14	22	25
Dec.	5	6	16	29	30	3	14	17	21	26	6	8	9	14	19	4	20	21	24	27

	1928					1929					1930					1931				
Jan.	11	12	13	14	31	1	17	18	19	23	11	12	14	26	27	3	7	8	24	30
Febr.	8	9	10	11	24	3	4	5	14	15	6	8	9	11	22	6	10	12	20	21
March	2	4	5	9	31	4	6	10	30	31	5	7	8	9	10	6	11	18	19	30
April	12	13	25	28	29	9	14	20	23	24	3	4	5	27	28	6	12	13	14	29
May	1	2	3	22	26	5	10	18	21	22	2	3	24	27	28	9	10	22	23	28
June	10	11	16	17	27	4	5	14	18	26	11	15	23	24	25	5	15	16	17	25
July	13	15	16	17	20	9	13	19	28	29	8	20	21	22	23	9	10	19	20	21
Aug.	10	14	15	20	22	7	8	9	25	28	2	3	4	17	28	2	12	17	18	30
Sept.	12	16	17	21	28	3	4	5	19	29	14	15	16	26	27	2	18	19	28	29
Oct.	9	10	11	23	28	1	2	15	26	27	11	13	16	23	24	9	10	11	16	25
Nov.	8	9	22	28	29	10	11	18	24	25	12	19	20	21	22	12	21	22	28	30
Dec.	4	17	19	20	23	1	2	19	20	21	8	16	17	18	31	19	20	21	26	27

Table H. Disturbed Days, 1920 - 1931

	1920					1921					1922					1923			
Jan.	9	10	11	21	28	1	10	17	24	31	8	9	24	25	31	13	20	21	29
Febr.	7	14	16	17	24	2	5	13	19	28	3	9	12	15	16	17	25	26	27
March	5	14	22	23	24	15	21	22	25	27	5	10	13	14	30	15	24	25	26
April	5	15	17	18	24	12	13	18	19	29	9	12	22	24	26	12	13	21	22
May	1	13	14	15	28	13	14	15	16	20	7	8	9	16	21	3	17	18	29
June	4	10	11	28	29	4	6	8	14	23	3	5	16	29	30	4	13	14	21
July	7	12	15	16	23	7	8	9	15	16	1	16	26	27	28	7	10	11	18
Aug.	8	9	12	21	22	3	5	6	26	30	11	12	13	14	23	3	4	6	13
Sept.	3	4	22	28	29	2	8	23	28	29	7	8	9	14	20	9	10	26	27
Oct.	1	10	24	25	27	7	8	11	12	21	5	6	7	20	31	15	16	17	18
Nov.	4	6	17	26	27	6	16	17	18	23	1	2	3	28	29	2	12	22	27
Dec.	4	5	6	26	27	12	13	16	28	29	5	10	14	15	26	23	24	25	26

	1924					1925					1926					1927			
Jan.	10	22	23	29	30	16	17	19	20	23	18	22	23	26	27	1	4	7	24
Febr.	5	6	20	21	23	8	9	24	25	28	11	18	23	24	25	9	10	24	25
March	7	8	20	30	31	1	5	9	10	15	5	6	9	10	18	9	16	17	27
April	6	7	17	25	26	3	9	10	15	20	6	7	14	15	16	9	11	12	14
May	21	22	23	24	28	4	5	28	30	31	4	5	6	10	11	3	5	7	20
June	10	11	18	19	20	13	23	24	25	28	1	2	8	9	23	2	5	11	12
July	9	20	25	26	27	15	22	26	27	28	5	7	27	28	31	1	7	21	22
Aug.	5	17	18	29	30	7	8	18	22	23	1	9	13	17	18	19	20	21	29
Sept.	1	7	8	24	27	1	14	15	21	24	9	14	15	20	21	4	6	7	9
Oct.	18	23	24	25	27	9	12	21	23	24	14	15	16	19	25	10	12	13	22
Nov.	1	13	19	24	26	1	2	9	10	14	1	3	21	28	29	18	19	21	29
Dec.	12	13	20	21	23	6	7	18	27	28	16	20	23	28	29	13	14	17	18

	1928					1929					1930					1931			
Jan.	1	23	27	28	29	5	8	9	10	14	3	4	5	6	7	10	16	17	18
Febr.	13	18	19	21	26	9	17	18	27	28	12	13	14	15	16	13	14	15	24
March	11	12	13	14	23	8	11	15	16	21	1	2	12	13	14	12	13	14	21
April	1	7	10	20	21	4	16	17	29	30	7	8	10	20	22	1	3	10	19
May	10	12	27	28	29	13	14	15	16	23	5	6	16	17	31	6	7	13	14
June	7	12	13	22	23	10	11	22	23	30	1	2	7	12	16	2	9	26	27
July	2	7	9	22	31	5	6	10	15	16	10	11	12	13	25	23	24	25	26
Aug.	4	5	7	26	27	1	11	14	15	18	6	7	8	9	12	8	9	20	21
Sept.	3	7	8	18	25	7	10	11	14	22	3	6	18	29	30	4	6	15	16
Oct.	2	19	22	24	25	7	8	16	17	30	3	14	17	26	27	2	5	12	29
Nov.	2	3	13	15	17	2	3	4	5	16	4	14	24	25	26	5	6	8	16
Dec.	1	5	6	12	13	4	5	16	17	22	3	4	13	20	21	2	3	4	5

Table G. Quiet Days, 1932 - 1937

	1932					1933					1934					1935				
Jan.	18	19	20	21	22	4	5	10	11	21	5	6	9	12	13	6	7	8	9	10
Febr.	2	16	17	27	28	6	11	13	16	17	1	6	14	23	26	4	11	19	27	28
March	1	15	16	25	26	5	6	7	9	16	12	13	14	19	20	4	10	27	28	29
April	12	19	20	21	30	11	12	13	28	29	13	17	18	23	29	2	6	7	27	28
May	8	9	18	19	20	9	10	12	24	26	14	15	16	27	28	6	8	9	24	25
June	3	4	15	27	28	5	6	16	18	24	2	3	21	22	26	1	2	15	16	25
July	3	13	14	28	29	13	14	15	21	30	10	18	19	22	23	5	6	17	18	30
Aug.	10	16	17	18	19	1	9	10	11	31	9	10	20	24	25	3	12	14	17	18
Sept.	3	10	11	16	17	3	5	6	23	24	9	10	13	14	23	2	8	13	21	22
Oct.	6	13	14	26	28	1	16	21	28	29	8	9	10	19	29	3	5	6	9	13
Nov.	6	9	10	22	24	14	15	17	24	26	1	20	21	22	30	4	10	17	25	26
Dec.	5	7	12	21	23	1	14	15	24	30	9	16	17	23	27	4	5	6	22	23

	1936					1937				
Jan.	3	4	6	7	16	1	15	16	23	25
Febr.	1	5	7	13	28	8	23	24	26	27
March	3	4	7	11	12	3	7	8	11	12
April	5	6	10	26	29	8	9	10	14	16
May	7	8	9	23	24	6	7	18	20	23
June	5	6	23	29	30	9	11	12	19	26
July	14	15	21	23	24	3	8	27	28	29
Aug.	7	11	18	19	23	13	16	24	25	30
Sept.	3	7	13	16	25	3	12	25	28	29
Oct.	2	3	22	27	28	17	18	20	29	30
Nov.	13	22	23	24	25	4	5	6	15	16
Dec.	9	10	19	22	24	4	13	14	27	28

Table H. Disturbed Days, 1932 - 1937

	1932					1933					1934					1935				
Jan.	2	25	26	27	28	1	19	22	27	28	1	2	14	18	29	17	23	24	27	28
Febr.	3	4	10	11	23	19	21	22	23	24	9	10	16	17	18	1	2	13	14	26
March	10	28	29	30	31	18	19	20	23	24	4	5	7	25	31	13	14	15	16	30
April	2	7	13	23	24	15	16	17	19	30	1	4	5	6	16	9	10	11	12	13
May	4	5	25	29	30	1	18	29	30	31	2	3	11	12	18	1	2	10	12	20
June	8	9	10	11	20	1	13	14	20	28	5	6	11	12	18	7	8	9	10	18
July	5	6	7	8	16	9	17	23	24	27	3	4	29	30	31	8	9	22	24	25
Aug.	2	3	27	28	29	5	6	13	18	21	3	4	27	28	29	19	20	21	27	28
Sept.	6	8	23	24	25	9	10	13	14	15	2	24	25	27	30	11	12	18	23	25
Oct.	15	16	20	21	23	5	7	9	13	14	15	20	24	25	26	11	20	21	24	27
Nov.	1	14	15	16	17	6	7	8	11	27	7	8	24	25	28	5	12	13	14	27
Dec.	8	14	15	16	17	3	4	5	9	10	3	4	24	29	30	1	14	26	27	28

	1936					1937				
Jan.	8	18	24	25	26	7	9	10	27	30
Febr.	16	17	19	21	22	3	5	9	19	21
March	20	21	23	24	25	1	5	15	27	31
April	18	19	20	21	22	24	25	26	27	28
May	11	12	18	19	29	4	5	27	28	29
June	1	2	9	10	19	5	6	13	20	27
July	2	6	10	11	29	14	19	20	22	24
Aug.	5	6	8	10	30	2	3	4	22	27
Sept.	11	23	26	27	29	1	10	11	14	30
Oct.	10	16	17	24	31	4	8	9	10	11
Nov.	3	11	15	16	29	18	19	23	29	30
Dec.	4	12	13	27	28	18	19	20	23	24

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1890																													
785	J	22	1	0	0	0	0	0	0	0	1	3	3	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
786	F	18	0	0	0	0	0	0	0	0	0	0	1	1	0	1	3	3	2	3	3	4	4	4	3	3	1	1	0
787	M	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2
788	A	13	2	1	1	1	0	0	0	0	0	0	0	0	1	0	0	1	1	3	1	0	0	0	0	0	0	0	1
789	M	10	3	4	4	2	0	0	0	2	4	5	2	3	2	1	1	0	1	0	0	0	0	0	0	0	0	0	1
790	J	6	1	2	3	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
791	J	3	0	1	3	4	4	6	6	6	4	3	3	1	0	0	0	0	0	0	0	1	3	3	3	4	4	4	6
792	J	30	6	4	5	4	2	2	3	2	2	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	5
793	A	26	0	5	6	6	8	6	8	7	9	6	6	7	5	4	4	4	4	4	1	0	1	2	2	4	4	2	1
794	S	22	1	0	3	4	4	5	4	4	5	3	1	1	0	0	2	1	1	0	0	2	1	0	1	1	0	0	0
795	O	19	0	4	5	6	7	6	6	7	6	6	5	3	3	0	0	0	0	0	0	0	3	4	3	4	3	4	2
796	N	15	1	0	0	1	0	0	0	4	3	5	6	4	5	4	4	4	3	3	4	0	0	0	0	1	1	1	1
797	D	12	0	4	5	5	6	2	5	4	5	1	1	1	0	2	2	1	1	2	0	1	1	0	0	0	0	0	1
1891																													
798	J	8	1	0	0	0	0	0	0	1	3	3	5	8	6	4	4	4	4	4	7	10	7	8	6	4	6	3	1
799	F	4	0	1	1	0	0	1	5	6	5	5	8	8	7	7	4	7	8	8	8	7	7	8	6	4	4	4	1
800	M	3	0	2	2	3	4	3	3	1	4	2	2	3	3	3	4	6	5	3	2	3	3	3	1	0	0	0	0
801	M	30	3	3	4	4	4	4	4	4	4	3	4	4	0	1	2	5	4	5	5	6	3	0	3	6	5	8	7
802	A	26	6	8	9	8	7	5	6	3	5	4	5	4	6	5	6	8	10	12	11	10	6	7	9	9	12	16	12
803	M	23	15	11	15	9	15	12	7	8	7	8	10	10	7	4	3	3	3	4	6	7	7	8	10	11	13	17	12
804	J	19	11	6	11	14	14	12	13	14	15	17	16	15	16	13	11	7	6	7	9	11	15	17	18	18	21	23	22
805	J	16	16	15	14	16	17	17	13	13	10	4	5	6	6	7	3	3	3	5	7	10	14	12	12	11	10	11	11
806	A	12	11	10	10	9	10	10	8	2	1	1	3	2	4	7	5	3	0	4	6	5	7	11	15	17	24	21	21
807	S	8	19	15	14	11	11	9	8	7	9	12	11	12	13	8	5	5	5	6	6	5	9	9	11	13	13	15	19
808	O	5	16	15	19	18	18	13	13	10	8	8	8	11	9	9	7	7	6	8	10	11	7	6	9	6	7	6	8
809	N	1	10	7	8	10	8	10	11	10	7	8	7	6	3	4	4	6	8	9	13	14	15	13	11	12	13	6	7
810	N	28	7	8	5	6	4	4	4	4	6	6	7	6	4	3	0	3	6	6	7	0	7	7	7	10	13	12	9
811	D	25	7	9	8	11	12	14	12	17	22	22	15	16	10	11	11	7	8	4	5	4	9	16	18	16	21	23	22

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1890																													
785	J	22	7	9	5	4	3	3	3	9	3	0	8	3	14	7	7	5	2	8	6	2	8	8	5	7	9	7	6
786	F	18	10	10	9	3	4	2	3	1	4	7	0	4	2	3	4	6	5	7	3	4	6	9	9	9	6	7	15
787	M	17	9	10	5	5	4	8	5	4	4	4	2	1	0	1	3	11	8	0	0	7	7	8	5	5	3	5	5
788	A	13	5	4	5	5	7	3	7	5	5	8	8	5	4	3	4	4	5	8	5	3	6	10	13	8	5	4	6
789	M	10	5	3	5	5	6	4	4	8	5	3	5	6	4	5	7	8	3	2	1	2	1	2	8	8	5	8	8
790	J	6	5	4	1	4	5	7	5	3	2	3	2	6	5	5	4	11	10	4	4	2	1	2	6	5	5	4	2
791	J	3	2	3	6	9	9	8	4	5	7	5	4	2	3	2	14	11	14	10	9	6	6	6	4	3	1	2	2
792	J	30	2	4	5	3	1	3	5	6	8	8	6	6	2	1	2	11	14	11	10	9	7	8	6	3	3	5	3
793	A	26	5	3	2	4	2	4	3	5	9	7	9	9	7	3	0	5	14	15	9	9	9	11	7	5	5	10	9
794	S	22	6	2	5	9	3	3	5	9	5	5	4	8	1	9	12	3	9	7	15	10	10	8	7	12	5	9	17
795	O	19	12	7	5	5	4	10	5	8	10	5	3	5	5	9	7	0	5	4	1	9	18	11	8	5	5	10	11
796	N	15	11	7	6	6	5	4	9	5	5	5	5	6	4	3	3	5	5	3	5	2	9	6	5	6	5	5	5
797	D	12	4	6	5	5	1	5	2	2	6	7	10	6	5	6	3	2	7	4	5	4	3	2	2	1	8	4	0
1891																													
798	J	8	1	2	7	8	7	2	5	4	16	12	10	10	6	4	6	6	12	4	1	3	9	8	3	1	7	3	3
799	F	4	2	6	7	5	4	12	8	12	15	13	15	14	8	10	7	5	2	2	4	6	4	8	10	5	2	1	19
800	M	3	16	12	15	8	7	5	5	4	3	10	8	6	11	12	12	10	8	5	4	4	11	14	8	8	8	3	1
801	M	30	8	15	13	11	8	6	4	3	14	18	13	9	10	15	12	8	5	6	15	12	2	12	9	9	8	8	5
802	A	26	2	2	2	9	2	2	5	8	12	8	11	8	7	8	5	6	5	13	19	20	18	8	7	8	8	8	5
803	M	23	5	4	1	5	7	11	10	9	6	2	4	6	8	9	8	3	5	5	1	2	1	1	17	8	7	7	7
804	J	19	8	8	7	5	6	8	6	7	9	1	1	3	6	5	9	9	3	12	10	3	2	4	3	2	3	9	2
805	J	16	6	11	5	2	6	3	2	3	14	9	5	3	6	5	5	0	8	12	13	9	5	1	2	5	6	8	8
806	A	12	6	6	7	9	9	5	3	10	8	8	7	2	2	2	4	2	7	16	12	10	10	9	11	9	3	2	2
807	S	8	6	13	18	18	14	7	8	9	5	6	1	2	1	7	5	10	8	2	10	9	18	12	11	9	9	7	5
808	O	5	4	2	6	12	8	11	7	9	8	5	2	0	0	10	10	11	5	1	12	17	11	12	9	9	9	7	5
809	N	1	3	2	3	3	4	2	2	3	0	7	5	3	4	9	10	10	11	5	7	15	16	9	5	4	5	8	7
810	N	28	8	5	2	2	3	2	4	3	4	19	12	12	8	10	8	8	10	6	2	1	1	8	9	9	10	8	2
811	D	25	4	3	2	4	3	15	4	5	2	4	16	16	18	7	8	3	4	9	7	5	2	4	10	14	10	8	6

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1892																													
812	J	21	23	17	20	18	15	13	10	10	13	7	16	15	13	14	11	16	14	13	14	14	17	21	23	25	24	27	26
813	F	17	24	23	19	20	16	14	9	12	7	4	5	4	4	5	4	3	5	6	7	6	5	7	8	10	11	8	10
814	M	15	8	7	13	10	7	11	14	15	17	17	20	20	19	14	15	10	10	7	8	6	7	5	10	10	11	11	10
815	A	11	12	15	10	8	7	6	10	13	14	18	22	25	33	34	27	21	19	21	16	15	11	16	18	15	19	12	17
816	M	8	13	11	10	11	10	10	13	14	14	12	11	14	14	16	17	20	23	18	21	25	27	24	26	25	23	20	17
817	J	4	14	7	7	10	11	12	13	14	13	16	12	11	16	19	17	25	24	25	24	24	18	20	17	16	13	11	4
818	J	1	6	8	8	13	14	13	16	25	25	25	22	16	18	17	13	11	18	20	21	13	7	6	9	11	16	18	18
819	J	28	18	21	20	23	25	27	26	26	22	16	16	17	18	21	21	26	26	26	29	29	23	24	21	19	27	25	26
820	A	24	16	14	8	12	16	12	13	12	10	14	11	12	9	4	14	5	12	12	15	14	13	15	14	9	9	9	12
821	S	20	13	12	12	12	14	15	17	20	19	20	21	21	21	24	26	27	19	16	15	15	13	8	7	8	15	13	6
822	O	17	9	13	13	15	16	17	18	10	9	4	4	10	15	19	19	19	19	19	17	17	12	10	10	11	11	10	7
823	N	13	10	9	10	11	12	12	12	11	8	13	15	16	14	13	17	18	22	22	21	23	26	27	32	27	16	22	23
824	D	10	24	19	21	12	9	10	7	7	4	4	3	7	9	9	18	19	17	18	19	16	14	17	24	18	15	18	18
1893																													
825	J	6	14	15	10	8	8	6	7	6	4	7	8	9	13	14	16	14	14	12	21	24	30	24	29	27	23	20	23
826	F	2	18	16	7	12	9	9	8	16	14	13	16	19	18	20	19	15	13	15	13	15	15	15	14	15	18	17	16
827	M	1	19	18	12	13	14	9	12	15	12	13	10	8	9	11	9	7	7	7	12	14	15	12	12	17	19	21	19
828	M	28	25	22	20	9	13	13	15	10	10	10	15	12	16	15	18	21	20	19	18	18	8	14	15	20	17	19	30
829	A	24	22	29	22	23	24	24	30	29	26	21	26	25	17	18	16	15	16	14	15	15	13	11	13	11	16	8	7
830	M	21	10	7	12	15	23	22	23	25	26	23	19	16	18	24	26	29	31	32	26	25	19	17	14	15	16	17	20
831	J	17	20	13	10	9	9	15	16	15	19	18	14	17	15	17	11	10	10	15	18	19	27	28	36	40	37	32	38
832	J	14	22	20	18	20	14	13	13	14	12	13	12	11	10	13	11	7	5	15	11	13	21	26	30	33	39	43	39
833	A	10	48	36	32	24	21	18	15	18	27	32	35	34	32	35	33	24	20	11	13	18	15	17	15	18	18	19	21
834	S	6	20	21	21	16	16	15	15	16	15	14	10	16	11	10	13	11	15	15	20	22	20	14	14	14	14	15	15
835	O	3	15	11	15	13	14	14	13	15	13	13	12	15	20	19	16	15	18	22	24	24	25	23	23	16	17	19	13
836	O	30	12	10	10	10	5	7	6	8	8	11	13	18	21	25	22	21	28	24	20	26	22	24	19	13	12	18	16
837	N	26	11	10	12	9	13	14	13	14	15	14	8	10	6	3	7	9	9	14	14	12	17	20	20	20	13	33	27
838	D	23	29	37	29	33	30	26	31	33	35	28	14	11	12	13	16	13	9	11	9	17	15	15	19	19	19	23	20

Table J. International Daily Magnetic Character-figures

Sunspots/5		•	0	1	2	3	4	5	6	7	8	9	10	11·	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1892																													
812	J	21	6	5	4	7	4	5	4	8	16	7	9	7	12	10	11	9	9	11	7	7	6	6	6	20	19	13	6
813	F	17	5	6	9	14	15	5	5	10	7	13	14	6	6	17	11	11	11	8	20	17	11	9	6	17	20	16	6
814	M	15	10	6	5	6	5	5	5	3	3	10	16	11	11	12	9	11	11	9	7	6	5	3	5	10	9	13	11
815	A	11	7	7	6	6	6	1	0	1	1	1	0	1	8	11	17	20	13	6	11	9	19	16	11	9	11	9	9
816	M	8	14	9	6	6	3	2	6	3	9	13	20	19	6	8	11	2	7	4	1	3	7	7	8	8	8	16	19
817	J	4	10	4	4	5	4	3	5	1	1	1	1	2	10	9	6	4	3	9	6	4	9	9	3	20	12	9	9
818	J	1	8	9	8	3	3	3	1	4	9	9	7	19	14	16	6	19	19	10	6	2	12	12	2	7	9	18	10
819	J	28	9	11	8	6	5	4	13	14	9	8	9	11	8	8	6	20	10	4	1	4	5	4	2	7	5	4	8
820	A	24	11	12	11	7	4	4	4	6	9	8	6	6	5	9	9	9	4	6	11	5	12	9	8	11	9	4	4
821	S	20	6	16	16	7	6	4	5	6	10	9	8	6	9	9	9	11	9	5	6	1	10	11	14	11	10	16	14
822	O	17	11	18	13	11	11	10	8	11	3	4	6	4	3	8	6	4	5	6	19	18	6	4	3	4	5	1	3
823	N	13	3	10	10	6	14	11	8	4	8	10	6	12	7	8	5	6	5	6	9	3	3	14	18	11	7	9	4
824	D	10	1	5	11	11	9	7	9	7	5	8	4	9	8	12	11	8	6	7	6	9	10	9	9	6	8	9	15
1893																													
825	J	6	11	4	7	14	8	11	9	6	4	3	2	2	8	10	8	14	13	7	6	2	7	4	5	12	6	5	1
826	F	2	11	10	16	18	11	11	9	6	4	3	4	1	11	11	16	14	10	9	8	6	5	2	1	6	2	5	2
827	M	1	11	4	7	9	6	7	5	9	6	3	6	10	7	13	16	11	8	2	0	0	0	0	4	6	16	18	11
828	M	28	12	11	4	4	5	6	9	3	8	6	8	8	2	4	5	12	10	8	8	8	7	7	8	6	2	1	3
829	A	24	9	6	16	11	10	5	4	3	1	6	7	9	5	11	12	12	9	8	8	8	6	6	6	4	10	5	5
830	M	21	4	3	6	7	9	6	4	1	1	10	3	3	7	7	11	9	10	7	1	14	9	9	7	1	8	10	6
831	J	17	2	15	16	10	8	3	3	1	4	5	7	12	13	11	8	8	6	4	2	4	4	4	1	4	4	5	4
832	J	14	11	11	18	7	8	6	5	12	12	7	9	8	6	6	10	7	3	1	2	5	5	6	7	18	18	9	5
833	A	10	6	9	12	12	7	8	5	1	19	11	9	5	8	8	1	4	1	1	1	6	9	7	7	7	7	2	11
834	S	6	10	4	14	13	7	9	5	5	10	8	7	11	8	10	6	6	4	4	3	5	18	10	9	9	14	8	14
835	O	3	11	6	10	11	10	8	6	14	6	8	9	9	8	7	8	9	4	3	0	2	4	11	13	12	9	10	13
836	O	30	7	8	16	19	16	8	4	7	5	9	10	9	2	6	6	5	4	4	6	6	4	1	1	9	4	7	6
837	N	26	8	12	11	10	9	9	3	4	5	16	11	1	6	9	4	4	4	4	5	6	6	1	1	4	2	1	1
838	D	23	6	12	11	6	9	9	10	11	5	4	9	18	19	11	10	7	3	4	6	14	13	12	8	1	1	1	4

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1894																													
839	J	19	25	27	20	19	13	18	29	27	20	18	14	7	9	11	10	10	12	16	20	16	16	20	15	22	15	20	14
840	F	15	15	16	16	20	20	24	27	25	16	17	22	15	18	17	15	10	5	9	11	9	7	12	11	15	17	14	11
841	M	14	15	15	9	5	6	6	8	7	7	4	6	7	11	10	17	18	20	22	22	21	19	22	18	22	24	19	21
842	A	10	20	16	15	13	13	12	7	6	6	8	10	13	17	23	23	15	20	21	18	21	18	18	20	19	22	21	20
843	M	7	17	20	20	15	14	16	15	21	27	26	26	29	38	32	29	23	22	18	14	14	14	18	18	16	20	16	13
844	J	3	19	23	24	21	22	18	22	25	22	20	27	29	24	32	27	28	29	23	19	19	15	14	10	13	10	10	17
845	J	30	15	18	17	20	15	19	28	28	25	29	25	30	26	32	23	26	32	30	28	23	21	22	22	24	22	15	15
846	J	27	15	11	11	12	9	12	23	17	21	19	19	18	21	18	15	16	14	13	14	16	21	20	20	17	17	16	13
847	A	23	11	5	2	5	8	9	8	10	12	10	11	10	8	10	16	17	17	23	21	24	24	25	17	21	20	16	10
848	S	19	8	8	8	11	10	13	7	6	6	11	6	12	12	15	15	16	23	27	30	29	29	29	20	21	21	19	17
849	O	16	11	4	6	5	9	11	13	9	10	13	10	9	12	13	14	10	8	10	14	13	14	20	15	16	14	15	14
850	N	12	15	12	11	7	5	6	9	9	10	12	13	10	14	10	12	15	12	11	8	18	15	16	20	18	22	21	17
851	D	9	10	11	11	7	7	7	8	9	6	10	11	9	14	11	13	12	11	14	11	13	8	10	15	7	8	10	16
1895																													
852	J	5	16	9	15	9	13	16	13	11	6	4	8	5	7	10	11	7	13	12	19	14	16	15	24	23	27	23	18
853	F	1	21	22	18	17	13	10	13	8	10	9	9	8	8	10	9	9	13	15	12	13	13	14	15	21	17	18	24
854	F	28	19	21	15	13	10	14	11	12	14	15	15	15	11	15	11	13	17	13	9	13	12	13	13	13	9	10	11
855	M	27	11	10	9	9	11	13	18	15	13	16	14	11	13	15	17	18	17	16	11	11	13	14	9	11	12	19	16
856	A	23	18	20	20	28	21	20	18	17	16	19	11	6	6	4	9	8	6	8	5	14	12	13	8	7	9	9	13
857	M	20	10	16	15	21	31	30	28	24	22	20	16	12	15	16	12	11	14	18	16	16	18	16	14	11	14	14	21
858	J	16	23	17	15	14	15	11	12	15	15	14	11	12	12	15	13	13	10	11	11	10	10	13	16	20	18	17	16
859	J	13	16	16	12	10	11	8	3	5	6	5	8	5	2	2	5	9	9	6	8	13	17	21	21	22	28	27	25
860	A	9	20	20	17	18	14	11	11	10	12	13	11	12	12	7	7	11	11	7	10	8	8	8	8	8	10	10	8
861	S	5	10	11	11	11	10	9	5	7	11	12	13	15	12	9	9	10	10	10	10	12	10	19	20	21	20	23	24
862	O	2	29	25	18	21	21	14	11	8	4	4	10	8	8	7	6	3	3	5	6	11	14	12	21	20	16	21	21
863	O	29	19	24	20	22	20	18	16	16	10	8	8	4	0	2	2	5	4	12	14	11	9	10	11	9	14	10	10
864	N	25	10	8	9	8	9	3	4	7	12	8	11	12	11	16	20	15	22	23	22	18	14	16	14	19	15	12	16
865	D	22	14	18	20	15	19	16	12	9	9	11	10	7	8	7	11	11	7	8	4	2	2	3	7	2	3	3	3

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1894																													
839	**J**	19	4	2	9	11	6	6	9	11	4	6	7	9	5	4	11	11	8	7	9	7	4	5	3	4	9	4	6
840	**F**	15	6	7	9	9	11	14	20	14	20	19	20	13	9	19	14	10	5	7	3	10	2	8	8	6	4	4	1
841	**M**	14	5	5	9	9	11	7	5	12	15	11	7	9	8	3	2	4	19	19	7	4	4	3	6	11	11	8	4
842	**A**	10	6	2	9	13	8	6	4	19	14	11	7	9	6	3	9	8	8	5	9	11	10	11	9	6	11	5	5
843	**M**	7	9	9	7	4	4	4	10	14	11	9	8	7	5	6	8	11	8	5	4	3	5	11	9	8	11	11	6
844	**J**	3	11	8	4	4	4	6	12	19	12	7	7	7	7	6	11	9	11	7	11	9	7	5	7	5	6	7	6
845	**J**	30	8	9	16	6	8	8	6	4	11	9	5	4	6	8	4	10	9	11	11	12	20	15	9	6	9	8	4
846	**J**	27	7	11	7	7	6	4	3	4	8	7	6	5	5	9	6	7	7	12	11	11	7	1	2	8	20	11	9
847	**A**	23	6	9	10	8	6	6	6	5	4	5	5	1	2	7	3	6	5	5	11	10	10	1	18	19	7	6	10
848	**S**	19	16	16	11	11	10	8	7	4	6	9	5	10	6	9	7	10	13	5	6	6	6	2	4	1	4	7	4
849	**O**	16	15	11	10	11	5	4	4	4	6	11	8	11	6	6	10	11	7	7	8	3	1	4	10	10	5	5	5
850	**N**	12	6	19	18	8	11	11	16	11	6	0	1	13	11	9	6	5	6	6	9	4	4	3	1	9	9	7	7
851	**D**	9	5	4	3	6	11	8	14	9	6	4	5	5	9	11	6	9	3	1	4	4	5	4	3	14	8	6	6
1895																													
852	**J**	5	3	12	10	8	10	9	8	9	3	3	5	11	12	15	12	10	8	5	8	8	4	3	1	2	4	3	2
853	**F**	1	9	9	5	3	8	16	12	15	18	16	7	5	7	13	17	16	12	8	6	11	5	2	6	16	4	1	5
854	**F**	28	8	8	8	11	11	10	12	7	16	15	7	5	2	17	17	11	12	12	9	8	7	6	12	7	4	10	11
855	**M**	27	9	7	4	11	10	11	7	7	8	12	12	8	5	7	9	18	15	10	8	8	11	6	7	8	7	5	2
856	**A**	23	17	6	10	8	8	3	2	4	7	11	8	1	8	7	9	9	8	17	7	3	4	10	6	3	5	8	4
857	**M**	20	5	8	8	4	4	4	2	5	11	17	11	10	9	13	15	12	8	9	8	2	8	6	9	3	2	2	2
858	**J**	16	9	11	7	4	2	3	6	8	7	1	2	7	8	7	11	13	9	5	5	5	12	2	6	5	4	9	12
859	**J**	13	13	13	12	6	6	4	3	8	6	5	5	5	1	12	14	9	8	8	5	5	2	2	2	8	5	0	3
860	**A**	9	9	17	9	6	5	6	5	5	5	9	5	5	3	1	7	5	3	6	2	3	6	3	5	3	2	5	15
861	**S**	5	12	2	0	0	4	5	2	5	3	9	14	8	5	8	12	12	3	8	6	4	5	5	2	2	7	15	10
862	**O**	2	8	4	11	11	8	5	9	5	3	6	15	16	16	12	12	16	7	8	8	5	5	7	5	2	11	16	14
863	**O**	29	12	11	8	8	12	5	7	7	3	2	12	18	17	17	15	12	4	12	6	4	4	2	5	2	5	15	17
864	**N**	25	8	8	8	9	10	7	5	4	5	5	3	2	17	14	12	11	10	7	5	4	8	2	5	7	5	4	12
865	**D**	22	8	8	13	7	5	5	2	1	5	2	1	5	16	17	14	12	10	9	12	10	8	7	8	10	6	8	15

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1896																													
866	J	18	3	5	5	5	3	5	4	4	7	11	11	10	10	10	12	11	12	12	11	10	9	13	7	7	6	7	9
867	F	14	13	8	6	3	4	6	10	11	20	25	22	16	22	18	19	17	18	14	22	18	15	14	11	10	6	8	4
868	M	12	6	6	7	10	3	3	6	7	11	8	11	13	17	16	13	6	9	8	14	21	17	23	19	23	19	16	13
869	A	8	12	12	9	10	5	3	2	0	0	0	3	3	7	8	8	9	6	9	10	9	9	5	7	6	5	6	6
870	M	5	7	5	4	7	5	4	5	5	6	4	3	0	3	4	4	6	5	6	4	3	4	7	10	14	13	13	10
871	J	1	12	12	13	7	7	7	5	4	9	12	16	16	18	12	14	12	11	8	4	3	5	10	13	16	14	14	11
872	J	28	8	6	6	7	8	6	8	9	9	11	9	8	8	8	7	14	17	11	11	13	15	17	16	17	10	11	8
873	J	25	8	7	4	4	4	3	2	2	4	4	4	3	2	0	0	3	5	4	4	6	5	6	9	8	8	10	8
874	A	21	8	8	8	7	7	5	6	5	7	12	10	10	6	5	4	6	5	8	11	16	17	19	23	19	21	21	24
875	S	17	22	24	18	21	16	11	10	11	7	5	3	5	6	6	4	3	4	2	0	3	2	4	2	5	5	6	4
876	O	14	6	5	5	9	13	14	10	10	12	10	10	9	7	8	6	4	5	4	5	4	6	8	6	5	7	9	12
877	N	10	15	15	17	16	19	10	9	10	6	4	3	7	6	6	5	4	5	5	6	4	7	6	10	10	7	13	12
878	D	7	11	9	13	10	7	7	6	6	8	2	9	9	11	7	15	10	8	6	9	8	8	10	10	12	6	11	9
1897																													
879	J	3	6	12	11	10	13	10	12	12	8	7	10	9	9	9	10	8	10	8	9	8	11	7	5	4	2	5	5
880	J	30	3	10	12	14	14	15	11	12	8	8	3	4	4	3	2	3	3	2	4	3	2	2	2	2	4	6	7
881	F	26	8	10	10	9	9	10	10	11	11	10	6	10	10	5	4	4	2	6	8	9	4	3	4	0	3	2	4
882	M	25	5	5	4	7	7	7	7	7	10	10	9	8	12	14	14	17	16	11	12	9	6	3	3	2	2	5	4
883	A	21	4	2	0	0	2	3	0	2	5	4	7	7	10	10	12	6	7	7	5	4	3	3	0	0	0	0	0
884	M	18	0	2	3	6	4	5	3	5	3	4	5	5	4	4	4	3	3	4	3	2	2	0	0	0	0	2	4
885	J	14	4	3	2	2	0	0	2	3	6	3	4	4	4	2	3	4	5	5	6	7	4	4	4	6	9	7	7
886	J	11	5	4	5	8	8	7	5	4	4	5	7	8	6	7	8	7	5	7	7	6	7	5	5	5	4	4	6
887	A	7	7	6	9	8	6	9	7	8	4	4	3	2	2	2	2	4	3	4	4	5	5	2	3	4	5	7	9
888	S	3	14	12	17	15	13	16	19	13	10	10	7	6	3	2	2	7	10	9	12	9	11	11	9	8	10	9	11
889	S	30	7	6	8	5	8	9	5	6	4	2	3	4	2	5	2	4	3	3	2	0	4	4	2	2	2	0	0
890	O	27	0	0	0	0	3	2	4	2	2	2	2	2	2	2	0	0	0	4	2	0	0	2	2	2	4	4	0
891	N	23	0	0	3	4	2	3	2	7	6	6	5	11	10	10	12	13	12	9	14	12	10	11	10	11	8	8	6
892	D	20	2	2	2	2	2	0	2	4	4	3	5	5	6	5	6	7	7	6	6	3	5	4	3	3	4	5	5

Table J. International Daily Magnetic Character-figures

		Sunspots/5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1896																													
866	J	18	9	16	14	8	8	6	6	9	5	8	5	8	16	15	14	12	14	17	12	6	4	11	9	7	6	6	11
867	F	14	14	12	7	8	5	7	3	7	4	4	4	4	9	11	19	16	8	7	9	19	12	7	12	6	5	6	5
868	M	12	15	10	11	9	5	2	1	1	9	2	8	6	4	9	17	14	15	9	6	11	8	5	8	9	9	4	3
869	A	8	8	8	8	7	8	4	2	6	0	9	8	6	1	14	14	13	12	11	8	6	5	6	2	6	18	19	5
870	M	5	1	4	7	4	2	5	8	10	5	5	6	7	14	16	10	14	9	9	11	8	6	1	1	1	3	2	1
871	J	1	1	2	4	4	5	2	1	8	11	6	4	5	2	14	5	14	8	7	4	2	3	2	0	1	3	8	8
872	J	28	4	8	7	4	2	11	12	11	9	4	5	0	2	10	12	8	7	7	6	0	2	0	2	5	2	11	9
873	J	25	10	6	11	1	3	4	1	13	14	9	8	2	11	13	9	8	9	7	5	2	3	3	1	9	9	5	8
874	A	21	14	9	8	8	8	4	1	1	11	9	3	4	5	12	9	9	9	6	5	4	4	4	7	8	6	7	8
875	S	17	11	19	4	16	6	8	8	10	4	4	2	1	7	4	11	6	6	3	4	2	1	9	12	12	19	19	9
876	O	14	9	10	10	6	2	9	7	4	3	4	4	2	1	1	1	4	9	5	2	1	0	6	11	16	19	14	12
877	N	10	11	4	2	4	6	7	8	5	10	6	5	8	1	2	1	0	3	6	2	2	2	8	4	15	18	11	8
878	D	7	5	1	2	6	6	1	14	12	11	7	4	0	0	1	1	1	6	5	5	5	10	10	9	4	1	12	19
1897																													
879	J	3	12	4	4	1	4	5	2	6	10	9	7	4	8	6	8	5	3	1	1	1	2	2	4	2	4	8	11
880	J	30	10	8	6	1	12	12	5	4	7	4	2	8	6	6	7	10	7	2	2	0	2	4	7	6	8	9	13
881	F	26	12	15	11	10	8	11	11	8	5	9	12	10	11	9	9	8	5	5	1	8	2	5	1	1	6	8	9
882	M	25	4	1	4	8	14	9	7	9	17	2	8	12	14	11	6	8	11	2	1	8	9	2	8	10	9	11	19
883	A	21	6	2	19	14	11	9	7	6	6	4	4	12	9	6	7	10	5	2	1	6	7	4	11	13	7	4	14
884	M	18	6	9	8	14	11	8	7	5	2	3	2	4	14	8	7	10	9	8	6	2	6	3	1	1	8	3	6
885	J	14	6	7	14	14	8	8	7	8	8	5	3	5	4	8	6	4	2	2	4	8	3	2	6	8	6	2	5
886	J	11	4	4	1	9	8	4	3	1	4	3	8	10	6	6	4	1	3	3	2	9	14	12	6	6	2	1	1
887	A	7	2	4	10	7	5	5	7	7	9	8	5	7	6	12	4	5	6	1	6	8	5	4	8	8	4	4	3
888	S	3	7	12	12	7	5	3	5	6	13	9	8	11	6	7	6	4	3	3	8	6	10	7	6	1	5	2	3
889	S	30	3	12	12	9	7	4	6	4	1	6	11	8	7	2	2	8	6	9	12	8	1	1	4	7	5	4	6
890	O	27	10	13	11	9	5	10	8	5	8	6	7	3	2	7	6	6	2	6	10	8	4	12	11	6	11	6	4
891	N	23	4	13	11	7	8	5	6	1	8	6	3	7	7	8	5	0	3	8	20	12	1	4	11	9	11	8	5
892	D	20	19	16	11	8	7	6	1	1	1	11	11	14	9	7	2	1	3	3	1	4	3	9	7	7	7	4	11

Table I. Daily Relative Sunspot Numbers divided by 5

unspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
898																													
893	J	16	8	10	13	13	9	10	10	11	10	11	5	5	4	2	2	0	0	0	0	5	4	7	8	6	10	11	10
894	F	12	11	13	10	14	14	12	19	14	10	4	4	6	4	4	5	4	5	5	4	8	10	11	9	13	16	18	18
895	M	11	18	17	18	20	16	13	9	6	3	0	0	5	0	0	2	2	2	2	2	0	3	2	3	4	3	3	6
896	A	7	6	7	6	8	4	4	4	0	2	0	0	0	0	0	0	0	0	2	2	3	4	8	8	7	8	6	3
897	M	4	6	2	4	8	9	10	5	5	5	9	7	7	6	4	4	3	5	6	6	4	4	5	6	6	5	2	4
898	M	31	9	6	9	9	5	3	5	5	5	0	2	2	2	2	3	4	5	5	7	8	3	2	2	5	3	7	9
899	J	27	7	8	5	6	3	0	0	0	0	0	0	0	0	2	0	0	3	0	2	3	2	2	2	3	3	4	3
900	J	24	4	4	4	2	2	2	3	11	11	13	13	12	11	12	9	8	9	8	8	12	10	9	8	6	7	4	2
901	A	20	0	2	4	4	2	0	4	4	6	2	2	4	5	3	4	4	8	8	12	10	11	12	13	14	14	11	11
902	S	16	8	8	8	6	3	2	5	4	4	3	4	3	7	8	9	11	7	8	13	14	10	11	9	8	6	4	6
903	O	13	4	4	5	6	4	3	7	2	2	2	4	5	6	7	12	12	14	9	9	13	13	12	13	14	15	14	11
904	N	9	10	11	7	7	5	4	3	3	2	3	3	3	2	3	2	2	3	3	3	4	4	5	4	5	5	5	5
905	D	6	5	6	5	5	4	3	2	2	2	0	2	2	3	4	2	2	2	0	0	0	0	0	0	3	6	4	3
899																													
906	J	2	7	5	6	7	12	8	8	9	5	6	5	4	3	4	4	3	2	2	0	0	1	3	2	3	3	4	3
907	J	29	3	4	3	4	6	6	5	4	5	2	4	3	3	2	0	0	2	0	0	0	0	2	2	1	2	2	0
908	F	25	0	0	0	3	4	4	5	3	0	0	0	0	2	3	2	2	2	2	4	6	7	7	9	8	9	11	6
909	M	24	4	5	6	5	3	2	2	2	3	2	4	3	3	2	2	0	2	3	6	5	6	4	4	4	3	3	3
910	A	20	2	3	3	4	3	3	4	4	3	2	2	2	2	2	2	2	0	0	0	0	0	0	2	2	1	2	4
911	M	17	0	3	3	5	3	2	2	3	3	3	3	2	2	2	2	2	0	2	2	3	3	3	5	6	5	6	8
912	J	13	7	6	7	6	7	6	5	3	0	1	2	2	4	6	8	7	10	5	5	3	6	6	6	5	4	4	5
913	J	10	4	6	5	5	4	4	3	0	0	0	0	0	0	0	0	0	3	3	2	5	3	1	0	3	0	0	0
914	A	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	4	4	3	0	3	2	0
915	S	2	0	2	2	2	0	0	0	0	0	0	0	1	3	2	0	0	2	2	2	2	2	2	1	2	6	7	8
916	S	29	7	4	3	2	0	0	0	0	0	0	2	2	3	2	0	0	0	0	0	0	0	0	0	0	3	4	8
917	O	26	10	14	13	11	5	4	2	2	0	0	0	0	0	0	0	0	3	2	5	2	3	4	5	4	4	2	1
918	N	22	0	2	1	3	3	0	3	2	2	2	2	2	3	3	3	3	4	2	0	2	3	3	5	7	6	6	5
919	D	19	5	0	0	0	1	0	0	0	0	2	2	2	2	2	2	1	0	0	0	0	0	0	0	0	3	7	6

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1898																													
893	J	16	17	15	13	11	11	9	5	0	4	7	9	7	9	5	6	5	0	2	2	3	12	7	1	4	6	2	20
894	F	12	15	10	16	13	13	11	6	5	10	11	8	1	4	3	1	1	3	4	11	1	4	9	10	3	7	7	9
895	M	11	16	10	7	7	20	20	11	12	11	11	7	5	6	5	3	7	7	6	4	6	5	4	3	4	11	7	10
896	A	7	15	10	7	7	6	14	11	11	11	8	7	9	3	4	0	1	6	4	3	2	6	5	1	7	11	5	9
897	M	4	14	10	5	5	5	6	5	12	13	8	5	7	5	5	5	3	5	1	4	0	0	1	5	7	9	12	16
898	M	31	11	9	6	5	3	0	5	14	10	6	8	7	5	3	5	9	4	3	3	9	3	1	6	9	7	9	14
899	J	27	10	7	7	9	6	0	7	5	7	7	8	7	6	3	5	5	7	2	1	2	4	0	7	13	11	14	11
900	J	24	10	9	7	11	7	4	7	4	4	8	13	7	8	7	5	6	4	3	5	5	6	1	5	15	11	11	7
901	A	20	9	9	11	9	3	0	7	9	11	8	4	5	6	10	17	7	6	0	1	9	20	19	7	5	7	9	5
902	S	16	5	11	5	3	6	4	5	7	11	9	4	5	12	15	7	8	6	3	1	3	3	7	3	3	3	3	3
903	O	13	7	5	7	3	3	2	7	11	10	11	5	5	17	10	10	13	17	15	7	5	6	7	7	3	4	7	5
904	N	9	7	3	3	6	3	1	7	5	11	11	6	9	17	18	8	6	5	9	7	3	3	3	6	6	9	4	12
905	D	6	9	10	6	7	3	5	2	5	11	11	12	6	6	12	7	8	7	3	3	3	1	8	7	4	5	5	4
1899																													
906	J	2	6	10	9	4	6	3	2	2	2	4	8	3	7	12	11	9	14	8	8	3	5	11	5	3	4	1	16
907	J	29	16	9	7	6	6	5	1	3	6	3	1	5	2	8	20	9	13	8	8	6	2	8	7	9	8	14	11
908	F	25	11	8	7	11	6	5	8	4	1	9	9	6	8	15	14	9	12	8	9	7	5	5	0	2	14	17	18
909	M	24	9	5	4	3	6	5	3	4	8	8	5	8	10	6	8	8	8	10	9	6	1	5	3	1	9	17	14
910	A	20	9	4	5	6	9	8	2	3	4	9	8	12	11	14	16	16	6	7	7	4	3	11	7	1	2	14	9
911	M	17	8	4	6	9	4	6	6	4	1	11	5	1	0	5	12	9	8	7	7	4	1	1	0	1	5	9	5
912	J	13	9	6	4	4	1	5	4	1	4	6	6	3	1	1	11	14	18	16	7	12	14	12	8	9	9	8	6
913	J	10	7	9	8	8	4	2	5	3	4	1	2	2	1	1	5	10	7	6	2	1	1	0	2	3	12	10	8
914	A	6	8	7	7	6	5	1	1	7	5	2	1	5	2	1	11	11	9	4	5	4	0	8	6	11	10	13	10
915	S	2	8	8	6	2	1	2	3	6	8	4	5	6	5	11	6	6	11	8	1	5	5	8	6	6	14	12	8
916	S	29	10	8	4	1	2	7	8	9	6	2	2	1	1	2	1	2	11	6	8	7	4	1	3	7	15	11	8
917	O	26	7	8	3	1	1	2	2	1	6	10	6	5	4	1	2	1	6	8	7	2	3	1	4	1	9	1	2
918	N	22	13	11	8	6	4	2	2	5	8	9	11	8	5	2	2	6	8	6	4	2	5	3	1	0	5	6	11
919	D	19	12	8	9	7	5	1	5	9	9	10	11	7	7	3	3	1	3	13	6	1	1	1	7	4	9	7	10

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1900																													
920	J	15	4	5	3	2	2	2	0	0	2	2	3	5	4	4	3	4	3	3	2	2	9	8	10	9	8	11	6
921	F	11	4	5	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	4	2	8	6	4	4
922	M	10	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	4	5	5	3	3	3	3	3
923	A	6	2	3	2	2	2	2	2	2	2	2	0	0	0	2	2	2	2	3	4	7	7	8	11	14	11	8	11
924	M	3	11	8	7	5	2	1	0	0	0	0	0	2	2	0	0	0	1	1	2	3	3	3	3	3	6	5	3
925	M	30	6	5	5	3	3	1	1	0	0	0	0	0	0	0	0	1	2	3	4	5	5	6	8	6	4	4	4
926	J	26	2	0	2	5	6	2	2	3	2	0	0	0	0	0	0	0	0	0	2	4	5	4	5	6	4	4	5
927	J	23	3	4	2	2	0	0	0	0	0	0	0	0	0	0	0	1	2	3	4	5	6	4	2	0	0	0	0
928	A	19	0	0	0	0	0	0	0	0	0	1	0	2	0	2	3	4	4	4	6	5	5	5	5	2	3	2	0
929	S	15	2	2	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	2	0	0	0	0	2	4	3	5	3
930	O	12	2	2	0	0	2	3	5	6	7	7	8	7	4	5	2	2	3	3	2	0	1	3	0	0	0	0	0
931	N	8	0	0	0	0	0	2	2	2	2	3	3	4	2	3	2	2	2	0	0	0	0	0	0	0	0	0	0
932	D	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0	0	0	0
901																													
933	J	1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
934	J	28	0	0	0	0	0	0	2	2	2	3	2	2	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0
935	F	24	0	0	0	0	0	0	0	3	4	5	4	2	4	5	3	0	0	0	0	0	0	0	0	0	0	0	0
936	M	23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
937	A	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
938	M	16	0	0	0	2	4	5	8	8	8	6	6	6	4	4	4	2	0	3	2	0	0	0	0	0	0	0	0
939	J	12	0	0	2	2	2	2	2	4	2	2	2	4	5	4	3	0	0	0	0	0	0	0	0	0	0	0	0
940	J	9	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	2	2	0
941	A	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	2	0	0	0	0	0	0
942	S	1	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
943	S	28	0	0	0	0	0	0	0	0	0	2	2	2	3	2	4	0	0	0	0	0	0	0	0	0	0	0	0
944	O	25	0	0	0	3	2	4	3	2	0	0	0	0	0	0	0	0	0	0	0	2	2	2	3	2	2	2	2
945	N	21	2	2	3	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
946	D	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	4	3	4	5	5	3	3

Table J. International Daily Magnetic Character-figures

[illegible]			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
900																													
920	J	15	1	7	8	6	17	11	1	7	3	4	6	12	8	6	5	2	2	2	2	2	16	9	1	1	7	10	6
921	F	11	9	4	1	6	8	4	4	2	1	4	7	3	4	8	5	6	4	2	10	8	6	2	2	4	3	15	14
922	M	10	8	1	7	20	10	5	6	2	5	4	2	1	7	6	4	3	3	1	1	6	8	5	4	2	1	7	8
923	A	6	4	2	3	8	10	7	6	5	3	2	4	7	4	2	3	5	1	3	5	3	4	5	2	5	9	11	5
924	M	3	8	5	20	8	7	2	0	1	2	6	7	3	1	1	1	7	7	1	2	3	3	2	4	4	1	1	6
925	M	30	7	4	3	6	4	3	6	5	2	4	2	2	1	3	2	2	1	1	2	2	2	2	3	1	3	2	1
926	J	26	0	6	7	7	5	1	2	1	2	3	2	2	1	1	4	2	2	1	2	1	2	4	3	4	8	4	2
927	J	23	2	8	7	4	3	5	3	3	3	9	3	7	1	0	1	8	7	2	0	3	3	6	4	4	2	3	4
928	A	19	6	8	6	3	1	1	7	2	9	4	2	1	2	3	2	5	5	6	4	1	3	5	6	2	2	4	3
929	S	15	6	7	4	3	3	2	2	6	7	5	2	2	5	6	5	3	1	1	3	9	6	1	2	4	3	7	2
930	O	12	1	2	3	1	4	1	3	1	9	5	2	3	3	12	11	8	4	2	6	1	8	7	4	1	1	2	0
931	N	8	1	1	1	1	6	6	4	3	2	3	7	4	3	3	5	5	6	5	2	4	2	3	1	4	7	2	2
932	D	5	1	1	7	2	2	8	4	1	1	5	2	2	2	2	3	1	1	2	1	1	3	3	12	9	4	3	2
1901																													
933	J	1	4	6	2	5	11	6	5	4	6	4	3	1	1	4	3	2	2	2	1	2	6	12	13	8	3	4	6
934	J	28	4	2	5	2	2	12	5	2	2	6	5	3	2	1	0	8	7	2	1	0	2	1	12	9	3	15	10
935	F	24	4	2	1	0	1	2	2	6	5	1	4	2	2	4	1	5	6	8	4	2	1	3	6	8	5	6	2
936	M	23	10	18	8	3	4	1	9	6	5	8	9	2	2	2	2	6	3	1	4	6	1	6	8	8	7	2	4
937	A	19	2	1	1	6	6	5	2	3	8	6	3	4	5	4	2	2	1	4	4	3	2	17	9	7	2	5	5
938	M	16	2	0	5	4	2	2	1	8	9	6	7	6	4	4	2	9	8	5	2	2	1	5	8	6	6	6	4
939	J	12	3	8	11	8	6	4	2	6	6	11	6	5	4	0	2	1	2	6	7	3	3	2	6	4	6	5	4
940	J	9	3	2	8	13	5	3	5	5	11	8	8	8	1	8	4	2	6	4	5	1	1	1	5	2	4	8	6
941	A	5	5	4	6	6	5	5	2	1	1	15	14	7	5	2	1	7	5	5	2	0	5	4	5	1	4	2	8
942	S	1	4	3	4	3	5	1	1	1	4	16	10	7	5	3	1	6	8	6	1	1	4	3	6	5	1	4	5
943	S	28	1	4	5	4	1	1	4	1	3	6	11	14	7	4	4	8	5	4	7	2	2	3	3	3	4	1	1
944	O	25	10	5	1	7	2	1	1	2	4	6	11	9	6	6	2	9	3	9	4	0	2	0	6	2	1	10	5
945	N	21	2	2	1	1	4	2	2	2	1	1	9	11	4	4	7	2	4	4	9	5	2	3	3	1	1	0	1
946	D	18	1	4	5	2	2	2	8	2	3	9	15	6	5	2	1	1	5	5	2	3	3	3	2	4	2	0	1

Table I. Daily Relative Sunspot Numbers divided by 5

nspots/5		5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
02																													
47	J	14	4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	F	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	5	6	10	10	12	10
49	M	9	7	5	5	4	4	2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	A	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	M	2	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	2	2	2	2	2	2
52	M	29	2	2	2	3	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0
53	J	25	0	0	0	0	0	0	0	0	2	2	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	J	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	2	2	2	3
55	A	18	2	0	0	0	0	0	2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	S	14	0	0	0	0	0	3	4	4	6	6	6	3	5	4	3	3	2	2	2	2	0	5	6	6	6	7	4
57	O	11	4	3	2	2	2	3	0	0	0	2	2	4	6	5	8	8	5	6	5	4	2	0	0	0	0	0	0
58	N	7	0	0	0	0	0	0	0	2	6	4	3	5	5	7	8	7	6	7	4	3	0	0	0	0	0	0	0
59	D	4	0	0	0	0	0	0	0	0	0	0	0	0	3	2	2	0	0	0	0	0	0	0	0	0	0	0	0
60	D	31	0	0	0	1	4	2	2	3	2	2	0	2	2	0	0	0	0	0	2	3	2	2	4	5	2	3	2
03																													
61	J	27	0	4	3	4	3	2	3	2	2	2	2	2	2	5	6	7	6	5	4	3	4	4	4	5	4	3	3
62	F	23	3	5	5	4	4	5	5	5	4	2	2	0	0	0	2	0	0	0	4	2	3	0	0	0	0	0	2
63	M	22	4	4	5	4	3	4	6	9	11	11	10	8	5	6	5	6	6	6	4	5	7	5	3	2	2	0	0
64	A	18	3	4	3	2	5	4	4	5	10	13	11	8	17	13	13	7	7	3	0	2	0	0	0	0	2	2	2
65	M	15	2	2	2	2	2	4	5	4	2	2	5	5	2	6	3	2	0	0	0	1	1	2	2	0	0	2	3
66	J	11	3	2	3	3	3	3	3	4	7	12	12	9	10	7	5	4	3	2	0	0	4	4	4	2	2	2	6
67	J	8	10	10	12	12	12	8	5	7	7	7	6	4	5	4	2	3	6	6	8	6	7	7	5	2	4	4	4
68	A	4	4	4	3	5	7	12	14	12	18	13	13	13	12	12	2	4	2	3	4	3	3	4	2	2	2	3	2
69	A	31	1	0	0	0	0	2	0	0	0	4	4	3	2	3	4	8	8	4	2	2	0	0	0	2	3	5	4
70	S	27	5	4	4	2	2	2	3	4	8	10	12	17	19	21	19	13	15	15	10	9	9	4	2	3	4	4	2
71	O	24	2	4	4	6	5	6	10	11	13	13	13	14	23	21	19	20	19	12	17	18	9	8	4	5	3	2	2
72	N	20	3	3	3	2	5	3	5	7	3	5	4	7	6	18	15	13	13	16	18	20	17	11	10	8	9	10	9
73	D	17	12	11	8	6	8	6	6	5	3	3	0	5	7	9	4	12	10	9	7	7	6	5	6	6	4	2	3

Table J. International Daily Magnetic Character-figures

		Sunspots/5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1902																													
947	J	14	1	13	13	7	5	2	2	0	0	1	10	2	9	4	3	4	2	1	3	3	5	4	4	4	12	10	7
948	F	10	5	6	5	5	5	2	7	2	2	2	9	9	3	3	7	10	6	1	1	4	2	1	0	4	6	4	8
949	M	9	2	1	10	8	6	0	5	3	5	3	0	1	5	3	5	12	10	6	3	1	4	1	2	5	2	7	6
950	A	5	2	2	1	7	6	10	18	5	5	3	1	1	5	4	6	10	10	6	4	4	0	1	1	3	2	5	3
951	M	2	2	1	4	5	2	5	10	15	7	2	0	4	5	3	1	4	6	5	5	5	2	3	4	2	3	4	3
952	M	29	5	5	6	5	1	2	4	5	6	4	1	2	4	5	5	2	5	11	5	5	4	2	2	6	6	2	6
953	J	25	5	6	5	5	9	6	4	1	4	5	3	1	3	9	5	5	4	10	3	1	7	5	2	5	3	2	1
954	J	22	5	6	14	11	8	3	2	3	1	1	3	3	5	4	1	2	2	2	7	5	6	1	1	1	2	8	5
955	A	18	4	2	4	15	11	5	5	7	5	4	4	2	1	5	2	8	5	5	4	4	3	1	1	3	3	8	5
956	S	14	1	6	3	6	9	11	11	5	5	5	1	2	1	3	3	6	6	4	2	3	5	4	3	1	2	2	1
957	O	11	13	4	6	2	3	0	4	6	3	4	3	2	7	10	7	4	10	5	7	10	15	5	4	2	0	0	7
958	N	7	2	1	1	4	1	3	6	9	6	4	3	5	4	2	6	8	12	15	10	8	2	1	1	2	4	5	3
959	D	4	2	1	2	2	1	9	6	6	6	7	2	3	5	3	0	5	3	3	6	12	7	6	6	5	5	3	4
960	D	31	1	1	3	2	8	8	3	1	3	5	6	4	3	4	2	1	4	1	7	6	4	3	2	8	7	2	12
1903																													
961	J	27	10	2	1	6	2	0	1	1	1	4	2	5	13	5	5	4	8	8	3	7	2	2	1	3	4	5	7
962	F	23	1	1	5	2	0	0	7	5	2	4	9	4	8	8	6	4	2	8	10	5	4	1	0	2	4	3	4
963	M	22	7	2	2	0	0	1	1	6	8	9	3	6	3	6	11	20	7	8	12	8	5	4	4	3	6	1	5
964	A	18	10	6	3	3	2	2	3	4	9	6	3	5	4	2	4	2	8	11	11	8	5	6	3	1	2	5	5
965	M	15	6	8	10	1	2	1	6	5	9	7	8	2	3	5	6	7	3	12	13	8	6	5	2	2	5	4	2
966	J	11	3	2	2	4	4	10	8	7	7	6	7	8	8	7	4	1	2	11	13	13	7	4	2	2	6	10	2
967	J	8	2	5	9	8	5	5	7	4	6	6	9	10	5	4	1	1	0	9	12	10	11	6	3	5	5	5	1
968	A	4	8	8	4	1	8	8	4	14	10	9	10	5	5	3	5	0	7	10	17	10	7	7	15	8	4	1	8
969	A	31	2	7	5	2	8	11	7	7	10	8	9	7	10	6	5	3	2	3	2	17	14	8	6	12	9	5	3
970	S	27	8	6	12	7	7	9	5	7	10	9	8	10	3	3	9	19	18	12	6	5	6	6	5	1	1	8	7
971	O	24	5	9	11	6	5	8	9	20	20	11	10	10	9	7	8	8	9	12	9	9	6	1	1	6	7	11	11
972	N	20	5	8	12	10	5	1	4	2	2	6	5	7	10	6	11	9	7	9	7	4	4	2	0	20	8	7	4
973	D	17	2	0	3	10	8	2	3	1	3	2	4	2	2	15	17	12	10	9	7	10	7	3	3	10	12	8	6

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1904																													
974	J	13	3	5	4	5	8	11	8	7	8	15	13	15	9	7	5	4	2	2	0	2	2	2	5	5	6	5	6
975	F	9	10	8	7	7	6	4	3	2	3	2	6	6	8	5	7	7	6	7	6	6	4	3	4	4	5	5	5
976	M	7	6	7	7	7	5	3	2	5	6	4	5	7	10	14	12	13	12	12	13	11	11	12	14	10	9	6	6
977	A	3	9	7	8	3	2	5	6	7	8	9	9	8	7	8	8	8	10	8	10	15	16	15	15	11	9	11	12
978	A	30	12	10	13	10	10	10	11	6	13	12	12	13	9	10	10	11	11	7	7	4	6	4	4	5	4	6	5
979	M	27	6	9	6	7	5	4	4	4	6	7	9	9	9	6	9	9	9	13	17	13	11	10	11	11	11	11	10
980	J	23	12	9	10	6	5	4	6	8	8	8	10	7	7	9	7	8	4	7	6	6	7	8	12	18	19	18	19
981	J	20	16	9	11	14	12	12	11	9	10	12	12	11	8	10	6	10	11	11	7	13	11	13	14	13	13	11	9
982	A	16	8	4	5	7	7	5	7	9	13	17	24	23	25	25	20	13	15	10	7	3	3	3	3	4	3	6	5
983	S	12	4	5	5	2	4	4	6	7	8	8	13	13	11	9	8	7	6	5	6	6	6	5	5	8	9	12	14
984	O	9	14	12	14	14	15	11	9	11	8	7	11	12	10	11	9	8	15	15	17	19	12	14	16	9	8	5	4
985	N	5	6	10	7	4	3	3	5	4	6	5	7	8	7	10	7	9	8	8	14	12	12	13	12	11	13	10	9
986	D	2	4	4	4	7	7	15	17	18	21	25	24	23	22	20	17	17	11	10	6	6	5	8	8	5	9	4	5
987	D	29	6	5	8	6	4	2	3	4	0	10	12	10	13	17	15	16	17	16	18	16	18	20	19	16	12	10	6
905																													
988	J	25	11	7	7	10	10	12	12	13	17	19	26	20	32	22	24	27	27	22	27	20	21	13	14	13	13	12	10
989	F	21	9	11	9	13	14	16	18	11	14	18	20	24	26	19	17	20	21	13	8	6	4	2	3	4	6	9	8
990	M	20	7	11	12	10	11	11	8	9	11	12	10	9	9	10	8	10	7	6	6	4	5	6	10	11	12	12	10
991	A	16	11	10	7	5	6	5	5	5	8	11	11	10	11	7	9	12	8	9	8	5	4	9	13	17	18	16	16
992	M	13	17	15	16	15	17	16	14	11	8	6	3	0	3	5	7	6	4	6	7	8	6	11	9	12	11	13	13
993	J	9	11	12	8	7	8	11	11	10	10	10	6	5	6	10	12	10	15	15	15	13	9	10	8	10	8	13	12
994	J	6	11	9	10	12	17	20	28	28	29	30	28	24	20	20	28	27	25	16	8	8	3	2	0	2	3	5	3
995	A	2	7	7	12	14	19	16	20	21	20	16	10	12	11	10	6	12	10	15	12	14	16	16	13	10	8	6	7
996	A	29	10	7	17	15	10	17	13	14	13	12	14	11	14	12	14	16	16	18	15	12	15	12	7	3	2	2	3
997	S	25	7	5	9	13	16	15	16	17	23	21	18	22	23	20	15	14	13	12	10	9	8	17	19	24	29	17	23
998	O	22	20	14	12	12	12	9	12	15	11	13	15	12	22	17	16	18	16	16	20	32	37	27	34	32	29	11	17
999	N	18	21	13	18	16	19	21	20	29	24	22	29	28	22	15	18	16	14	14	11	13	12	13	7	7	12	11	7
000	D	15	9	7	7	11	11	13	15	13	10	15	14	14	11	9	10	10	9	10	8	7	4	5	6	8	6	7	6

Table J. International Daily Magnetic Character-figures

		Sunspots/5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1904																													
974	J	13	2	0	7	15	4	3	1	2	7	8	2	7	6	3	3	17	12	10	7	9	8	3	7	11	11	10	8
975	F	9	10	5	3	3	8	5	8	10	9	1	3	1	1	2	4	5	5	3	1	0	3	4	5	9	13	9	1
976	M	7	4	3	5	2	9	8	3	2	1	1	0	1	2	4	1	1	1	2	3	10	5	3	8	7	5	19	12
977	A	3	8	9	3	3	7	6	7	9	8	3	6	2	5	1	8	12	13	3	2	4	0	4	6	7	1	2	7
978	A	30	7	8	5	7	4	1	3	1	6	1	2	3	16	17	10	7	4	8	7	8	4	4	4	5	8	2	2
979	M	27	9	15	9	6	5	5	4	1	5	5	13	8	2	3	3	7	3	1	3	15	17	8	7	2	5	4	2
980	J	23	2	1	1	3	10	5	1	1	8	5	4	2	3	16	12	6	7	7	2	2	7	12	7	6	6	3	3
981	J	20	6	3	3	6	4	2	6	7	7	6	7	5	2	10	15	11	5	5	2	1	1	6	1	2	2	3	5
982	A	16	5	7	6	2	3	8	7	6	0	0	0	1	1	7	11	7	3	3	3	5	7	7	6	8	8	6	9
983	S	12	7	6	3	5	11	3	2	0	1	1	4	4	8	19	8	1	2	3	4	3	7	2	2	4	5	15	13
984	O	9	7	5	2	1	10	8	4	6	2	1	2	3	16	9	5	2	7	4	8	9	6	7	7	3	12	8	13
985	N	5	12	6	2	1	1	1	3	1	1	2	6	15	11	10	1	0	1	7	2	2	13	9	7	3	7	4	7
986	D	2	3	13	7	7	5	2	2	10	3	3	1	1	9	9	12	3	4	6	2	5	2	1	1	2	7	7	6
987	D	29	8	6	2	3	2	3	10	19	10	5	2	2	7	8	7	3	11	7	2	12	6	4	8	7	12	3	9
1905																													
988	J	25	8	5	4	7	10	3	12	8	10	20	15	15	7	7	5	5	7	2	5	5	13	8	8	6	3	1	3
989	F	21	6	8	13	7	7	7	5	1	7	20	10	8	13	8	17	9	5	5	3	5	5	7	10	10	3	2	4
990	M	20	3	2	3	5	3	2	2	8	1	6	6	5	19	11	9	8	9	8	5	2	2	1	1	7	3	7	7
991	A	16	3	1	0	2	7	9	2	1	0	2	9	7	4	9	8	8	6	5	3	1	1	2	4	3	3	2	7
992	M	13	6	2	1	2	6	7	8	4	2	4	6	5	3	6	11	7	8	6	6	3	4	6	7	12	10	4	6
993	J	9	13	14	10	4	1	3	7	3	3	1	0	2	6	10	14	7	3	3	2	1	3	6	7	1	2	1	7
994	J	6	15	10	9	8	3	1	4	8	4	1	1	3	3	4	1	2	3	13	14	5	1	4	7	2	2	1	3
995	A	2	18	12	8	5	6	13	6	2	1	6	6	8	7	2	8	4	3	5	7	3	5	6	5	7	5	8	9
996	A	29	12	12	11	7	5	17	11	10	2	3	6	7	8	5	4	2	2	3	1	1	15	14	6	6	7	3	4
997	S	25	8	9	12	7	4	10	7	1	1	3	6	13	8	7	4	5	5	5	10	7	1	2	7	8	1	4	3
998	O	22	2	1	4	5	8	3	15	0	1	0	1	1	2	10	7	7	7	5	4	5	2	20	15	10	20	20	10
999	N	18	6	5	2	6	4	6	7	3	5	7	5	2	2	1	7	7	15	7	3	2	2	1	1	1	11	12	6
1000	D	15	3	2	2	4	6	10	4	1	0	1	3	2	1	5	7	4	1	1	4	3	4	3	6	2	8	8	2

Table I. Daily Relative Sunspot Numbers divided by 5

nspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
06																													
01	J	11	6	7	10	10	8	8	5	2	2	3	10	11	16	18	18	21	20	18	13	13	7	14	8	8	6	6	8
02	F	7	3	2	4	4	3	3	4	6	3	3	5	7	8	6	11	13	11	10	10	8	6	9	5	10	5	3	2
03	M	6	6	5	7	11	11	10	7	7	10	12	10	10	12	15	19	29	23	21	21	27	23	23	19	14	18	18	24
04	A	2	23	15	13	10	14	17	17	13	13	13	12	9	8	8	13	10	6	7	6	5	4	4	5	10	17	13	14
05	A	29	9	11	9	11	8	8	7	7	11	12	8	9	9	10	14	22	16	16	10	7	8	9	7	7	10	14	19
06	M	26	20	22	17	16	14	12	14	15	18	15	16	21	13	14	14	12	10	8	7	5	6	5	3	4	3	7	13
07	J	22	9	7	9	14	18	19	26	29	36	25	25	27	28	28	27	22	22	21	22	19	15	14	15	19	21	22	25
08	J	19	24	24	21	24	22	23	17	15	14	16	18	20	19	11	9	10	7	4	2	3	4	6	7	7	8	10	9
09	A	15	6	10	7	5	6	8	9	12	11	13	16	16	16	14	21	22	18	17	13	11	11	8	10	16	14	14	10
10	S	11	14	12	12	11	17	14	15	15	13	13	11	11	12	7	9	8	9	7	10	6	6	6	7	11	9	4	6
11	O	8	8	6	6	4	2	2	3	4	2	0	0	0	2	3	4	2	5	5	3	0	5	3	2	4	5	3	5
12	N	4	5	4	4	3	5	6	10	9	14	16	18	11	11	5	5	5	2	2	3	7	9	12	13	17	14	11	10
13	D	1	18	14	10	11	10	5	6	11	9	6	7	12	10	17	17	12	17	19	17	23	23	18	16	14	7	12	12
14	D	28	14	15	14	15	17	14	21	15	20	21	8	10	11	12	15	19	17	22	23	24	20	19	14	14	11	12	12
07																													
15	J	24	13	14	12	14	11	12	17	21	19	20	18	20	27	24	21	26	26	21	35	30	31	28	25	19	20	14	13
16	F	20	15	18	22	21	25	20	22	18	17	20	20	18	22	24	18	21	19	21	18	16	17	16	10	14	18	11	10
17	M	19	8	7	8	7	7	8	6	4	5	2	5	3	6	11	16	18	19	18	22	18	16	14	10	10	8	8	14
18	A	15	10	5	6	6	7	9	13	10	7	8	10	10	9	7	5	5	4	2	6	8	10	14	16	17	16	17	13
19	M	12	16	13	12	13	8	9	6	4	9	8	8	9	9	5	4	4	4	6	3	4	3	6	6	4	5	7	5
20	J	8	3	6	5	5	7	12	15	14	13	19	20	17	18	11	12	10	7	9	5	2	2	3	3	4	6	8	7
21	J	5	7	5	6	7	6	6	10	13	11	12	21	17	15	13	11	13	14	11	14	10	10	10	8	13	14	11	9
22	A	1	10	10	9	9	13	11	12	11	9	11	11	10	10	13	15	14	13	14	16	14	14	14	9	10	8	5	7
23	A	28	12	13	13	10	13	14	14	15	14	18	20	24	21	22	20	23	23	18	16	11	14	15	15	15	16	14	14
24	S	24	15	19	21	22	19	19	18	13	17	12	11	11	6	6	8	12	18	16	19	18	19	16	17	20	20	16	19
25	O	21	22	22	18	13	10	6	7	6	5	5	10	8	9	7	11	10	11	13	17	15	25	22	28	18	18	16	12
26	N	17	15	14	18	13	12	11	11	11	12	6	7	6	2	3	3	2	3	4	6	8	10	9	11	16	17	14	14
27	D	14	14	16	18	14	13	12	12	11	11	10	9	8	5	5	4	8	8	6	5	9	10	12	16	13	14	12	9

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1906																													
1001	J	11	4	6	8	9	8	3	1	1	7	1	8	8	1	0	0	3	7	6	1	1	16	12	8	8	9	7	12
1002	F	7	9	7	8	9	6	4	0	6	14	11	5	6	17	4	5	2	8	19	17	17	8	15	12	4	4	15	11
1003	M	6	8	12	12	11	9	4	9	10	10	4	3	9	10	0	2	1	1	1	10	13	8	5	6	6	2	1	2
1004	A	2	8	10	8	1	0	0	7	11	12	10	3	6	5	1	1	5	4	5	4	7	9	11	11	9	3	2	12
1005	A	29	14	9	3	3	1	1	1	7	4	13	10	3	8	4	7	11	16	5	3	4	8	12	10	5	2	6	5
1006	M	26	1	10	6	4	2	1	13	15	11	10	5	3	7	10	7	4	8	3	7	3	9	10	7	4	0	1	1
1007	J	22	1	1	5	7	7	2	3	2	2	8	7	4	4	9	10	2	2	3	9	18	13	10	5	2	1	1	3
1008	J	19	3	5	9	2	9	3	6	8	10	9	12	15	8	6	4	5	2	0	1	15	17	10	6	5	13	12	7
1009	A	15	4	7	2	0	6	3	5	3	2	5	8	8	11	8	7	3	9	9	10	12	15	10	7	4	1	1	1
1010	S	11	2	5	4	6	9	10	8	7	3	9	8	18	17	11	10	11	6	5	7	10	12	9	9	6	6	2	1
1011	O	8	1	1	3	4	10	9	6	4	5	2	0	8	5	9	12	8	3	3	6	12	8	5	7	4	4	0	1
1012	N	4	5	7	4	5	10	5	10	10	5	7	2	5	3	5	9	9	4	13	14	3	6	8	3	3	2	2	3
1013	D	1	3	8	7	3	1	3	6	16	16	4	4	5	3	4	10	17	13	3	3	3	11	20	14	10	2	9	7
1014	D	28	6	2	3	3	3	5	2	4	10	3	9	14	5	8	16	10	5	14	13	7	3	3	2	3	1	7	8
1907																													
1015	J	24	9	6	6	9	9	5	8	6	6	8	3	2	2	4	15	14	19	18	14	10	8	13	9	5	7	2	4
1016	F	20	9	8	11	10	11	11	4	2	5	10	7	2	7	7	7	6	3	3	17	13	17	6	2	1	1	1	2
1017	M	19	4	5	17	13	4	2	9	4	3	4	3	1	0	0	0	1	3	10	10	2	3	2	2	8	2	7	11
1018	A	15	6	12	8	11	8	3	1	4	4	3	7	6	9	6	9	2	1	3	9	5	4	11	7	3	2	9	10
1019	M	12	8	11	10	11	8	5	14	14	9	5	7	6	9	7	2	1	9	14	2	8	6	5	8	8	2	8	6
1020	J	8	2	5	10	10	9	6	4	2	2	2	9	17	12	8	14	6	5	7	4	7	8	3	3	10	11	9	2
1021	J	5	8	8	10	9	6	14	16	8	2	3	1	2	2	3	4	2	1	7	5	3	11	8	10	16	10	1	6
1022	A	1	11	10	9	9	5	3	4	4	8	7	3	1	2	8	4	6	5	7	7	9	12	9	8	10	8	5	3
1023	A	28	2	1	15	9	2	3	4	8	9	7	4	2	1	17	12	12	5	1	2	11	14	11	10	9	5	1	1
1024	S	24	1	6	10	7	7	8	12	11	11	8	3	3	1	1	8	8	4	6	3	18	18	15	10	5	0	0	3
1025	O	21	6	14	8	5	8	6	12	8	10	6	2	1	7	11	10	6	2	7	9	5	13	12	9	4	5	3	5
1026	N	17	4	2	1	0	18	12	5	4	3	5	9	8	1	1	1	1	1	8	8	8	10	9	1	9	12	11	11
1027	D	14	4	2	2	5	4	4	7	2	3	2	1	2	8	7	6	4	6	3	3	8	7	7	9	6	8	9	14

Table I. Daily Relative Sunspot Numbers divided by 5

unspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
908																													
028	J	10	9	11	11	12	9	8	8	9	9	3	5	4	8	10	8	6	7	4	2	2	3	7	6	5	5	9	6
029	F	6	6	7	8	10	10	10	13	13	11	7	6	8	6	2	6	6	5	6	6	4	9	9	5	4	2	2	3
030	M	4	5	5	5	7	7	5	8	9	9	9	8	8	11	8	10	10	11	11	9	6	4	3	0	2	5	4	3
031	M	31	3	4	4	12	20	22	22	22	21	20	18	13	10	7	4	4	3	4	5	5	13	10	10	11	9	11	8
032	A	27	10	18	17	18	17	14	13	11	11	10	13	17	11	16	12	11	8	5	7	4	5	5	4	6	3	5	4
033	M	24	2	2	3	5	9	8	8	16	20	22	24	19	19	17	15	13	7	10	6	5	5	5	6	5	5	8	12
034	J	20	7	7	5	5	8	7	8	8	7	7	8	6	9	9	12	13	9	8	3	6	2	4	2	3	4	10	10
035	J	17	13	14	14	14	15	19	14	11	9	3	3	2	3	5	7	13	17	22	22	28	23	27	26	27	22	12	7
036	A	13	12	16	18	16	20	18	16	13	17	15	14	12	10	12	21	21	19	30	27	27	32	27	25	21	17	16	15
037	S	9	20	20	16	14	11	14	16	10	14	18	18	15	16	17	15	18	20	17	15	14	16	17	19	15	13	14	14
38	O	6	9	10	10	11	11	11	7	5	2	2	2	0	0	0	3	3	7	6	5	4	4	5	7	6	5	5	3
39	N	2	5	3	5	6	9	13	10	17	19	15	16	17	14	18	16	13	7	9	7	8	9	6	7	8	7	5	5
40	N	29	4	6	6	7	6	7	6	7	4	6	8	10	6	6	6	4	4	5	8	9	8	7	6	13	13	17	15
41	D	26	11	10	11	10	12	11	12	13	8	8	8	4	11	12	10	11	7	6	5	5	6	7	7	8	8	8	8
09																													
42	J	22	13	14	15	20	15	21	26	27	19	21	16	14	12	7	5	3	3	2	2	2	7	16	10	9	11	12	11
43	F	18	9	6	7	6	6	11	13	14	22	20	18	18	20	28	22	12	18	6	5	6	5	2	7	6	4	4	5
44	M	17	4	7	9	9	13	18	22	21	26	22	16	20	23	27	20	12	13	9	6	9	7	6	5	4	2	2	2
45	A	13	3	2	2	5	6	6	10	8	14	15	11	12	7	6	5	4	6	9	9	9	7	4	5	4	5	8	8
46	M	10	13	11	12	13	12	13	11	8	9	6	6	8	10	8	7	5	5	5	5	5	2	2	2	2	0	4	5
47	J	6	8	6	7	7	3	2	3	4	5	6	9	9	10	6	4	6	5	6	4	4	5	5	7	3	3	3	2
48	J	3	2	2	2	5	6	3	3	4	0	0	4	4	5	6	7	9	16	13	14	21	21	16	19	15	10	6	6
9	J	30	4	5	6	6	7	6	6	5	5	3	4	5	5	8	6	8	9	8	6	4	5	3	3	3	4	4	4
0	A	26	5	2	0	4	4	5	6	4	3	3	5	8	9	12	12	14	15	15	11	5	5	4	5	6	8	8	9
1	S	22	9	11	13	10	6	4	9	9	9	6	9	12	12	14	16	15	11	23	23	17	22	24	20	18	17	12	9
2	O	19	9	8	6	4	6	7	10	9	5	5	5	9	13	13	12	14	14	19	19	15	13	8	5	6	6	4	7
3	N	15	8	7	7	9	9	10	11	10	10	10	14	14	16	17	22	20	25	22	17	17	10	11	7	4	4	3	0
4	D	12	0	3	3	4	5	8	12	15	16	15	16	12	12	15	13	17	18	16	16	13	11	11	10	8	5	4	4

Table J. International Daily Magnetic Character-figures

		Sunspots/5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1908																													
1028	**J**	10	7	9	7	2	3	6	4	6	6	1	0	7	3	2	6	9	5	10	9	12	6	6	4	2	11	11	13
1029	**F**	6	14	13	7	7	9	12	11	8	2	3	5	4	1	3	1	1	13	12	11	4	5	9	6	11	16	13	11
1030	**M**	4	13	8	7	5	13	11	8	7	5	0	1	2	11	8	4	6	11	9	5	4	2	5	18	19	17	12	11
1031	**M**	31	10	7	3	4	3	8	12	12	10	4	2	1	2	1	6	13	10	10	8	5	0	4	11	11	10	9	10
1032	**A**	27	10	8	7	3	11	12	9	9	9	6	3	6	3	10	11	8	10	8	2	3	7	4	4	4	7	5	11
1033	**M**	24	12	14	16	12	11	10	10	8	7	4	14	11	2	9	6	5	3	1	1	8	8	5	6	5	9	10	10
1034	**J**	20	10	7	3	2	10	8	10	9	5	6	7	4	1	2	2	6	8	8	2	2	1	3	7	6	3	14	15
1035	**J**	17	11	7	1	2	3	2	2	9	12	6	3	3	2	3	1	6	6	9	3	3	4	10	13	15	6	11	16
1036	**A**	13	12	10	6	1	7	8	16	4	16	10	3	4	8	6	2	2	9	6	7	1	2	6	12	18	10	7	11
1037	**S**	9	11	12	18	20	8	5	4	13	11	6	6	2	1	4	5	8	7	3	5	14	20	19	11	4	8	9	13
1038	**O**	6	12	6	2	0	1	1	13	13	10	3	3	5	10	2	1	2	1	1	2	2	5	5	0	2	4	14	11
1039	**N**	2	8	6	1	0	1	6	18	13	11	9	6	3	9	5	2	17	10	5	3	3	1	1	5	3	4	4	10
1040	**N**	29	7	2	2	2	3	14	14	9	6	4	2	3	1	4	8	3	1	4	3	6	6	1	1	4	6	2	8
1041	**D**	26	11	4	4	3	3	4	16	13	20	14	7	1	0	2	2	2	2	1	11	9	6	2	3	7	5	2	3
1909																													
1042	**J**	22	1	3	7	12	9	12	11	16	17	19	9	14	10	7	4	13	7	7	6	7	1	1	3	3	2	3	4
1043	**F**	18	2	2	4	11	12	13	11	2	9	7	3	4	6	9	9	12	11	6	8	2	3	2	4	6	6	2	2
1044	**M**	17	5	13	18	7	12	11	8	2	11	11	13	15	15	5	4	5	3	2	2	2	1	0	1	3	10	10	10
1045	**A**	13	6	2	1	10	9	9	6	3	0	0	3	7	10	11	11	2	3	4	8	4	3	3	3	2	1	1	2
1046	**M**	10	4	7	4	2	20	18	8	4	18	14	5	3	7	4	0	1	9	8	7	3	1	7	8	7	3	2	4
1047	**J**	6	2	4	3	1	3	7	2	3	8	9	7	6	4	3	1	11	11	10	7	2	3	1	9	11	9	6	4
1048	**J**	3	7	4	1	1	2	7	6	1	3	10	10	8	9	5	2	0	1	2	8	6	10	11	3	3	7	9	9
1049	**J**	30	5	3	4	10	10	4	4	1	10	7	11	10	9	5	6	2	0	0	2	4	9	6	1	5	3	5	6
1050	**A**	26	9	8	10	14	14	10	6	6	12	10	9	9	9	10	9	4	1	0	1	3	6	2	2	0	0	4	15
1051	**S**	22	9	3	5	20	12	8	9	8	18	6	12	9	8	2	3	7	11	9	4	6	7	1	2	5	2	4	16
1052	**O**	19	18	8	7	5	17	16	9	4	2	2	5	5	3	2	1	1	1	0	6	2	7	2	2	6	1	3	7
1053	**N**	15	9	10	4	4	3	12	8	9	3	3	2	4	11	3	6	16	13	11	4	6	2	7	1	0	1	2	1
1054	**D**	12	3	11	13	8	10	9	9	6	6	11	9	6	1	3	4	4	3	4	3	9	6	2	2	6	7	5	2

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1910																													
1055	J	8	4	4	3	2	2	2	2	2	2	2	2	4	6	6	8	8	10	9	8	7	7	7	7	7	6	5	2
1056	F	4	0	0	0	0	0	0	2	4	5	6	5	7	10	9	7	14	8	14	16	12	18	14	9	7	8	3	6
1057	M	3	5	8	9	5	5	6	6	5	6	6	4	5	5	6	4	4	5	5	4	2	3	3	7	4	5	3	2
1058	M	30	3	3	2	2	2	0	0	0	0	0	2	2	2	3	2	4	6	4	5	4	3	2	2	2	2	2	0
1059	A	26	0	0	0	3	3	2	2	2	3	5	4	4	4	5	4	3	5	4	6	7	10	7	9	10	9	8	10
1060	M	23	5	2	3	3	3	3	3	2	4	4	6	4	6	2	2	4	3	2	3	2	2	2	2	0	2	2	2
1061	J	19	0	0	0	0	2	2	4	7	7	6	3	4	2	5	5	8	7	6	3	5	4	3	4	5	2	2	2
1062	J	16	2	3	2	2	2	2	0	0	0	0	0	4	7	4	4	4	4	5	5	5	6	5	4	6	6	5	5
1063	A	12	3	2	2	2	2	2	3	3	0	0	0	0	0	0	0	0	0	0	2	3	5	3	4	5	6	5	4
1064	S	8	4	4	4	5	2	2	0	0	0	0	5	5	6	8	5	5	7	8	11	10	14	13	19	17	16	10	9
1065	O	5	8	5	3	7	5	4	4	3	2	3	0	3	6	9	12	9	10	11	15	14	20	14	9	7	6	4	4
1066	N	1	3	2	3	0	0	0	0	0	0	0	0	0	0	0	0	3	5	4	4	3	2	2	2	0	0	0	0
1067	N	28	0	0	0	0	0	0	0	0	0	0	0	3	3	3	4	4	4	3	3	0	3	2	2	2	3	2	0
1068	D	25	0	0	0	2	0	0	0	0	0	2	2	3	2	3	2	2	2	2	2	2	2	0	0	0	0	0	0
1911																													
1069	J	21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	5	4	5	5	5
1070	F	17	4	4	3	3	2	0	0	0	0	2	3	4	3	3	6	6	4	4	4	2	3	3	3	2	0	0	0
1071	M	16	0	0	0	0	0	0	0	0	2	0	0	0	2	2	3	3	6	8	9	7	9	7	4	3	4	3	2
1072	A	12	2	2	2	2	0	0	0	0	0	2	3	4	4	5	6	5	4	6	3	3	5	4	4	5	4	5	4
1073	M	9	4	0	0	0	0	0	0	0	0	0	0	2	2	2	2	0	2	3	3	3	2	2	2	2	3	2	2
1074	J	5	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	2	2	2
1075	J	2	4	2	2	0	0	0	0	0	0	2	2	2	3	3	2	0	0	0	0	0	0	0	0	2	2	0	0
1076	J	29	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	3	4	2	2	0	0	2	0	0	0	0
1077	A	25	0	0	0	0	0	0	0	2	3	3	4	4	3	3	3	2	2	0	0	0	0	0	0	0	0	0	0
078	S	21	0	0	0	0	0	0	0	0	0	0	0	2	4	2	2	2	2	2	0	0	2	2	0	0	0	0	0
079	O	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	2	2	2	0	0	0	0	0	0	2	2	0
080	N	14	0	0	0	0	0	0	0	0	2	2	3	2	2	2	2	2	3	3	2	0	0	0	0	0	0	0	0
081	D	11	0	0	0	0	0	0	0	0	0	3	2	2	3	2	0	0	0	0	1	0	0	0	0	0	0	0	0

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1910																													
1055	J	8	1	8	3	1	9	8	6	1	0	13	4	7	2	7	10	9	14	14	13	8	5	3	1	1	3	10	10
1056	F	4	9	4	1	1	1	6	4	4	2	3	11	9	6	13	10	3	14	12	9	7	10	12	9	8	8	8	8
1057	M	3	7	11	10	11	10	3	7	7	4	1	10	11	9	11	11	7	4	11	10	8	3	1	0	1	14	19	13
1058	M	30	12	15	13	12	7	11	5	5	1	1	3	0	1	10	5	2	1	4	9	13	9	2	1	7	11	9	6
1059	A	26	7	16	13	11	11	6	8	8	7	4	4	7	1	6	5	5	3	11	9	9	7	9	11	11	7	3	4
1060	M	23	6	11	15	10	7	9	7	6	9	4	2	1	2	1	4	2	11	10	8	8	5	7	1	0	0	2	0
1061	J	19	5	17	12	10	9	7	8	9	4	3	9	1	0	1	7	9	13	11	10	7	8	9	4	0	0	0	5
1062	J	16	3	4	5	7	5	2	7	9	9	7	4	1	3	10	7	4	11	10	9	10	9	1	0	1	4	15	9
1063	A	12	3	6	11	7	2	4	11	12	10	12	18	10	11	5	2	4	13	14	11	9	9	7	3	2	4	11	9
1064	S	8	5	3	9	9	2	9	10	5	10	4	2	1	10	10	11	9	10	15	10	11	10	18	11	9	12	12	16
1065	O	5	9	15	8	8	4	7	9	14	15	9	4	1	0	0	16	13	13	9	10	10	13	12	16	12	9	9	4
1066	N	1	3	10	3	7	6	3	9	11	10	10	5	2	1	4	4	10	12	16	14	9	8	9	5	3	9	7	9
1067	N	28	11	12	11	9	12	10	8	9	10	10	7	2	1	1	1	10	9	13	7	1	7	8	8	7	11	7	7
1068	D	25	10	2	4	15	14	9	7	6	10	10	8	10	8	2	11	10	9	8	1	9	4	6	9	1	9	4	3
1911																													
1069	J	21	2	4	7	17	14	11	8	11	8	8	12	10	12	5	5	9	9	9	9	5	8	4	3	12	11	3	9
1070	F	17	9	8	5	3	18	18	13	11	10	9	11	12	10	4	8	9	12	8	6	8	2	1	1	1	5	9	8
1071	M	16	6	1	0	2	19	17	12	14	11	12	12	12	11	10	5	7	6	5	8	8	1	2	5	15	17	12	6
1072	A	12	7	2	0	2	17	13	11	9	10	12	11	8	7	3	2	3	8	5	12	1	4	3	0	6	8	14	8
1073	M	9	5	6	10	3	1	12	16	11	9	7	9	8	10	3	6	3	10	8	5	6	4	10	11	9	1	1	12
1074	J	5	10	7	3	1	11	12	11	8	9	9	6	4	1	0	0	6	10	8	9	2	0	0	1	7	2	3	13
1075	J	2	9	8	7	4	6	10	10	2	6	7	4	2	1	1	1	11	10	12	5	3	3	1	8	1	1	4	16
1076	J	29	11	10	4	2	1	8	9	8	6	1	1	2	1	1	1	8	1	2	5	5	2	11	8	3	1	18	14
1077	A	25	11	11	9	8	1	2	9	2	1	0	1	5	2	3	1	1	6	9	8	5	1	8	11	7	3	10	17
1078	S	21	16	11	9	2	0	1	5	2	1	1	1	8	5	3	0	2	5	8	9	16	15	5	6	6	1	7	11
1079	O	18	14	9	7	7	6	1	7	7	3	4	0	6	1	1	0	1	10	7	7	3	1	8	11	7	3	8	17
1080	N	14	14	9	5	7	1	6	5	2	0	0	1	5	3	1	1	2	2	7	1	2	1	1	15	5	2	1	6
1081	D	11	19	5	1	6	1	1	10	6	1	0	1	0	1	1	8	12	8	5	1	3	9	7	0	1	3	7	3

(T 52)

Table I. Daily Relative Sunspot Numbers divided by 5

unspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
912																													
082	J	7	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
083	F	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
084	M	1	0	0	0	2	0	2	2	3	4	3	3	4	3	3	2	2	2	2	0	0	0	0	0	0	0	0	0
085	M	28	0	0	0	0	0	0	0	0	0	0	0	2	5	5	4	4	4	3	2	0	0	0	0	0	0	0	0
086	A	24	0	0	0	0	0	0	2	2	3	3	2	2	2	0	2	2	0	2	0	2	0	0	0	0	0	0	0
087	M	21	0	0	0	0	0	0	2	3	2	2	2	3	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
088	J	17	2	3	2	2	2	2	2	3	2	2	2	2	0	0	0	0	2	2	2	2	3	3	3	3	2	0	0
089	J	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
090	A	10	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
091	S	6	2	2	4	3	2	4	4	4	4	4	4	3	5	4	2	2	0	0	0	2	3	2	2	2	2	0	0
092	O	3	0	2	3	5	5	5	4	3	2	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0
093	O	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	2	2	0	0	0	0	0	0	0
094	N	26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	4	3	3	4	4	6	4	4	2
095	D	23	2	0	0	0	0	0	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	2	2	2	2	0
913																													
096	J	19	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
097	F	15	0	0	0	0	2	3	2	3	3	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2
098	M	14	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	2	0	0
099	A	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	M	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
101	J	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
102	J	30	0	0	0	0	0	0	0	0	0	2	2	2	3	2	2	0	0	0	0	0	0	0	0	0	0	0	0
103	J	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
104	A	23	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	2	0	0	0	0	0	0	0	0	0
105	S	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0
106	O	16	0	0	0	0	0	0	0	0	0	2	2	3	2	2	3	2	0	0	0	0	0	0	0	0	0	0	0
107	N	12	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0
108	D	9	3	4	5	4	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	5	2	3	4

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1912																													
1082	J	7	3	3	4	5	8	11	12	6	0	0	8	7	4	5	2	10	3	5	1	0	1	5	1	3	1	1	7
1083	F	3	6	3	1	1	2	4	2	9	7	10	11	3	2	9	10	6	3	1	1	3	8	9	7	11	1	3	2
1084	M	1	4	1	6	0	8	9	9	15	10	9	2	4	1	6	7	2	0	1	0	1	9	8	1	0	1	6	0
1085	M	28	7	11	1	1	1	3	4	3	13	10	8	1	1	10	1	2	2	8	13	10	10	6	6	5	0	1	3
1086	A	24	2	3	1	2	1	1	3	1	5	6	5	13	9	8	7	1	2	6	13	14	10	3	1	2	2	4	3
1087	M	21	2	1	1	3	1	1	2	0	8	6	8	10	8	8	2	0	1	6	12	9	9	6	3	1	2	1	1
1088	J	17	3	2	1	1	2	6	9	6	2	2	8	9	8	3	3	2	9	13	12	7	6	5	1	1	0	0	1
1089	J	14	6	0	5	5	2	1	7	4	1	1	0	3	7	8	1	0	3	11	9	3	2	0	9	17	5	0	0
1090	A	10	3	2	0	0	5	4	4	9	10	9	2	6	11	9	4	3	1	8	7	4	3	3	3	1	6	7	5
1091	S	6	4	1	6	7	2	4	6	3	3	0	0	18	11	4	5	1	7	9	16	3	6	0	0	2	3	10	0
1092	O	3	6	3	0	1	4	3	2	4	12	8	9	16	13	9	7	0	0	7	3	4	3	2	3	0	3	7	1
1093	O	30	2	1	4	5	0	1	7	6	1	4	8	12	10	0	2	14	8	9	6	4	2	3	0	8	6	0	1
1094	N	26	8	3	0	0	0	3	10	7	0	0	10	14	6	5	2	4	1	8	4	1	0	0	3	1	0	0	12
1095	D	23	13	9	5	2	0	0	2	5	1	0	10	14	8	4	1	0	3	7	10	1	1	3	6	6	1	5	13
1913																													
1096	J	19	10	8	3	4	2	1	8	2	3	6	3	13	7	3	1	0	0	0	2	6	6	7	2	1	10	10	16
1097	F	15	12	9	9	5	8	4	6	4	1	0	11	11	4	0	0	0	4	3	1	1	6	8	3	0	9	3	3
1098	M	14	16	12	13	12	4	3	1	9	5	10	4	2	1	2	3	8	9	9	9	2	4	6	2	1	2	9	19
1099	A	10	11	8	13	7	6	9	10	9	1	2	0	0	1	5	3	1	1	9	7	1	0	6	4	6	14	14	13
1100	M	7	11	7	5	4	1	1	3	0	5	1	2	7	5	1	0	0	0	3	5	2	7	5	5	2	2	14	10
1101	J	3	8	8	2	0	0	0	1	2	1	0	4	5	7	4	2	1	9	2	5	2	6	7	1	7	1	9	10
1102	J	30	5	6	3	4	1	2	3	4	2	0	5	2	13	9	9	9	2	1	1	1	11	8	3	2	5	11	5
1103	J	27	2	1	2	2	1	1	2	3	5	2	5	6	3	9	10	10	9	8	2	7	9	2	5	2	1	2	2
1104	A	23	10	4	4	5	4	6	0	1	1	5	4	5	2	4	10	10	15	14	11	8	9	6	1	2	4	5	5
1105	S	19	6	1	2	13	10	3	5	5	1	1	0	7	1	0	0	7	12	14	14	13	11	11	6	7	7	4	5
1106	O	16	3	1	12	9	8	2	3	0	0	4	0	4	1	1	5	9	7	14	11	1	1	7	10	9	3	1	4
1107	N	12	3	5	0	0	0	3	3	4	1	2	2	1	1	1	5	8	11	5	1	4	5	2	12	8	2	8	2
1108	D	9	6	0	1	2	0	3	1	2	0	3	7	2	1	2	0	2	10	11	8	1	2	1	1	2	6	4	7

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1914																													
1109	J	5	2	0	2	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1110	F	1	0	3	3	4	2	2	2	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
1111	F	28	0	0	0	0	0	0	0	0	0	0	0	0	3	2	2	2	4	2	2	0	0	0	2	0	0	0	0
1112	M	27	0	0	0	2	3	3	3	4	4	5	3	5	5	4	4	2	2	2	3	0	2	2	2	5	5	4	4
113	A	23	4	2	3	3	5	9	10	7	6	5	4	2	3	2	2	0	0	0	0	0	0	0	0	0	0	0	0
114	M	20	0	2	4	3	2	0	0	0	0	2	0	0	0	0	0	0	0	0	0	3	2	2	2	4	7	8	10
115	J	16	7	7	5	7	5	4	2	0	0	0	0	0	0	0	0	2	0	2	2	2	2	4	3	2	2	4	3
116	J	13	0	0	0	0	3	2	0	0	0	2	0	0	0	0	0	0	2	2	2	0	0	0	0	0	0	0	0
117	A	9	0	0	0	0	2	2	2	3	5	5	5	6	4	4	4	4	2	3	0	0	0	2	2	0	0	0	0
118	S	5	0	0	0	0	2	4	4	6	6	7	7	10	7	6	5	4	2	2	0	2	2	2	2	2	2	2	2
119	O	2	2	0	0	0	0	0	0	0	2	2	0	2	0	2	0	0	0	2	3	5	2	3	2	4	3	6	6
120	O	29	5	2	3	3	3	4	4	4	3	7	6	6	4	3	3	5	3	4	0	0	2	2	2	3	4	3	6
121	N	25	3	4	3	3	5	7	5	11	11	11	8	6	6	6	6	5	8	7	7	6	4	2	4	3	3	2	2
122	D	22	2	2	3	3	2	2	2	2	6	4	5	7	8	8	7	6	4	5	4	3	4	5	6	7	7	9	10
15																													
23	J	18	5	7	5	5	3	5	2	2	2	2	4	4	3	3	2	6	4	10	12	10	13	17	18	18	19	13	10
24	F	14	6	6	6	4	2	2	0	3	4	8	9	6	11	12	16	16	14	12	11	14	15	14	12	10	11	10	5
25	M	13	6	8	7	7	11	7	7	6	5	3	3	2	3	2	3	3	5	11	10	10	13	19	16	19	14	12	12
26	A	9	11	9	7	5	5	2	2	0	0	2	5	8	8	8	7	6	4	8	9	11	10	16	20	21	20	17	15
27	M	6	16	12	8	6	2	0	0	0	0	0	0	0	0	0	2	2	6	5	7	7	7	8	8	12	9	4	3
28	J	2	7	7	2	2	2	3	4	2	2	2	2	5	13	15	24	28	30	26	36	31	29	30	30	20	17	15	12
29	J	29	13	15	10	7	6	5	7	11	8	9	12	13	17	18	17	12	12	9	13	7	10	16	16	21	20	26	20
30	J	26	29	23	24	17	22	20	19	22	21	25	22	21	18	13	10	8	5	7	8	9	9	10	10	14	21	13	15
31	A	22	10	14	16	13	17	12	15	10	15	15	13	16	14	13	15	14	19	16	11	8	7	4	6	5	4	6	4
32	S	18	8	7	8	9	8	9	11	12	14	13	15	11	9	10	7	4	4	4	9	4	10	7	9	9	3	13	10
33	O	15	13	16	17	23	20	23	18	12	11	8	6	8	11	10	13	12	15	7	8	9	7	5	6	10	8	11	10
34	N	11	10	9	8	7	8	9	7	10	15	17	13	12	13	11	5	5	10	5	6	7	6	5	7	7	6	8	10
35	D	8	8	5	10	6	6	5	14	9	11	7	8	9	7	5	4	3	3	4	6	7	7	8	14	13	11	12	12

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1914																													
1109	J	5	12	6	9	8	1	0	4	8	9	6	8	9	6	2	1	1	6	13	4	0	0	0	1	7	0	1	2
1110	F	1	1	5	10	9	8	10	6	2	2	1	1	2	6	5	9	4	8	9	2	2	1	11	7	5	1	1	1
1111	F	28	10	9	11	8	9	4	13	10	3	3	6	9	9	2	5	12	7	10	8	5	6	2	1	4	8	9	6
1112	M	27	7	1	1	2	3	10	7	5	2	2	19	14	12	6	7	4	1	2	1	1	10	4	3	8	1	1	1
1113	A	23	6	7	1	0	9	4	0	0	1	1	0	0	5	9	5	3	0	1	9	2	1	0	9	8	12	3	0
1114	M	20	0	1	1	0	2	8	11	4	5	0	1	12	15	8	8	3	3	4	7	9	1	2	3	0	1	1	6
1115	J	16	1	0	2	10	8	3	1	0	1	11	11	12	11	8	7	0	0	0	1	17	8	5	1	5	8	1	1
1116	J	13	1	2	4	4	3	6	1	4	6	8	8	9	13	9	7	9	18	14	13	9	10	13	9	5	10	5	4
1117	A	9	1	1	8	3	4	1	2	1	4	6	9	9	3	1	12	5	8	6	3	10	14	11	6	1	0	5	8
1118	S	5	8	8	0	2	8	7	5	5	1	2	2	8	3	8	2	2	0	2	12	9	4	2	18	10	9	9	11
1119	O	2	9	9	5	5	8	9	6	9	7	4	2	3	1	4	6	10	6	6	3	9	7	2	1	0	0	10	18
1120	O	29	18	9	2	12	5	12	13	9	2	7	1	0	5	14	9	3	10	8	8	8	5	5	0	0	1	0	1
1121	N	25	1	11	11	5	7	4	1	0	5	4	6	1	9	8	10	6	1	3	0	1	0	10	8	1	7	1	0
1122	D	22	8	1	9	2	0	9	15	3	8	6	13	0	1	6	15	8	8	7	3	1	1	8	8	8	3	1	3
1915																													
1123	J	18	0	0	6	3	4	1	5	14	11	9	7	4	6	1	6	7	1	4	9	5	1	13	11	4	1	6	3
1124	F	14	0	4	0	2	1	15	13	11	11	13	12	10	10	4	3	1	1	1	2	4	7	12	14	11	7	6	5
1125	M	13	3	1	1	7	9	9	9	15	16	15	10	7	11	8	3	2	4	8	2	5	11	9	3	2	4	11	15
1126	A	9	4	1	2	1	0	3	11	10	5	10	10	9	8	13	10	1	2	13	3	1	1	5	12	13	9	8	5
1127	M	6	1	0	0	3	1	1	7	6	7	7	11	13	2	8	9	9	7	3	6	6	3	13	0	0	5	6	1
1128	J	2	1	0	0	0	1	6	9	5	1	5	14	14	9	2	9	20	17	1	2	10	12	7	4	9	7	6	4
1129	J	29	7	1	3	12	7	0	2	11	0	7	11	9	11	10	2	1	0	0	0	1	0	1	1	8	3	1	6
1130	J	26	8	12	2	9	7	1	8	12	10	7	0	8	12	8	1	8	2	1	0	0	0	2	9	6	9	5	7
1131	A	22	4	3	1	8	16	10	9	12	8	3	0	1	1	0	5	1	1	0	1	6	2	3	7	1	2	9	11
1132	S	18	1	1	0	4	16	18	13	11	12	11	13	13	12	7	1	3	2	0	1	4	1	0	11	7	1	1	15
1133	O	15	19	14	9	2	16	14	12	12	19	15	16	11	5	3	3	7	8	16	8	3	2	16	20	9	9	5	9
1134	N	11	8	9	3	0	12	17	15	15	11	11	10	10	3	1	3	6	9	9	0	1	0	1	3	1	1	19	11
1135	D	8	5	6	5	8	7	0	9	14	9	4	1	3	3	0	1	9	6	9	11	9	2	3	3	3	1	0	4

Table I. Daily Relative Sunspot Numbers divided by 5

unspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
916																													
136	J	4	17	18	21	16	14	8	12	6	5	6	6	5	4	6	7	7	6	5	10	9	5	8	8	13	11	7	9
137	J	31	9	18	19	14	10	13	17	16	19	15	20	17	19	15	13	10	10	6	8	6	9	13	7	7	5	5	6
138	F	27	4	8	6	9	13	16	21	18	20	16	19	12	16	11	10	8	3	7	10	13	16	9	9	9	16	20	14
139	M	25	19	18	15	15	15	11	22	21	28	29	29	29	22	15	16	14	14	15	12	12	12	12	10	10	8	8	5
140	A	21	8	8	9	6	14	15	14	14	16	18	21	21	19	21	14	16	15	11	12	11	14	12	7	7	7	12	8
141	M	18	8	10	10	13	14	18	18	22	25	28	32	16	18	13	8	5	5	10	11	7	5	3	2	8	6	7	8
142	J	14	8	7	8	10	9	12	18	22	28	35	35	37	26	23	23	17	14	12	13	12	8	6	7	8	9	5	5
143	J	11	4	2	2	3	3	3	6	7	9	14	21	26	25	23	28	24	22	12	13	6	5	8	6	5	3	5	10
144	A	7	7	9	9	6	6	8	7	10	11	10	10	11	13	18	15	11	9	3	0	0	0	4	6	3	4	5	4
45	S	3	4	6	7	4	10	10	9	10	10	12	17	16	17	16	15	17	19	15	17	9	4	3	2	5	4	5	4
46	S	30	4	2	0	6	4	6	8	15	22	27	19	22	23	23	20	16	13	13	12	9	6	5	2	4	4	4	6
47	O	27	5	7	9	7	9	13	15	16	23	20	25	23	22	22	15	16	16	14	11	12	11	10	6	2	2	4	11
48	N	23	13	13	16	13	13	6	13	12	11	14	16	12	14	21	17	18	15	12	14	11	8	9	5	4	6	9	8
49	D	20	6	2	3	8	12	7	9	9	14	15	16	18	18	14	12	11	12	15	23	17	19	17	21	22	19	13	11
17																													
50	J	16	11	11	12	13	9	12	12	12	14	16	19	19	25	17	17	12	10	9	12	15	17	18	25	20	21	25	25
51	F	12	24	24	20	20	19	20	17	11	12	10	10	4	7	3	5	5	6	13	12	20	27	25	27	22	24	22	19
52	M	11	22	21	22	24	19	18	19	18	17	16	16	14	14	20	20	23	22	19	16	16	13	13	7	6	3	2	6
53	A	7	5	2	3	3	6	16	22	30	32	30	30	22	22	19	14	18	21	18	21	21	20	13	12	22	27	19	23
54	M	4	24	20	18	19	14	12	12	16	18	19	19	19	22	28	25	33	24	19	25	28	28	32	28	33	34	29	23
55	M	31	30	28	22	18	18	13	15	16	18	21	20	25	28	28	28	26	30	33	30	29	19	19	20	19	22	21	22
56	J	27	27	33	28	24	22	21	20	22	26	23	30	28	25	26	33	44	31	35	27	24	26	16	16	14	11	17	15
57	J	24	19	27	24	25	25	29	27	25	23	18	19	22	27	40	54	53	44	49	50	49	45	47	42	35	28	29	32
58	A	20	25	27	26	21	17	17	23	20	22	21	22	24	31	24	21	24	30	26	21	17	21	22	18	19	19	22	20
59	S	16	20	24	28	30	37	42	42	42	42	42	35	26	14	17	12	9	13	10	8	12	13	14	12	14	10	13	8
60	O	13	13	12	9	11	14	15	17	19	22	16	24	28	27	24	21	12	13	12	15	17	16	20	16	15	14	17	21
61	N	9	16	19	15	16	16	15	17	18	20	20	20	18	19	25	20	25	21	22	23	27	29	33	24	28	28	17	13
62	D	6	12	13	15	10	9	15	24	30	31	33	28	31	25	34	32	37	32	33	39	42	38	36	30	29	26	24	23

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1916																													
1136	J	4	8	6	5	1	1	7	13	19	16	11	2	1	4	0	3	1	12	6	11	12	7	11	7	2	4	8	1
1137	J	31	3	1	1	1	5	10	1	1	12	7	4	2	10	8	7	5	1	10	11	9	8	1	9	7	6	1	8
1138	F	27	12	3	1	1	5	12	7	6	10	10	19	19	17	9	8	0	2	1	3	14	10	9	11	9	3	1	13
1139	M	25	12	8	0	2	17	15	12	9	3	1	0	1	6	7	11	5	0	3	2	1	7	11	10	5	9	3	8
1140	A	21	3	7	3	1	19	14	14	15	18	9	9	10	8	5	9	11	7	7	7	6	10	9	3	2	1	8	7
1141	M	18	2	5	3	14	17	13	11	8	3	2	3	7	14	12	8	0	1	1	2	8	9	11	5	1	4	9	9
1142	J	14	4	3	0	7	10	11	10	9	13	11	3	9	8	7	8	9	10	16	9	7	10	10	8	3	15	13	10
1143	J	11	9	7	7	4	0	3	11	9	6	4	2	3	11	8	2	2	0	1	0	3	1	1	12	6	3	11	15
1144	A	7	11	10	7	5	6	4	5	7	0	0	0	1	10	10	8	15	15	9	0	14	19	7	10	7	4	1	8
1145	S	3	13	14	12	6	8	6	2	10	12	14	9	3	8	11	11	8	0	0	1	5	7	8	6	9	13	3	1
1146	S	30	15	14	12	6	2	8	20	17	14	11	8	12	10	15	10	2	0	1	1	3	8	10	10	9	8	7	2
1147	O	27	1	1	9	3	2	4	8	12	14	14	13	9	7	7	5	6	18	9	4	7	5	5	9	7	4	3	6
1148	N	23	6	1	11	11	12	10	10	11	14	14	12	9	3	3	3	3	5	3	2	10	8	7	13	9	9	5	2
1149	D	20	2	1	0	0	3	7	4	12	11	8	11	10	8	5	1	20	16	12	10	10	8	9	8	13	11	5	1
1917																													
1150	J	16	5	5	3	7	10	11	13	11	7	11	9	7	1	3	5	8	1	4	2	5	7	7	9	8	1	4	5
1151	F	12	0	2	6	19	15	7	11	11	15	8	9	8	7	6	7	4	7	7	1	3	11	15	10	8	13	7	5
1152	M	11	3	4	10	2	8	8	1	2	2	10	10	7	6	4	11	4	8	0	1	0	4	6	9	10	6	10	12
1153	A	7	6	8	14	1	0	3	3	0	8	12	9	9	1	1	2	7	5	7	9	14	1	1	5	10	12	15	14
1154	M	4	9	9	1	4	0	6	3	3	3	2	9	3	12	8	5	1	2	7	8	6	4	7	7	10	13	11	8
1155	M	31	3	0	0	7	10	0	5	15	10	6	5	3	2	12	9	4	5	4	1	0	0	0	8	13	17	12	5
1156	J	27	1	4	4	1	2	13	11	5	1	1	8	2	1	5	10	8	15	6	3	0	0	0	2	0	11	12	2
1157	J	24	3	3	2	10	11	15	9	18	11	3	3	1	1	0	8	7	20	17	6	4	19	19	16	9	6	4	1
1158	A	20	12	19	14	14	5	10	17	5	1	1	7	2	1	11	11	9	17	7	3	5	11	3	1	3	4	2	3
1159	S	16	3	5	8	11	10	8	7	0	5	2	1	3	8	8	11	10	12	13	11	9	9	3	10	7	4	10	2
1160	O	13	14	16	4	1	3	1	1	1	0	1	7	8	13	1	4	16	16	12	10	7	2	2	0	1	4	3	3
1161	N	9	1	1	4	16	10	13	1	1	3	4	10	8	1	3	0	4	12	14	14	9	5	2	4	9	11	11	9
1162	D	6	5	7	12	8	1	6	5	1	9	4	20	16	16	11	10	4	0	3	3	7	12	5	2	5	5	1	1

(T 58)

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1918																													
1163	J	2	18	17	13	16	10	13	14	13	15	19	14	20	21	20	22	24	20	21	22	22	24	26	23	28	24	22	20
1164	J	29	24	20	21	14	13	17	10	9	5	6	5	6	4	14	17	18	23	16	19	23	23	18	18	15	11	9	11
1165	F	25	14	12	11	13	10	12	8	11	7	7	10	11	11	13	21	28	22	22	15	21	23	22	21	20	13	8	12
1166	M	24	13	16	17	17	12	13	11	14	19	17	20	24	19	26	19	23	18	16	17	15	14	14	23	16	14	10	15
1167	A	20	12	14	11	15	10	9	13	14	18	20	19	17	24	22	26	23	24	19	18	21	23	20	20	20	15	14	11
1168	M	17	10	7	7	7	6	10	6	7	9	13	12	12	13	22	28	33	31	35	31	25	25	16	15	9	10	4	4
1169	J	13	4	3	3	3	3	3	7	10	10	10	8	8	7	8	5	10	15	14	20	21	23	30	29	29	27	27	21
1170	J	10	20	21	18	14	14	19	17	14	12	15	14	19	23	25	29	28	29	26	31	25	22	19	25	23	25	26	25
1171	A	6	20	15	17	10	13	13	21	25	19	22	29	26	26	19	27	25	26	28	17	20	18	17	21	15	15	15	15
1172	S	2	11	10	8	8	9	13	18	17	19	20	13	17	19	16	14	16	16	12	19	15	22	18	16	18	15	17	28
1173	S	29	29	23	14	17	17	15	13	14	15	12	13	13	15	15	13	10	12	12	16	19	27	21	30	25	26	25	22
1174	O	26	20	21	16	14	20	17	12	11	10	11	8	6	6	7	10	6	7	7	8	13	16	20	40	43	42	33	38
175	N	22	26	27	16	22	11	16	15	12	12	8	9	7	9	8	8	9	14	13	6	6	9	11	8	7	4	10	12
176	D	19	14	12	14	19	17	24	23	23	24	19	17	8	6	7	4	4	2	4	6	6	12	16	16	22	19	18	19
919																													
177	J	15	15	18	11	7	8	8	8	5	3	3	5	8	11	10	10	12	13	16	21	30	30	34	24	23	24	27	24
178	F	11	20	17	15	11	12	12	13	9	10	9	5	6	9	9	9	12	11	12	16	16	13	8	12	14	15	17	14
179	M	10	26	22	20	20	22	19	17	15	14	11	7	6	6	6	9	9	9	12	12	11	14	14	13	11	18	18	14
180	A	6	13	16	15	15	14	8	6	6	9	6	3	6	4	4	4	5	7	6	12	14	15	13	16	15	16	10	13
181	M	3	18	20	23	26	22	22	24	20	17	14	14	12	14	16	22	30	32	32	25	22	19	19	13	13	11	9	5
182	M	30	10	12	17	21	20	24	25	27	22	18	14	18	16	19	25	24	20	25	22	23	30	29	29	31	31	25	23
183	J	26	25	26	24	16	11	11	15	19	14	13	8	10	6	7	13	13	16	17	19	26	16	15	12	13	11	10	13
184	J	23	14	18	19	15	18	12	7	8	5	6	3	5	10	12	10	14	13	14	15	15	17	14	21	23	22	22	21
185	A	19	21	21	23	20	15	13	14	13	8	7	8	8	11	11	8	11	12	12	9	10	11	10	10	11	13	11	11
186	S	15	11	7	8	8	6	10	11	13	15	18	13	16	11	11	17	15	14	11	10	6	7	6	8	11	14	16	17
187	O	12	24	19	18	13	12	8	7	4	4	5	3	3	4	5	7	10	12	20	19	23	22	20	19	17	13	9	8
188	N	8	8	8	8	7	8	5	3	3	3	2	4	4	3	3	4	5	10	13	8	9	12	13	13	16	11	15	15
189	D	5	9	6	5	9	9	9	7	10	8	8	7	8	8	7	4	2	4	5	8	6	4	4	6	5	3	7	6

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1918																													
1163	J	2	2	10	9	10	10	4	1	2	7	2	13	10	9	9	4	1	1	0	3	9	4	1	7	5	4	7	4
1164	J	29	14	18	16	12	10	2	2	13	13	8	1	8	11	12	19	14	11	13	11	5	1	1	5	7	0	11	8
1165	F	25	0	1	6	11	6	7	11	6	1	1	11	20	9	11	11	11	2	4	15	13	4	4	0	8	10	10	11
1166	M	24	3	0	5	9	4	7	5	6	4	6	5	9	15	17	11	8	7	8	18	11	0	1	3	3	4	13	14
1167	A	20	1	6	7	10	2	12	14	4	6	4	13	11	5	4	7	10	4	1	1	2	4	9	7	6	9	11	17
1168	M	17	19	13	11	10	9	5	4	4	2	1	1	1	10	9	2	1	0	0	0	4	6	6	5	13	19	12	12
1169	J	13	9	11	12	11	9	4	1	3	9	5	0	2	1	9	3	1	0	0	9	8	8	5	4	2	3	11	10
1170	J	10	8	11	7	8	10	9	5	2	5	0	0	1	1	3	3	14	9	9	15	13	12	10	4	9	9	6	8
1171	A	6	9	10	11	10	6	9	3	5	7	20	17	3	1	0	5	1	5	3	11	15	13	12	7	3	1	12	15
1172	S	2	8	10	12	12	9	8	11	6	6	1	1	5	5	2	11	11	12	14	12	20	12	7	9	4	2	6	11
1173	S	29	10	12	12	12	12	12	12	9	7	16	11	3	1	1	0	1	11	20	15	11	12	11	7	8	6	7	9
1174	O	26	2	1	11	6	6	14	7	6	1	2	1	0	1	2	3	9	16	17	14	12	15	12	10	6	8	6	5
1175	N	22	6	16	13	5	1	2	6	18	12	17	7	13	7	1	0	12	20	15	14	11	9	10	7	4	3	8	6
1176	D	19	14	11	10	8	10	8	20	15	3	1	1	1	7	2	1	14	19	16	12	9	8	5	1	1	4	12	12
1919																													
1177	J	15	7	14	13	14	12	9	7	8	5	2	0	0	0	7	8	5	16	14	12	11	9	10	7	1	2	5	0
1178	F	11	0	2	13	14	10	11	5	7	2	7	16	14	14	5	0	2	12	19	14	15	11	10	8	10	8	3	1
1179	M	10	0	4	9	7	12	5	9	9	0	13	17	17	17	11	0	7	4	12	14	10	10	10	6	1	1	9	0
1180	A	6	11	14	12	7	7	4	7	4	0	4	14	18	14	10	10	9	9	9	8	3	0	4	2	8	6	4	19
1181	M	3	18	8	10	7	3	1	5	5	0	6	16	12	9	6	11	9	6	8	15	13	6	15	10	12	9	1	1
1182	M	30	2	2	5	5	4	4	5	5	1	1	11	13	11	10	6	2	3	1	6	4	1	1	2	6	11	9	10
1183	J	26	10	4	7	7	1	9	5	8	3	1	0	5	11	9	6	6	4	6	5	0	1	14	11	4	2	2	11
1184	J	23	16	11	3	2	2	1	4	3	4	8	8	6	6	8	2	5	7	3	4	20	20	1	2	6	5	6	9
1185	A	19	16	10	4	3	7	5	6	10	5	11	8	6	2	0	15	11	10	2	14	11	4	11	7	7	1	9	9
1186	S	15	9	9	8	10	18	15	10	7	9	16	11	7	3	1	2	1	18	14	17	15	20	15	9	8	11	5	7
1187	O	12	2	1	0	9	12	11	10	4	5	2	13	13	3	0	11	12	13	9	6	7	1	3	2	15	5	2	1
1188	N	8	2	1	1	12	11	4	1	3	14	12	10	1	1	7	13	7	6	5	5	1	1	3	6	2	2	10	9
1189	D	5	9	6	4	7	5	6	5	7	10	15	20	9	1	5	8	9	10	11	9	11	6	0	2	1	1	2	1

Table I. Daily Relative Sunspot Numbers divided by 5

spots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
20																													
90	J	1	10	9	8	11	7	4	4	4	3	9	10	9	7	7	8	8	6	9	8	6	7	14	16	23	26	23	20
91	J	28	23	12	11	7	5	4	7	5	6	5	6	7	7	6	11	9	17	18	20	16	16	16	16	19	17	18	19
92	F	24	11	14	10	8	6	6	4	3	6	8	11	8	7	11	18	23	21	21	14	7	7	9	19	16	27	27	34
93	M	22	30	33	23	16	16	10	5	7	3	4	3	2	3	2	3	4	5	4	0	2	3	5	4	5	6	6	4
94	A	18	4	5	2	2	4	3	0	2	3	2	4	6	5	5	5	3	8	9	10	8	10	7	10	9	7	9	7
95	M	15	7	4	2	2	4	2	2	6	5	4	5	9	10	12	12	13	13	11	11	12	8	8	6	2	7	8	8
96	J	11	10	7	8	6	7	7	7	3	5	6	9	6	10	9	12	9	10	11	11	10	8	8	8	10	9	9	8
97	J	8	7	5	6	4	4	4	4	2	2	5	6	5	4	3	5	7	6	8	7	7	8	6	4	3	6	6	6
98	A	4	3	2	2	2	0	2	2	2	3	7	8	11	10	8	5	4	4	4	5	4	3	2	2	2	2	4	5
99	A	31	5	9	7	8	9	11	9	8	8	4	4	5	7	7	8	9	7	0	0	0	0	2	4	7	7	13	17
00	S	27	17	16	12	13	10	9	9	12	9	6	9	11	9	13	14	15	14	15	16	11	9	7	8	8	8	10	12
01	O	24	13	13	12	9	9	7	5	6	7	7	9	11	12	11	10	10	6	4	3	3	2	3	3	4	3	2	2
02	N	20	2	2	4	4	5	10	9	7	7	8	7	5	4	4	3	4	3	3	3	4	3	4	4	4	4	5	3
03	D	17	3	7	9	12	11	8	9	9	11	15	10	12	7	6	10	8	5	6	9	6	5	4	4	4	6	8	11
21																													
04	J	13	13	16	16	11	10	5	7	9	7	6	6	4	3	4	3	4	2	3	2	5	5	6	5	6	4	5	8
05	F	9	8	7	8	7	7	5	5	4	5	6	8	10	7	8	9	8	3	3	4	4	4	3	2	2	0	0	0
06	M	8	2	3	5	8	9	7	11	7	6	7	7	5	2	6	5	8	8	7	8	11	11	9	7	6	6	5	7
07	A	4	7	5	5	7	5	4	6	6	10	8	6	8	10	11	9	12	10	9	10	8	6	7	7	5	5	2	0
08	M	1	0	4	2	0	0	0	0	2	5	8	12	10	15	13	10	8	7	10	8	4	5	2	3	2	3	2	5
09	M	28	4	2	3	0	5	6	5	5	4	5	7	14	12	12	11	9	9	5	4	0	2	2	3	2	3	3	4
10	J	24	4	6	9	14	15	15	17	22	17	19	18	20	16	12	10	12	10	10	12	9	6	3	4	3	3	4	4
11	J	21	4	2	2	3	5	6	5	7	7	8	7	5	4	4	2	2	0	0	0	0	0	2	4	3	3	3	3
12	A	17	4	6	6	6	7	9	5	7	8	10	17	12	7	5	6	4	2	4	2	0	0	2	0	0	3	3	4
13	S	13	6	5	7	6	6	7	7	8	6	5	4	5	4	4	4	4	4	2	0	0	0	0	0	0	0	4	2
14	O	10	2	2	4	4	5	4	3	2	2	2	4	3	4	8	13	12	12	11	8	5	3	2	2	3	0	0	0
15	N	6	0	0	0	0	0	0	2	3	3	3	3	3	5	8	11	10	11	10	9	6	5	5	5	3	3	0	0
16	D	3	0	0	0	0	0	0	3	2	6	5	8	9	9	10	12	13	12	10	10	7	5	3	3	2	4	2	2
17	D	30	0	0	0	0	3	2	5	5	6	5	6	6	5	5	3	3	4	4	3	2	3	3	3	3	3	0	0

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1920																													
1190	J	1	9	9	3	1	0	4	9	5	11	11	13	7	0	6	8	6	8	3	2	2	12	7	9	7	3	5	0
1191	J	28	10	8	10	3	1	0	1	1	1	4	10	5	1	1	5	8	7	9	6	16	14	9	1	2	7	1	1
1192	F	24	16	8	5	8	5	0	5	1	0	17	19	11	6	9	1	8	6	9	2	18	10	11	5	3	1	3	7
1193	M	22	20	20	18	11	6	8	6	1	1	1	1	2	4	10	13	10	9	7	4	5	1	2	2	2	19	8	14
1194	A	18	12	9	10	8	2	8	11	1	2	2	0	5	10	12	9	10	4	1	1	0	5	11	2	0	1	16	11
1195	M	15	12	8	4	3	2	4	5	1	1	3	5	6	9	13	10	6	2	3	1	2	10	3	8	1	0	8	16
1196	J	11	11	3	2	0	4	2	2	2	4	5	2	1	2	7	3	4	2	8	8	7	5	0	1	1	1	10	12
1197	J	8	10	6	1	5	11	3	7	14	10	6	9	2	5	1	8	11	7	5	5	0	0	0	1	1	2	0	7
1198	A	4	11	6	0	5	11	11	8	4	14	9	10	6	2	1	9	8	6	12	13	6	3	2	2	1	1	4	10
1199	A	31	4	10	4	16	14	9	2	7	14	14	10	9	1	8	8	11	9	11	7	4	2	2	17	9	3	3	1
1200	S	27	11	19	17	11	13	9	3	9	6	9	9	6	7	15	4	4	3	1	2	4	6	2	2	1	1	7	9
1201	O	24	10	13	9	11	9	9	0	7	7	7	7	10	10	13	8	1	1	1	2	8	5	1	8	4	11	9	2
1202	N	20	4	10	8	1	1	1	16	12	2	2	1	0	8	9	17	14	11	7	9	7	3	1	1	8	8	8	5
1203	D	17	1	5	3	9	1	0	5	9	11	17	11	6	1	1	7	9	2	3	8	7	3	4	2	8	10	3	6
1921																													
1204	J	13	1	2	8	7	14	7	4	7	6	2	3	9	3	7	3	4	3	2	8	9	9	2	4	12	9	4	1
1205	F	9	1	3	3	1	9	8	2	1	8	5	9	6	8	4	1	3	4	4	4	8	10	9	6	3	1	2	3
1206	M	8	1	9	10	3	6	2	9	13	10	2	3	1	1	14	13	3	9	13	12	13	7	12	8	2	2	1	8
1207	A	4	1	1	3	1	6	8	8	6	12	16	10	8	6	4	14	12	10	12	9	8	4	3	1	1	6	17	5
1208	M	1	4	2	8	8	0	1	0	6	9	7	4	14	19	20	20	20	16	9	16	18	13	8	6	3	1	4	6
1209	M	28	8	6	1	2	5	4	8	10	1	11	8	12	10	9	4	2	4	9	2	4	6	1	2	6	4	8	9
1210	J	24	4	1	7	3	3	7	3	5	1	2	7	1	7	10	10	12	4	1	5	7	7	9	10	3	3	7	6
1211	J	21	1	6	8	4	1	6	4	6	7	7	2	0	7	12	9	11	11	8	7	2	3	10	5	2	6	10	10
1212	A	17	7	1	1	3	5	2	1	2	0	11	9	2	0	12	8	2	18	7	8	4	1	7	11	6	3	1	0
1213	S	13	1	2	4	6	2	5	6	1	5	2	11	0	0	0	2	14	15	7	9	3	2	8	10	8	10	19	10
1214	O	10	6	14	11	2	5	6	1	1	1	1	4	11	6	4	5	2	2	10	9	8	2	9	7	0	1	0	10
1215	N	6	13	7	7	7	10	1	2	8	7	4	16	17	12	10	5	8	7	11	5	4	2	1	3	0	1	2	7
1216	D	3	7	3	1	0	0	2	3	6	8	14	16	8	6	13	10	5	1	0	0	9	10	7	1	8	9	17	13
1217	D	30	8	4	2	4	1	2	4	7	11	15	13	11	8	4	2	1	0	12	11	9	8	8	0	0	1	18	15

Table I. Daily Relative Sunspot Numbers divided by 5

unspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1922																													
1218	J	26	0	0	0	0	0	0	0	3	0	0	0	4	9	6	6	7	10	15	13	8	6	5	3	2	4	2	2
1219	F	22	3	2	4	5	8	14	17	22	26	25	24	24	23	22	16	14	15	18	20	16	10	6	4	4	4	3	0
1220	M	21	0	0	3	6	7	8	7	7	7	6	6	6	6	8	4	4	4	3	2	0	0	0	0	0	0	0	0
1221	A	17	0	0	0	0	0	2	2	4	5	4	5	3	3	7	8	7	6	5	3	3	2	2	2	0	0	0	0
1222	M	14	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	3	4	2	0	0	0	0	0	2	2	2
1223	J	10	0	4	5	4	4	4	2	2	0	0	0	2	2	2	2	2	0	0	0	0	0	0	0	0	0	3	5
1224	J	7	4	6	6	4	2	2	0	0	0	0	0	0	4	6	5	6	6	4	4	3	2	2	0	0	0	0	0
1225	A	3	2	2	2	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	4	5	5	5	3	4
1226	A	30	3	3	1	0	0	0	0	0	1	0	0	0	0	0	2	2	3	2	2	2	2	2	2	2	2	2	0
1227	S	26	0	2	2	2	2	2	2	2	2	0	0	0	0	0	2	5	4	2	2	0	3	2	3	3	4	3	0
1228	O	23	0	0	0	0	0	0	0	2	2	3	3	3	0	2	2	3	2	3	3	3	3	3	3	2	2	2	0
1229	N	19	2	3	3	0	0	0	0	0	0	0	3	2	3	6	5	5	5	5	4	3	4	4	0	0	0	0	0
1230	D	16	0	0	0	0	0	0	2	3	4	6	8	9	10	10	10	7	6	7	2	3	0	0	0	0	0	0	0
1923																													
231	J	12	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	2	3	3	0	0	0	0	0	0	0	2
232	F	8	0	0	0	0	0	2	2	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
233	M	7	0	0	0	0	0	0	0	0	0	2	0	0	0	2	0	3	2	2	2	2	0	0	3	3	4	3	2
234	A	3	0	0	0	0	0	0	0	0	0	0	2	2	0	0	3	3	2	3	3	3	3	2	2	2	3	3	2
235	A	30	0	0	0	0	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	0	2	0	2	2	3	3	0
236	M	27	0	2	2	2	2	4	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	2	2	2
237	J	23	2	2	8	5	5	4	7	8	4	4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
238	J	20	0	0	2	2	2	2	2	0	0	2	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
239	A	16	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	3	3	4	5	4	4	4	7	5	5	2
240	S	12	2	2	2	3	2	3	3	2	0	0	0	0	2	3	3	5	4	4	4	2	2	2	2	2	0	0	0
241	O	9	2	2	2	2	2	2	2	2	2	2	2	2	2	5	6	6	7	5	5	4	3	3	4	5	6	5	6
242	N	5	6	4	4	3	2	2	3	3	2	2	2	2	0	0	0	0	0	0	0	2	2	3	2	0	2	0	0
243	D	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	2	3	4	0	0	0
244	D	29	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1922																													
1218	J	26	9	5	2	3	4	13	7	6	14	11	8	9	2	10	12	5	2	12	8	10	14	15	9	5	5	7	8
1219	F	22	4	1	1	0	8	10	10	12	10	11	7	15	3	1	1	1	12	8	11	18	19	7	4	11	9	11	9
1220	M	21	4	7	1	1	12	5	4	4	8	14	11	10	8	2	0	0	0	1	11	13	12	8	13	8	8	7	6
1221	A	17	5	5	1	5	11	16	10	13	11	14	11	6	9	6	2	0	0	3	7	7	13	13	13	11	8	2	2
1222	M	14	3	1	14	7	3	6	4	12	9	8	8	8	9	7	4	0	0	0	2	11	11	7	13	10	5	3	3
1223	J	10	1	2	7	6	4	0	13	11	9	6	4	5	4	6	1	2	2	7	9	14	14	12	9	10	5	3	4
1224	J	7	5	1	2	5	1	0	2	10	10	12	8	8	9	5	1	1	5	8	1	15	14	15	12	9	7	6	3
1225	A	3	2	4	9	5	2	3	11	11	16	14	13	13	9	5	1	1	3	6	9	8	12	9	7	8	9	3	8
1226	A	30	8	8	5	3	5	4	3	7	14	15	12	11	6	5	9	19	11	4	6	4	2	14	11	2	2	1	3
1227	S	26	1	11	11	4	10	2	8	10	9	19	13	12	11	9	7	2	5	5	7	4	1	9	2	1	10	4	0
1228	O	23	3	9	9	3	7	5	4	9	16	12	12	12	7	2	0	0	3	4	11	2	1	0	2	6	2	4	1
1229	N	19	2	2	6	4	1	2	1	0	10	10	13	9	8	5	1	1	9	9	3	0	4	10	8	4	3	11	9
1230	D	16	0	0	1	1	1	1	0	1	4	8	13	3	3	5	6	2	2	2	3	6	5	5	1	1	3	5	3
1923																													
1231	J	12	3	10	7	4	5	3	1	1	13	10	9	9	8	2	2	0	1	11	9	6	8	5	7	7	2	6	8
1232	F	8	6	1	8	4	1	1	8	1	7	11	9	4	3	1	1	1	1	18	16	17	10	4	3	2	2	1	1
1233	M	7	6	1	1	1	1	2	2	8	10	9	6	6	4	6	6	2	0	19	18	14	11	8	6	3	4	2	1
1234	A	3	1	2	0	1	3	6	6	9	8	9	11	8	3	1	1	1	2	9	12	12	10	8	2	1	1	1	2
1235	A	30	2	1	1	10	8	5	2	4	8	3	3	1	1	1	4	3	2	16	13	9	7	5	1	2	1	3	2
1236	M	27	3	1	12	11	3	2	2	5	10	9	7	2	1	1	1	3	8	17	12	9	3	1	1	3	6	10	4
1237	J	23	3	0	1	2	7	7	2	12	5	8	2	1	1	8	9	5	3	14	9	6	2	0	1	2	3	9	7
1238	J	20	3	0	6	11	0	1	1	6	3	2	1	1	1	3	11	9	3	8	3	2	3	2	1	5	9	7	7
1239	A	16	3	1	0	3	3	0	2	2	7	5	1	3	1	0	1	5	6	4	7	3	1	0	0	0	9	9	5
1240	S	12	2	5	7	3	0	4	7	2	3	0	0	3	5	3	17	20	12	8	7	4	0	0	0	3	4	1	5
1241	O	9	2	8	9	7	1	7	19	20	16	14	10	6	2	3	0	0	3	4	9	2	1	0	8	8	13	8	1
1242	N	5	1	3	6	1	1	1	1	9	7	2	0	3	2	1	1	0	6	8	5	1	0	1	11	8	11	8	4
1243	D	2	0	3	10	3	1	2	2	10	8	2	2	6	8	5	2	0	3	4	2	2	1	10	11	10	16	11	9
1244	D	29	4	3	2	0	2	12	3	2	2	6	6	4	16	8	2	0	0	6	8	6	2	5	0	2	13	16	11

Table I. Daily Relative Sunspot Numbers divided by 5

nspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
924																													
245	J	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	0	0	0	0	0
246	F	21	0	0	0	0	6	7	5	5	4	3	3	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
247	M	19	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	2
248	A	15	0	0	2	7	8	9	9	9	7	6	6	3	2	0	0	0	0	0	0	0	0	2	3	4	3	3	5
249	M	12	7	7	9	10	10	9	9	7	5	5	6	5	5	3	3	4	4	3	3	6	6	10	10	9	10	13	9
250	J	8	6	5	3	4	2	5	7	6	4	4	4	4	4	5	3	3	2	4	3	4	3	3	3	6	7	7	7
251	J	5	7	8	8	10	11	11	11	11	8	10	7	6	4	2	2	0	3	5	5	3	2	4	4	5	3	4	5
252	A	1	5	6	8	7	6	6	3	3	5	2	2	3	4	3	2	3	3	3	3	3	4	2	3	2	2	3	3
253	A	28	4	8	12	9	9	8	12	8	8	6	4	4	2	3	4	7	6	5	3	3	2	3	4	4	5	5	5
254	S	24	7	4	4	3	9	9	7	6	9	8	9	8	8	9	9	8	6	5	5	4	6	4	4	4	5	3	3
255	O	21	4	4	4	3	6	6	5	5	5	4	3	3	2	0	0	0	0	0	0	0	0	3	3	3	3	4	8
256	N	17	5	9	9	8	14	10	11	11	11	10	7	5	3	2	0	0	3	0	0	0	2	2	4	5	5	6	6
257	D	14	6	5	5	4	6	6	4	7	10	8	8	4	2	0	0	2	0	0	0	0	2	2	0	0	0	0	0
25																													
58	J	10	0	2	3	3	0	0	3	3	3	6	4	3	4	2	2	0	0	0	0	0	0	0	2	4	4	2	2
59	F	6	2	5	8	7	8	8	11	15	9	10	7	7	3	6	3	3	4	3	3	0	2	0	3	5	2	2	3
60	M	5	0	0	0	0	0	0	2	4	4	4	6	4	6	7	7	5	4	4	3	4	6	7	9	8	7	5	5
61	A	1	2	2	2	3	4	4	5	5	7	12	14	11	12	9	7	7	9	7	12	12	8	9	8	6	7	6	5
62	A	28	2	2	4	4	5	7	7	7	9	8	7	9	8	7	8	6	8	10	10	16	17	14	14	16	15	11	8
63	M	25	7	6	9	8	6	6	4	8	17	19	16	19	15	18	24	23	22	21	18	18	11	7	4	2	3	2	0
64	J	21	0	4	3	2	3	3	4	3	3	4	5	6	8	10	14	10	10	13	10	9	9	9	10	5	5	10	10
65	J	18	8	7	9	9	5	0	4	7	7	7	8	10	7	9	5	5	6	0	2	5	6	8	12	14	12	10	10
66	A	14	7	2	2	2	2	6	5	6	4	7	6	12	13	13	15	16	16	19	16	12	16	15	12	15	9	8	13
67	S	10	10	7	6	4	6	9	12	8	10	12	13	15	16	15	16	15	17	18	15	16	16	15	12	13	11	12	7
68	O	7	6	9	9	11	10	10	12	15	17	19	25	22	25	25	24	23	19	19	20	12	10	9	7	6	7	6	7
69	N	3	5	4	7	4	6	5	8	10	8	10	8	8	9	10	11	11	12	13	17	22	25	24	28	24	20	14	16
70	N	30	13	14	13	15	13	14	19	23	16	12	12	18	20	27	25	23	32	28	27	23	27	25	21	22	21	23	21
71	D	27	18	20	18	17	19	19	15	12	8	7	12	10	16	18	19	17	14	12	11	13	16	19	16	19	17	21	19

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1924																													
1245	J	25	8	9	3	3	20	18	6	3	3	5	2	15	10	8	2	1	8	7	1	2	0	0	8	6	0	3	17
1246	F	21	12	10	12	8	8	8	3	0	0	0	8	10	4	7	9	13	11	10	7	5	5	1	1	1	5	0	5
1247	M	19	9	10	10	10	9	3	4	2	3	1	8	17	10	1	0	7	1	1	11	10	3	2	1	0	2	0	2
1248	A	15	5	2	7	6	5	4	6	4	3	4	12	12	5	4	6	1	3	1	2	2	2	0	0	3	4	1	3
1249	M	12	9	5	0	1	8	4	0	2	4	18	20	17	10	8	4	6	14	7	6	1	2	1	0	3	2	0	1
1250	J	8	1	10	20	19	8	6	1	6	8	4	15	16	12	9	9	9	4	4	3	4	3	4	7	3	4	0	1
1251	J	5	1	6	8	2	15	9	6	2	4	1	10	6	6	8	7	10	10	3	0	0	10	13	14	7	2	0	0
1252	A	1	5	4	4	8	10	8	5	4	1	1	1	0	2	1	1	3	12	12	4	0	0	2	3	1	0	2	1
1253	A	28	2	12	11	7	14	1	4	8	9	10	17	17	9	8	1	11	8	2	6	1	2	4	5	1	3	4	11
1254	S	24	14	6	4	12	8	3	2	1	1	1	10	5	4	9	6	2	1	1	0	1	0	6	9	5	11	1	5
1255	O	21	7	4	18	19	11	2	10	4	0	0	8	10	7	6	2	0	9	3	0	9	7	2	3	10	6	5	1
1256	N	17	1	0	16	3	6	1	1	19	6	10	4	9	5	1	0	1	2	2	0	0	6	6	3	2	6	14	7
1257	D	14	3	6	0	6	5	6	12	12	4	8	4	0	1	2	1	0	0	4	1	1	5	0	7	0	2	1	2
1925																													
1258	J	10	0	0	1	10	1	1	15	14	10	18	13	9	2	11	9	0	1	1	0	1	3	0	7	1	0	0	1
1259	F	6	3	5	10	15	3	2	6	5	4	1	6	7	3	7	7	0	0	0	7	8	1	0	8	9	1	1	4
1260	M	5	9	4	1	1	15	10	4	2	1	1	14	4	2	0	3	8	2	3	6	5	1	5	9	0	5	1	1
1261	A	1	7	8	11	1	9	7	7	6	10	11	8	9	1	1	10	6	1	1	0	11	3	2	2	1	0	4	7
1262	A	28	3	5	2	1	0	4	18	10	3	4	6	8	1	0	0	1	0	0	0	1	8	8	1	1	4	2	5
1263	M	25	4	5	8	13	5	14	13	9	9	10	8	5	10	3	2	1	3	5	3	19	8	5	9	9	6	2	0
1264	J	21	0	3	13	19	13	5	12	13	8	8	7	5	4	2	5	3	1	2	9	10	6	1	1	7	13	1	0
1265	J	18	1	4	1	9	10	7	4	9	17	14	11	4	0	1	5	2	10	9	3	6	13	13	9	6	1	0	0
1266	A	14	3	1	3	8	11	8	3	3	15	18	6	7	5	1	0	4	9	6	16	16	10	8	7	5	11	2	5
1267	S	10	0	0	0	1	17	18	11	9	5	2	2	19	11	8	18	6	5	1	1	0	0	8	3	1	11	9	7
1268	O	7	4	12	16	10	11	15	9	7	9	7	4	0	3	8	17	15	19	18	7	4	9	2	0	0	9	13	11
1269	N	3	2	1	0	2	2	11	17	15	11	4	9	14	7	1	1	1	1	3	0	1	2	9	5	1	0	0	0
1270	N	30	0	5	4	1	4	9	14	9	4	5	6	5	2	5	1	9	6	0	10	3	7	0	1	6	8	4	0
1271	D	27	15	17	5	6	5	7	9	4	10	3	4	10	6	4	4	7	7	14	13	13	11	4	14	11	3	0	18

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1926																													
1272	J	23	21	18	25	15	16	8	11	10	5	8	9	8	9	7	6	7	9	7	9	20	19	29	30	33	29	26	17
1273	F	19	20	14	12	9	10	10	13	12	11	9	10	14	21	21	24	20	20	21	17	13	10	13	9	12	18	22	16
1274	M	18	15	14	10	8	3	9	7	8	9	10	4	6	7	8	7	6	6	6	5	5	6	6	10	6	7	8	12
1275	A	14	15	13	14	14	13	15	12	9	7	3	3	4	4	3	4	9	8	13	11	12	14	18	11	15	21	19	18
1276	M	11	19	18	17	12	18	17	13	15	16	13	15	14	8	6	7	5	5	9	11	11	10	13	15	10	20	16	17
1277	J	7	19	18	19	19	19	13	15	16	12	13	10	11	10	11	9	8	11	12	12	18	19	21	23	22	21	16	16
278	J	4	13	9	10	7	7	5	4	6	4	2	3	3	2	o	o	3	5	9	11	11	20	15	25	23	23	22	20
279	J	31	20	16	13	18	21	17	16	8	10	14	13	12	14	15	11	10	9	8	8	11	11	15	12	15	16	15	13
280	A	27	11	11	11	10	9	9	5	6	5	8	9	9	6	6	7	6	7	8	11	15	15	18	23	23	22	21	18
281	S	23	17	14	14	14	21	16	12	10	6	5	8	7	8	11	14	24	14	17	29	27	31	27	22	23	17	12	11
282	O	20	7	10	8	8	8	13	19	17	19	10	13	8	11	13	12	15	12	9	6	7	7	10	8	8	8	8	8
283	N	16	8	8	6	4	11	21	23	23	21	22	22	18	18	13	15	23	17	12	11	18	14	14	13	8	14	18	19
284	D	13	24	16	22	20	19	19	18	15	18	21	14	22	19	19	11	17	13	6	10	14	14	13	17	18	16	23	21
927																													
285	J	9	25	24	25	16	11	14	14	16	18	23	18	16	14	16	13	17	9	9	8	10	13	22	30	25	28	32	31
286	F	5	35	24	33	24	26	25	22	17	12	20	15	12	16	16	13	14	10	11	11	9	11	12	16	13	9	10	10
287	M	4	14	12	12	15	18	19	10	11	10	15	14	21	19	25	26	30	23	27	25	17	12	10	5	8	4	7	4
288	M	31	5	16	17	18	16	21	20	20	16	28	21	33	28	25	28	27	32	25	24	19	14	12	14	13	10	11	18
289	A	27	19	8	11	10	10	11	14	15	20	21	20	16	19	22	21	29	20	12	12	14	18	20	14	15	17	18	18
290	M	24	21	18	18	18	9	12	7	6	7	9	9	9	11	13	16	16	12	11	13	15	15	11	7	5	11	8	7
291	J	20	6	7	9	9	13	15	18	16	24	23	23	20	24	21	23	20	15	13	9	11	11	9	4	3	7	7	9
292	J	17	10	9	5	7	9	10	10	11	14	14	15	7	9	8	9	10	12	7	10	11	10	8	4	4	4	7	7
293	A	13	11	12	14	12	16	20	16	12	10	13	14	14	12	12	12	11	14	14	12	13	13	11	8	10	9	10	8
294	S	9	9	10	14	19	20	21	31	26	23	17	17	17	11	9	15	18	17	12	16	8	6	6	9	7	11	13	17
295	O	6	17	17	18	19	23	23	15	18	16	16	11	9	7	11	8	14	14	12	13	14	10	5	5	6	8	15	13
296	N	2	12	7	5	9	8	7	3	10	15	19	21	23	20	19	17	18	17	13	16	16	14	17	18	18	14	14	9
297	N	29	12	12	7	6	6	4	7	13	12	15	18	17	17	11	13	10	4	4	4	2	2	3	4	3	7	12	13
298	D	26	7	12	13	14	15	16	19	20	15	17	18	21	17	19	19	17	15	16	10	13	13	17	16	13	15	11	14

Table J. International Daily Magnetic Character-figures

		Sunspots/5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1926																													
1272	J	23	17	4	3	20	19	9	5	3	5	7	11	12	9	4	0	0	0	0	10	13	9	10	9	12	3	13	15
1273	F	19	7	4	8	9	20	20	15	10	3	4	10	11	9	7	20	13	9	2	18	14	12	8	3	3	1	8	8
1274	M	18	14	10	11	12	10	4	6	3	2	5	10	10	10	2	5	2	4	2	3	12	12	11	11	7	4	8	6
1275	A	14	20	20	15	11	4	5	1	9	11	10	10	6	9	5	1	0	0	0	0	8	17	14	12	9	6	9	15
1276	M	11	11	10	11	3	0	3	9	9	6	10	8	3	3	1	3	0	1	1	1	2	0	19	18	4	1	3	5
1277	J	7	9	14	10	7	3	0	2	3	7	6	3	4	2	0	2	2	10	8	0	0	1	5	7	1	2	6	3
1278	J	4	4	11	7	10	6	2	2	1	6	5	1	3	4	5	9	4	1	1	1	0	7	2	3	10	14	2	7
1279	J	31	16	13	5	8	5	3	2	1	0	9	7	1	8	15	7	7	8	9	10	9	4	1	1	0	3	3	3
1280	A	27	1	2	2	3	5	0	2	1	0	0	8	5	16	17	11	10	7	2	15	18	14	7	10	14	17	19	11
1281	S	23	8	4	4	1	0	0	0	0	1	3	9	10	4	4	8	5	3	0	7	4	12	19	20	18	9	6	13
1282	O	20	7	1	0	0	5	15	3	5	5	8	2	3	10	9	11	3	1	6	0	0	0	0	5	6	1	0	0
1283	N	16	0	0	4	5	1	12	6	7	6	1	3	1	15	16	7	2	2	7	7	2	2	3	0	0	7	6	5
1284	D	13	4	1	9	10	7	3	1	10	7	3	16	9	4	3	9	9	10	2	7	13	7	1	15	10	8	19	11
1927																													
1285	J	9	1	1	8	8	5	7	3	2	6	6	8	1	0	0	1	11	11	11	3	2	6	3	1	4	2	9	4
1286	F	5	4	0	0	8	15	12	5	10	11	6	4	11	7	9	7	4	1	0	0	14	11	10	6	11	12	3	4
1287	M	4	2	5	6	5	7	14	13	8	5	8	8	11	18	16	11	9	10	2	1	1	0	1	13	17	16	9	8
1288	M	31	6	5	5	3	10	7	2	9	9	14	8	15	10	9	19	9	1	0	1	1	1	0	0	9	12	9	2
1289	A	27	1	0	3	5	4	5	13	10	18	6	16	10	9	3	0	0	3	5	12	9	1	1	10	14	6	4	3
1290	M	24	2	3	1	10	12	4	2	2	9	10	6	3	11	6	3	0	1	7	10	10	3	5	6	2	6	1	0
1291	J	20	0	1	3	2	0	1	13	7	3	3	7	10	6	1	2	6	7	10	9	1	0	2	2	3	2	1	1
1292	J	17	9	2	7	7	16	20	11	8	4	7	8	2	1	3	3	7	10	9	8	8	1	2	3	2	2	3	2
1293	A	13	0	4	8	7	2	2	9	20	20	8	5	2	0	0	3	2	15	15	9	12	5	6	14	9	12	12	12
1294	S	9	14	16	10	5	9	10	8	2	2	2	2	7	4	1	0	1	10	10	8	7	10	9	3	9	8	1	9
1295	O	6	10	15	13	10	18	9	20	17	7	4	3	1	3	5	5	1	20	19	10	8	9	3	3	7	8	1	0
1296	N	2	0	1	5	2	1	0	6	4	2	2	2	5	0	0	4	2	15	8	7	8	1	0	4	0	3	5	1
1297	N	29	8	9	8	9	2	0	9	10	7	7	7	6	1	3	17	14	12	7	13	12	11	0	0	1	5	1	1
1298	D	26	3	1	13	5	1	8	11	2	1	6	3	3	4	2	4	2	0	0	1	0	3	2	1	5	4	8	5

Table I. Daily Relative Sunspot Numbers divided by 5

nspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
28																													
99	J	22	9	13	21	24	28	27	20	19	14	19	18	10	13	9	10	7	7	7	5	7	6	8	7	6	15	15	15
00	F	18	14	21	27	23	26	26	23	22	24	25	21	19	18	13	10	10	13	15	19	19	25	17	21	21	18	19	22
01	M	16	24	24	25	29	21	17	19	14	16	15	10	14	13	15	13	14	20	19	16	22	24	29	24	29	26	23	24
02	A	12	22	20	20	18	18	13	6	9	7	7	5	6	6	9	11	11	16	16	21	26	29	25	23	24	24	28	27
03	M	9	27	24	17	15	11	6	4	4	3	3	3	1	3	5	8	8	10	8	11	21	27	32	31	25	26	23	23
04	J	5	19	20	21	16	9	6	6	4	7	7	7	11	14	15	19	18	19	19	26	30	33	27	28	32	27	25	25
05	J	2	26	26	17	12	10	12	19	18	17	14	25	26	25	28	26	29	24	26	28	26	21	15	12	12	13	13	14
06	J	29	19	21	22	21	24	24	22	16	13	15	12	15	16	15	15	19	19	14	16	12	10	11	12	15	17	21	19
07	A	25	21	23	23	21	17	17	17	13	10	17	14	13	10	12	21	21	18	17	18	15	12	12	10	11	15	17	21
08	S	21	23	22	22	27	29	34	29	23	22	23	14	14	13	10	9	6	13	12	11	15	19	15	18	17	17	17	16
09	O	18	17	14	14	15	11	15	16	11	9	6	6	5	8	10	11	10	10	10	12	14	14	11	17	18	15	17	18
10	N	14	15	16	13	10	11	12	8	8	6	7	4	2	6	5	6	2	6	10	11	13	19	23	19	19	17	14	17
11	D	11	16	21	18	19	21	18	12	10	4	6	5	4	4	3	2	3	6	7	10	12	14	16	15	14	15	10	9
29																													
12	J	7	13	16	17	16	9	17	17	20	16	23	18	18	16	19	16	14	21	15	14	13	11	4	5	6	7	7	8
13	F	3	7	9	10	10	17	19	24	25	18	17	14	15	14	11	10	11	7	12	12	12	12	13	12	14	14	11	9
14	M	2	10	12	14	11	15	15	18	21	19	19	19	16	14	13	12	8	5	4	3	3	4	5	6	5	7	8	4
15	M	29	4	8	11	10	11	10	7	8	13	11	13	13	16	14	15	12	15	12	13	10	10	12	13	12	10	10	8
16	A	25	8	9	9	8	8	11	13	19	19	15	15	17	14	16	16	13	11	10	16	15	15	15	11	10	6	7	6
17	M	22	7	6	10	13	12	12	12	7	7	7	6	6	6	10	11	12	17	14	12	14	17	18	17	16	16	16	13
18	J	18	16	14	18	21	22	21	17	16	17	13	13	16	17	15	17	18	12	14	14	16	15	12	15	19	16	18	16
19	J	15	19	18	14	14	15	17	20	16	13	10	8	10	14	12	9	9	12	14	14	10	8	9	12	13	10	12	13
20	A	11	13	16	16	21	27	25	23	22	22	19	14	15	14	11	11	10	7	7	6	6	2	2	4	6	6	10	10
21	S	7	12	11	12	12	8	7	11	5	7	6	8	4	4	9	11	9	7	3	2	5	5	4	9	8	10	11	13
22	O	4	11	11	13	15	14	16	17	16	16	16	14	12	8	8	6	4	4	7	5	7	4	7	8	14	20	12	16
23	O	31	14	14	16	15	19	18	20	16	19	16	19	15	14	17	20	19	13	14	14	10	9	9	9	11	12	17	18
24	N	27	24	26	30	26	28	25	25	20	18	19	14	12	12	12	21	26	28	28	28	28	29	28	28	27	27	26	26
25	D	24	24	27	22	19	19	15	13	8	8	9	11	8	10	11	13	14	13	12	13	14	19	18	20	22	22	27	19

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1928																													
1299	J	22	4	8	5	6	5	16	11	9	7	1	7	8	8	7	4	4	5	1	1	2	0	9	10	8	7	7	2
1300	F	18	9	9	9	11	5	7	0	9	10	8	7	3	1	0	1	0	0	1	3	1	0	5	18	13	13	12	7
1301	M	16	7	6	3	4	3	5	6	9	8	8	5	1	3	2	4	1	10	7	9	8	6	8	13	9	7	10	1
1302	A	12	1	1	1	7	9	7	1	6	9	10	5	5	1	1	2	2	0	0	1	1	0	0	3	12	5	6	9
1303	M	9	3	14	13	14	11	9	10	11	9	10	8	4	2	1	2	3	3	1	17	19	17	9	8	9	8	7	8
1304	J	5	11	8	11	10	7	0	1	12	13	11	6	3	3	3	3	8	6	17	12	9	7	4	3	6	5	4	4
1305	J	2	13	9	8	7	7	15	20	12	10	9	6	1	3	1	2	2	7	4	2	10	13	6	7	5	7	6	9
1306	J	29	4	2	11	7	4	3	11	17	11	11	3	2	1	4	11	6	1	1	3	3	5	2	1	1	0	8	5
1307	A	25	5	17	16	9	3	0	4	4	8	15	4	7	5	16	15	11	8	7	3	7	9	4	0	1	15	14	6
1308	S	21	2	9	8	11	16	8	6	1	2	6	5	12	8	4	10	8	8	5	0	4	4	5	9	8	7	10	8
1309	O	18	20	12	8	9	11	4	17	18	11	10	3	7	8	6	6	13	17	11	1	5	4	0	0	12	12	9	16
1310	N	14	6	13	12	13	8	7	2	1	0	2	8	6	3	4	0	0	4	9	4	2	1	12	15	8	5	3	4
1311	D	11	9	11	9	7	4	3	0	3	0	1	8	7	1	7	8	9	3	2	6	4	2	0	1	8	10	13	7
1929																													
1312	J	7	1	11	15	11	8	1	5	11	5	3	0	0	0	3	6	6	0	1	0	3	1	2	7	2	4	5	2
1313	F	3	1	0	2	13	9	9	13	11	8	4	3	1	1	10	18	14	13	6	6	11	8	4	6	11	19	19	8
1314	M	2	6	9	4	11	2	7	13	8	4	14	20	13	9	17	14	10	5	6	11	14	10	6	8	6	4	6	7
1315	M	29	6	2	2	6	4	4	14	6	3	3	4	1	5	7	3	3	1	9	16	13	7	3	1	5	2	0	0
1316	A	25	4	4	4	9	10	11	8	8	8	5	3	3	4	4	4	1	3	9	15	11	11	11	7	2	4	4	1
1317	M	22	2	11	5	8	4	4	10	7	7	5	5	6	3	1	0	2	5	8	10	15	12	6	3	0	3	4	3
1318	J	18	1	3	4	6	12	12	10	6	0	3	9	2	10	9	6	5	4	13	12	6	3	2	19	10	3	2	11
1319	J	15	16	16	10	4	0	9	7	7	2	8	4	3	1	0	0	3	10	18	12	3	5	10	0	0	0	0	1
1320	A	11	12	5	2	18	15	10	7	12	11	4	3	1	1	3	0	4	3	1	2	2	6	5	5	1	1	1	5
1321	S	7	16	7	10	16	14	12	11	15	10	9	5	5	1	4	10	16	7	5	7	7	11	2	1	5	2	3	6
1322	O	4	9	4	5	18	18	15	10	7	9	11	5	2	18	17	14	13	10	5	7	5	9	5	0	2	3	10	15
1323	O	31	10	12	13	19	14	13	12	12	6	9	1	1	2	4	3	10	16	2	1	3	9	7	3	2	1	2	8
1324	N	27	10	6	6	5	1	2	14	16	15	12	8	8	8	7	11	10	3	4	3	17	14	7	1	1	1	14	9
1325	D	24	6	4	3	2	4	3	4	7	7	3	13	15	16	16	13	9	4	3	0	1	7	3	7	3	10	6	8

Table I. Daily Relative Sunspot Numbers divided by 5

spots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
0																													
6	J	20	13	15	12	13	10	8	7	7	10	12	14	13	10	15	20	23	24	22	23	17	21	14	16	11	10	8	8
7	F	16	5	5	5	4	2	2	3	4	4	4	3	4	6	5	5	6	11	11	10	10	10	11	6	4	6	6	4
8	M	15	6	9	11	9	9	7	7	7	2	5	4	5	8	7	6	11	11	12	12	9	9	12	12	13	7	10	10
9	A	11	12	10	12	9	7	5	4	3	2	6	6	7	6	5	5	5	5	8	7	10	11	11	10	9	6	8	5
0	M	8	6	5	4	5	5	5	4	7	9	7	5	7	8	7	7	9	14	10	9	12	8	11	10	7	7	6	6
1	J	4	7	6	7	14	13	13	11	11	9	8	5	5	4	4	3	2	3	2	2	2	2	4	3	7	6	6	7
2	J	1	5	8	7	6	6	6	4	4	8	5	3	6	4	5	5	5	6	2	2	2	5	3	5	3	3	4	4
3	J	28	7	7	2	5	4	5	2	2	2	4	4	4	4	4	5	5	5	3	2	2	2	5	5	6	7	6	8
4	A	24	8	9	7	8	9	11	10	10	12	10	10	14	14	15	14	9	8	9	8	7	8	5	6	4	2	0	2
5	S	20	2	2	2	3	4	4	7	7	7	6	4	6	6	6	6	9	9	8	8	7	12	11	10	12	8	7	5
6	O	17	3	5	5	3	2	3	4	4	6	8	13	11	10	10	10	12	9	5	6	4	2	2	2	2	3	2	3
7	N	13	0	3	3	5	6	7	9	12	13	14	14	10	12	16	15	14	13	11	9	7	7	4	2	2	2	0	2
8	D	10	4	5	4	3	5	5	4	6	11	10	9	7	6	6	9	11	11	7	4	2	3	3	0	2	0	2	4
1																													
9	J	6	3	3	3	3	3	2	2	4	9	9	6	4	5	4	5	5	5	5	4	4	2	2	1	2	0	0	0
0	F	2	2	6	6	5	6	4	6	6	6	4	4	5	4	9	9	10	9	10	13	18	16	20	19	14	16	14	10
1	M	1	7	7	5	7	8	7	6	8	7	7	8	8	9	10	10	9	11	10	8	8	6	5	4	4	2	2	2
2	M	28	3	4	2	6	7	7	6	8	5	7	9	9	10	6	8	8	9	8	8	7	9	5	4	4	8	9	6
3	A	24	6	8	5	4	3	4	4	4	2	2	2	2	4	4	6	4	7	7	6	7	4	8	8	6	7	7	5
4	M	21	6	6	7	7	7	7	7	4	4	4	4	3	6	7	8	6	8	7	9	7	10	4	3	0	2	0	0
5	J	17	0	2	0	0	0	2	0	0	0	0	2	2	5	5	5	5	5	4	4	4	8	7	6	10	7	6	6
6	J	14	5	2	2	2	2	0	0	2	2	2	2	2	0	2	2	5	5	5	5	7	7	6	6	4	2	0	1
7	A	10	0	2	0	2	0	0	0	3	3	3	2	2	0	0	0	0	2	6	8	5	6	6	7	6	6	4	3
8	S	6	3	3	4	3	5	6	5	6	6	4	6	6	6	3	3	2	5	5	4	2	2	3	4	2	3	2	5
9	O	3	3	4	3	3	2	2	2	0	2	0	2	2	2	0	0	0	2	2	2	2	5	4	3	3	3	3	2
0	O	30	3	4	2	3	4	4	5	6	3	3	4	0	0	0	0	0	0	0	2	3	2	4	8	9	7	8	9
1	N	26	9	7	7	7	6	4	3	2	0	0	2	3	3	5	6	7	8	8	7	5	5	3	2	2	2	2	3
2	D	23	4	5	7	6	6	3	2	3	2	3	6	3	2	2	0	0	1	0	2	0	0	0	1	2	3	3	2

Table J. International Daily Magnetic Character-figures

		Sunspots/5	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1930																													
1326	J	20	11	9	7	5	4	3	0	0	3	8	11	7	11	10	12	6	5	2	4	3	1	1	3	17	19	17	16
1327	F	16	15	10	11	9	8	4	3	8	10	13	9	7	13	14	14	9	5	0	1	0	0	0	0	11	19	17	16
1328	M	15	14	12	11	12	11	8	8	11	7	14	7	9	11	12	11	6	5	8	6	5	2	3	13	16	17	13	15
1329	A	11	14	13	12	9	10	10	8	6	13	17	14	15	13	11	8	8	5	6	11	12	5	1	4	11	18	15	15
1330	M	8	11	12	8	5	11	11	4	7	16	17	12	11	10	10	11	6	3	8	6	2	3	4	12	17	14	14	13
1331	J	4	12	5	6	14	10	8	3	2	15	13	7	3	19	12	11	8	7	8	3	1	1	1	5	11	13	12	9
1332	J	1	6	9	10	12	11	5	5	2	12	17	16	15	14	7	8	12	10	5	5	2	3	2	3	10	16	13	7
1333	J	28	7	10	9	6	5	4	3	2	9	17	17	17	15	12	12	13	11	12	11	7	4	5	10	5	11	12	11
1334	A	24	10	6	6	6	3	4	6	7	13	10	16	11	11	14	8	6	11	6	6	5	3	3	2	2	6	20	13
1335	S	20	4	11	4	7	8	4	1	1	11	17	18	11	9	15	12	10	9	7	9	6	4	2	2	0	15	3	2
1336	O	17	17	9	9	12	7	4	2	1	12	17	16	12	13	13	11	7	8	4	12	5	2	3	7	7	3	1	0
1337	N	13	5	15	11	4	4	4	0	0	0	0	8	16	18	12	10	5	6	4	3	2	19	15	3	2	5	0	2
1338	D	10	1	1	8	11	7	3	0	0	0	8	15	14	10	9	7	6	5	5	0	6	1	0	7	4	0	0	0
1931																													
1339	J	6	0	0	0	11	11	5	3	2	0	6	14	14	13	8	9	3	2	2	1	12	10	8	9	8	1	5	7
1340	F	2	9	7	7	4	0	4	2	2	1	2	0	18	15	13	7	6	2	2	1	0	3	4	18	14	12	10	3
1341	M	1	2	8	9	3	4	2	6	8	7	9	2	12	14	10	4	2	2	0	1	8	13	8	6	3	7	11	6
1342	M	28	5	5	1	4	9	3	9	6	2	1	1	3	7	10	8	0	0	1	2	1	3	9	13	13	3	4	3
1343	A	24	2	5	6	1	2	0	2	1	4	2	2	4	11	17	4	0	0	10	8	12	13	14	8	5	6	2	5
1344	M	21	3	1	1	1	7	10	5	0	7	2	1	13	17	9	3	2	6	3	10	13	8	10	10	5	5	2	1
1345	J	17	0	2	3	2	8	7	4	3	1	15	14	14	7	2	2	9	5	6	2	2	4	3	1	0	7	6	6
1346	J	14	8	8	7	3	2	1	0	1	1	17	10	13	9	5	13	8	6	2	4	2	7	4	6	4	10	14	12
1347	A	10	8	8	3	5	4	3	10	2	2	9	15	12	3	4	9	14	9	9	9	3	2	4	8	1	11	15	8
1348	S	6	13	11	11	9	7	5	6	4	9	15	15	13	4	2	10	11	9	9	9	5	6	7	0	2	11	12	16
1349	O	3	9	12	15	8	5	4	3	4	3	15	13	7	9	4	11	12	11	8	9	10	9	6	3	8	11	13	20
1350	O	30	18	11	6	6	8	12	14	14	10	16	11	9	6	0	8	11	13	14	10	13	11	8	3	0	4	4	4
1351	N	26	15	11	3	4	2	10	15	14	14	13	9	4	3	4	10	13	11	8	8	9	8	8	1	0	0	0	3
1352	D	23	9	1	10	1	0	11	12	12	10	10	14	9	2	2	4	9	11	11	11	12	12	7	8	8	8	5	1

Table I. Daily Relative Sunspot Numbers divided by 5

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
932																													
353	J	19	2	0	2	4	4	4	6	9	8	4	4	4	4	4	4	4	2	2	2	2	2	0	2	2	2	0	0
354	F	15	0	0	0	0	0	0	0	0	4	5	6	6	8	7	6	7	10	8	5	5	5	4	2	2	2	0	0
355	M	13	0	0	0	0	2	3	2	2	2	2	0	2	0	2	3	3	2	2	2	2	2	4	2	0	0	0	0
356	A	9	0	0	0	0	0	0	2	2	2	2	0	0	4	7	6	5	7	7	7	6	5	3	2	0	0	0	0
357	M	6	0	2	3	3	2	2	2	2	5	8	7	9	8	6	8	6	7	7	7	5	5	4	4	2	2	2	2
358	J	2	0	2	3	3	5	7	8	7	7	7	3	2	2	2	4	4	5	6	6	7	6	7	6	8	6	6	7
359	J	29	5	5	5	5	6	7	7	5	3	3	3	2	2	2	2	0	0	0	0	2	0	2	0	2	0	0	0
360	J	26	0	2	2	2	2	2	2	3	2	2	2	2	2	2	0	0	2	0	0	0	0	0	0	0	0	0	0
361	A	22	0	0	3	4	5	6	4	4	2	2	2	2	2	0	0	0	0	0	0	0	0	3	2	0	0	0	0
362	S	18	0	2	0	4	2	0	2	2	0	1	2	2	2	2	2	2	0	0	2	2	0	0	0	0	2	2	2
363	O	15	2	2	2	3	5	6	4	4	4	3	3	2	2	2	2	2	2	2	2	3	3	0	0	0	0	0	2
364	N	11	2	2	2	2	2	0	3	5	6	7	5	3	0	0	0	0	0	0	2	3	4	3	3	2	2	2	2
365	D	8	3	3	3	3	5	5	4	3	3	4	4	4	4	0	0	0	0	0	0	2	3	2	2	2	0	0	2
933																													
366	J	4	3	4	4	4	4	4	4	4	6	9	8	6	3	2	0	0	0	0	0	0	0	0	0	0	2	3	5
367	J	31	7	9	10	14	12	13	14	11	11	10	7	7	4	3	2	0	0	0	0	0	0	0	0	0	0	0	0
368	F	27	2	3	2	2	2	3	3	3	3	3	2	2	2	2	0	0	0	0	0	0	0	0	0	3	3	3	5
369	M	26	6	6	6	5	3	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	4	2	2
370	A	22	0	0	0	0	0	0	0	2	0	0	2	0	0	0	0	0	0	1	2	0	0	0	0	0	0	0	0
371	M	19	2	3	3	3	4	3	2	0	0	0	0	0	0	0	2	2	2	2	3	1	0	0	0	0	0	3	4
372	J	15	4	3	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	2	4	4	4	3	2
373	J	12	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
374	A	8	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2
375	S	4	2	0	3	4	3	3	2	0	0	0	0	0	0	0	0	0	0	2	3	3	2	0	0	0	3	3	2
376	O	1	0	0	0	0	0	0	0	0	0	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	2	4
377	O	28	4	3	3	2	2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
378	N	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
379	D	21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	3	3	4

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1932																													
1353	J	19	0	1	0	0	2	9	14	12	14	14	8	7	9	2	2	17	17	10	10	9	11	9	14	13	11	8	7
1354	F	15	4	0	0	2	8	12	5	12	14	10	8	4	0	0	0	0	14	14	13	13	12	11	11	13	16	12	5
1355	M	13	3	6	1	2	5	14	6	8	12	13	10	7	2	0	4	16	17	17	16	12	15	12	10	12	9	15	11
1356	A	9	9	6	2	2	14	11	9	6	8	12	2	0	0	10	14	14	12	12	10	8	6	2	5	10	11	12	11
1357	M	6	9	4	1	0	7	11	5	8	8	10	11	4	1	1	0	10	7	10	8	11	7	8	11	19	19	8	2
1358	J	2	2	0	0	1	3	9	11	11	10	9	6	6	2	0	1	0	1	5	11	7	8	4	2	5	6	1	1
1359	J	29	2	2	1	1	0	8	11	15	11	10	9	8	2	4	0	1	1	14	8	4	5	2	5	4	1	4	1
1360	J	26	5	2	0	0	3	9	9	12	14	11	11	5	1	1	3	0	4	11	11	5	2	0	1	1	1	1	10
1361	A	22	10	8	1	4	1	17	19	17	13	7	8	8	1	3	8	15	9	12	10	1	1	2	2	4	2	0	1
1362	S	18	9	12	10	6	11	16	16	18	11	10	3	8	6	4	8	6	7	6	2	3	4	9	10	6	4	2	3
1363	O	15	18	13	12	6	5	15	15	10	12	10	7	1	11	1	3	9	3	12	8	5	8	6	1	5	3	0	1
1364	N	11	3	8	7	12	11	16	12	9	6	6	2	0	2	0	10	2	1	7	10	4	6	5	4	4	0	4	0
1365	D	8	13	10	8	3	1	11	18	16	15	13	8	8	2	0	2	1	2	9	8	9	9	2	8	9	12	8	2
1933																													
1366	J	4	0	0	12	8	3	1	0	1	1	1	7	12	5	2	1	12	9	1	14	11	11	9	10	14	12	9	7
1367	J	31	5	1	8	3	3	3	1	5	4	3	2	0	3	0	8	9	1	0	3	17	14	17	16	15	15	11	12
1368	F	27	6	3	4	4	5	2	0	1	1	2	0	4	8	4	8	6	2	1	4	15	15	16	15	15	15	15	10
1369	M	26	6	11	9	8	6	6	5	4	9	8	5	6	10	8	7	7	1	0	1	5	12	13	14	11	12	12	12
1370	A	22	12	11	6	5	8	4	3	1	13	19	11	8	10	8	10	7	1	0	1	1	1	4	10	9	6	8	11
1371	M	19	7	2	2	3	2	1	3	0	7	6	11	12	12	11	5	4	1	0	0	1	10	8	3	1	4	16	12
1372	J	15	8	1	2	1	8	12	6	3	1	1	10	6	9	10	8	4	5	4	5	4	3	4	2	9	13	9	9
1373	J	12	4	1	1	0	3	10	10	3	5	1	3	15	16	5	7	11	2	2	1	3	0	1	1	1	18	13	4
1374	A	8	3	1	1	0	2	15	10	8	4	10	13	10	11	13	2	10	11	9	9	3	1	1	2	1	6	8	1
1375	S	4	4	0	2	4	8	19	12	7	8	17	14	15	10	9	9	9	6	6	5	3	4	9	6	8	9	7	4
1376	O	1	1	3	2	7	13	9	16	10	12	11	9	11	13	13	6	1	6	12	2	4	1	2	4	8	9	9	3
1377	O	28	1	0	2	2	4	6	9	11	7	16	15	15	10	8	11	4	2	1	1	2	0	5	6	6	10	4	5
1378	N	24	1	3	0	11	6	4	2	1	4	12	13	14	8	10	2	15	13	6	2	3	1	1	2	4	9	6	3
1379	D	21	4	3	2	0	3	4	4	8	5	0	2	16	12	3	1	0	1	2	3	2	3	3	1	1	10	8	6

Table I. Daily Relative Sunspot Numbers divided by 5

unspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
934																													
380	J	17	3	1	0	0	0	0	0	0	0	0	0	0	2	3	3	3	2	2	0	2	3	4	2	3	2	2	2
381	F	13	4	4	2	3	3	3	2	0	0	0	0	2	2	0	0	0	0	0	0	2	0	2	2	4	5	3	3
382	M	12	2	0	0	0	0	0	2	0	0	0	0	0	0	2	2	0	0	2	2	0	0	3	3	2	2	0	0
383	A	8	0	0	0	0	0	0	2	5	7	7	6	5	5	5	4	4	4	3	2	4	3	2	0	2	0	0	3
384	M	5	4	5	6	7	5	4	2	2	3	5	6	5	7	9	10	8	6	7	7	5	4	4	2	4	2	0	0
385	J	1	0	0	0	0	0	0	0	0	2	0	0	0	0	0	3	6	7	6	5	3	4	2	2	2	2	2	0
386	J	28	0	0	0	0	0	0	0	0	2	2	3	3	4	5	5	5	5	4	5	4	4	4	2	2	0	0	2
387	J	25	2	0	0	2	0	0	0	0	0	0	0	0	2	2	2	2	3	3	5	5	5	5	6	4	5	3	0
388	A	21	0	0	2	2	0	0	2	0	0	2	0	0	0	2	0	0	0	0	0	0	2	0	2	2	2	4	2
389	S	17	2	0	0	0	0	0	2	0	0	0	0	2	3	5	3	0	0	2	0	0	0	0	0	0	2	2	2
390	O	14	2	2	2	2	2	3	0	0	2	2	0	2	2	2	2	2	4	2	3	3	4	3	3	3	2	2	2
391	N	10	2	2	2	0	0	0	0	0	2	0	0	0	0	0	0	2	3	3	5	7	7	6	4	3	3	3	2
392	D	7	3	3	0	0	0	0	0	0	0	0	0	0	0	3	5	5	6	5	10	8	6	7	7	8	7	7	6
935																													
393	J	3	6	5	4	5	4	3	3	2	3	5	5	6	4	4	3	3	2	2	2	3	4	7	7	7	6	6	3
394	J	30	2	3	2	2	2	2	2	6	10	6	5	6	5	6	4	6	5	4	2	2	4	6	5	6	4	4	5
395	F	26	4	4	4	4	4	4	3	4	3	2	2	4	6	7	9	12	15	14	10	9	7	8	7	5	4	3	2
396	M	25	2	2	2	0	0	0	0	0	0	0	2	0	0	0	3	5	5	4	5	7	6	8	5	4	4	3	4
397	A	21	3	4	2	2	2	0	0	0	2	4	4	6	10	12	12	12	12	13	11	9	10	9	9	10	7	6	2
398	M	18	0	0	0	0	0	0	4	2	2	2	2	4	6	8	7	8	7	7	7	9	5	10	12	15	15	14	11
399	J	14	9	10	9	8	9	8	14	10	7	6	8	9	11	12	11	9	9	9	6	6	5	6	6	5	10	9	8
400	J	11	9	12	13	11	10	10	10	10	11	12	8	7	6	6	5	4	3	2	0	2	2	2	3	5	5	4	6
401	A	7	9	10	6	7	6	8	8	7	6	6	4	6	5	11	11	8	7	7	7	6	3	6	6	5	8	6	8
402	S	3	8	10	10	10	10	10	11	7	5	5	7	6	6	7	7	7	7	4	4	5	9	12	12	14	16	15	16
403	S	30	13	15	15	14	11	13	12	11	8	3	2	5	8	13	12	14	14	19	17	16	13	16	14	13	13	13	12
404	O	27	8	6	5	3	3	5	4	9	10	10	9	12	12	14	13	14	14	20	20	23	21	21	19	19	14	15	11
405	N	23	13	9	9	12	12	12	13	11	10	12	13	13	11	14	16	16	17	13	16	17	13	12	14	12	12	10	6
406	D	20	6	9	9	10	11	16	13	16	13	13	14	15	12	14	13	14	13	11	8	8	9	6	7	8	9	13	14

Table J. International Daily Magnetic Character-figures

Sunspots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
1934																													
1380	J	17	3	10	4	3	5	9	9	7	4	6	2	7	9	7	7	1	7	4	11	7	2	3	7	17	13	7	6
1381	F	13	5	1	10	17	13	11	4	5	3	5	1	8	7	2	3	5	4	12	7	15	17	12	14	5	7	10	10
1382	M	12	2	2	1	8	8	5	8	3	1	3	11	10	8	13	6	5	8	9	6	15	14	9	6	13	14	11	6
1383	A	8	3	2	1	1	2	1	2	5	10	1	0	5	6	3	6	1	4	3	2	1	2	0	2	2	13	13	7
1384	M	5	5	3	2	1	3	6	14	13	6	1	1	0	1	17	9	4	7	8	4	5	7	2	1	0	1	2	1
1385	J	1	1	0	1	6	14	13	4	8	6	1	9	12	2	7	8	5	3	9	2	3	0	0	2	1	1	0	8
1386	J	28	4	2	1	6	2	15	8	7	4	2	2	5	0	5	2	3	6	7	7	2	0	0	1	1	0	0	4
1387	J	25	3	1	1	1	8	18	11	10	7	13	11	7	6	5	4	1	3	6	10	9	7	4	3	8	7	8	1
1388	A	21	5	9	5	0	2	11	12	13	13	10	7	9	13	11	8	1	4	5	7	0	1	6	6	0	1	1	9
1389	S	17	11	2	9	8	5	9	1	15	18	9	13	8	6	11	5	4	2	5	6	6	7	0	0	0	0	7	8
1390	O	14	3	13	3	6	4	0	9	6	3	2	17	16	12	6	6	0	1	1	0	1	5	2	7	1	17	14	6
1391	N	10	3	3	2	6	4	1	1	3	5	4	0	0	0	0	12	8	2	1	7	5	0	11	8	12	18	10	2
1392	D	7	8	5	1	1	8	3	3	6	8	0	0	6	5	4	11	5	0	11	10	7	1	2	18	15	9	10	7
1935																													
1393	J	3	7	9	3	0	1	3	2	2	7	6	3	4	9	6	16	10	5	4	7	8	13	12	7	7	16	14	3
1394	J	30	6	9	13	16	11	1	3	2	7	6	3	6	1	3	17	15	9	10	5	8	2	7	12	5	7	11	8
1395	F	26	13	0	0	11	10	10	2	8	3	4	3	4	3	8	3	16	19	14	13	10	7	7	8	12	6	5	10
1396	M	25	6	3	1	0	1	13	5	1	0	2	4	2	0	0	8	13	18	19	16	13	6	5	10	6	9	5	4
1397	A	21	1	1	7	5	3	2	0	0	1	4	17	13	5	1	1	0	2	0	0	12	9	10	7	2	5	10	1
1398	M	18	6	5	14	7	5	1	0	1	4	4	2	3	7	4	1	0	2	9	8	5	18	17	16	14	13	7	6
1399	J	14	4	0	1	8	15	12	8	7	2	2	1	1	1	4	8	8	6	3	5	1	1	0	0	8	15	10	3
1400	J	11	7	3	2	9	6	4	0	1	9	4	5	11	9	14	17	6	5	2	5	0	6	6	4	0	1	6	9
1401	A	7	5	2	3	3	1	0	4	0	3	6	1	0	10	10	10	9	9	6	8	5	12	9	3	5	7	9	1
1402	S	3	4	10	7	5	8	3	7	11	19	16	1	5	10	9	11	13	10	5	1	1	16	12	17	12	9	7	6
1403	S	30	13	9	7	2	3	0	1	9	6	3	10	13	6	2	10	12	10	10	9	6	18	16	11	4	18	10	5
1404	O	27	16	12	7	10	13	6	11	9	1	14	9	3	4	4	0	9	16	13	13	2	3	1	7	9	10	4	3
1405	N	23	2	3	1	0	12	4	10	12	13	11	10	1	2	1	7	5	6	7	7	7	9	13	13	12	7	6	4
1406	D	20	4	4	1	0	6	11	13	13	14	9	7	6	2	1	1	1	2	1	1	12	10	12	8	11	12	6	2

Table I. Daily Relative Sunspot Numbers divided by 5

spots/5			0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
36																													
07	J	16	12	12	16	17	18	20	20	21	22	17	15	15	10	10	8	9	9	9	10	9	14	12	13	19	17	15	12
08	F	12	19	14	16	16	19	12	11	12	15	23	20	18	22	18	20	19	17	13	15	14	12	11	14	12	17	18	18
09	M	10	19	16	14	12	14	12	13	12	14	18	23	21	20	17	11	10	13	16	20	20	21	22	19	20	19	15	17
10	A	6	19	18	16	18	20	19	19	17	18	15	18	18	15	17	15	14	10	12	11	15	16	11	10	7	5	6	9
11	M	3	10	12	10	8	10	10	10	8	11	10	15	15	14	14	13	10	8	6	9	10	9	17	15	16	13	14	14
12	M	30	10	16	16	20	13	13	13	14	14	15	13	8	7	8	9	7	4	11	14	12	21	18	24	20	16	15	18
13	J	26	23	21	14	14	16	16	15	9	10	11	8	6	10	9	10	10	14	14	16	14	14	14	11	10	9	6	6
14	J	23	6	8	9	10	8	10	12	13	19	14	13	15	13	18	18	18	18	22	18	17	14	18	21	25	19	21	23
15	A	19	18	13	15	13	13	13	18	15	18	18	22	25	24	22	14	16	12	14	13	10	11	9	8	10	14	19	16
16	S	15	13	15	13	14	12	15	16	24	26	18	18	17	19	19	18	22	23	20	21	23	25	28	28	25	22	17	17
17	O	12	17	16	19	25	21	21	17	16	17	13	11	13	11	7	8	11	16	18	19	19	24	28	30	29	32	31	26
18	N	8	28	26	30	30	27	28	24	24	19	19	13	12	8	6	8	10	9	14	20	29	44	40	33	39	36	36	33
19	D	5	32	30	24	27	21	22	17	16	15	15	8	9	15	18	17	15	18	24	26	30	31	31	33	27	34	41	38
37																													
20	J	1	29	32	22	22	17	17	19	21	20	17	20	19	16	19	18	21	22	22	21	26	26	33	31	36	37	40	36
21	J	28	41	40	47	47	43	45	37	31	30	26	20	18	15	17	14	16	18	18	18	21	19	18	23	23	26	38	31
22	F	24	38	33	34	30	30	31	31	22	13	16	15	21	23	22	20	20	12	9	5	4	5	5	8	7	9	13	15
23	M	23	22	19	13	16	24	27	24	27	29	28	26	23	28	30	23	24	19	17	15	17	13	8	6	11	15	13	16
24	A	19	19	26	26	30	29	32	38	32	33	30	25	19	18	19	16	12	12	10	10	10	14	21	20	19	21	25	28
25	M	16	37	33	32	32	36	31	39	41	43	35	26	19	15	17	21	20	16	18	19	24	26	25	21	13	15	20	20
26	J	12	27	34	37	39	35	38	39	37	37	38	40	33	27	22	24	19	16	16	19	14	19	15	13	19	22	29	37
27	J	9	37	39	41	45	38	43	41	36	31	34	31	30	30	29	28	26	25	23	29	25	26	28	27	36	36	42	40
28	A	5	41	36	27	31	35	37	28	29	23	25	26	24	15	17	18	20	16	21	28	30	28	29	29	26	22	22	26
29	S	1	22	22	25	17	16	17	21	21	24	24	22	20	22	21	20	18	12	17	18	18	15	21	26	24	26	21	16
30	S	28	21	26	25	27	34	43	42	35	37	35	32	31	33	30	28	29	28	23	26	26	25	23	18	15	12	13	15
31	O	25	14	12	12	21	18	23	27	21	23	21	14	14	10	7	10	14	17	16	17	20	22	15	14	19	18	15	17
32	N	21	13	13	12	10	13	14	20	13	14	10	3	7	7	6	11	10	14	10	12	12	15	14	22	23	29	31	23
33	D	18	22	25	22	18	18	22	24	25	23	25	21	23	22	23	24	22	18	16	16	21	21	18	12	19	20	21	22

Table J. *International Daily Magnetic Character-figures*

1408	F	12	2	0	11	9	15	15	6	16	7	13	13	11	6	8	12	11	1	0	2	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1409	M	10	3	0	1	3	2	7	2	7	8	8	12	14	11	14	15	13	10	11	6	5	2	5	11	9	13	5	0
1410	A	6	2	5	9	3	1	4	9	11	9	12	6	11	16	15	15	17	17	14	8	6	1	3	6	1	4	6	2
1411	M	3	1	11	3	2	2	1	0	12	13	15	6	9	8	13	9	15	13	10	8	3	1	0	2	7	4	6	13
1412	M	30	11	10	12	15	8	4	2	0	2	10	18	12	8	4	6	10	10	9	4	9	19	11	2	3	0	6	2
1413	J	26	8	4	3	1	1	2	18	8	2	12	15	10	8	3	15	15	9	9	1	1	9	11	9	4	5	1	2
1414	J	23	0	1	5	2	6	8	17	7	8	7	6	5	2	8	9	0	9	7	9	1	5	3	4	4	2	2	0
1415	A	19	1	4	3	2	1	2	6	4	8	4	5	12	7	1	4	1	6	4	1	0	7	8	6	12	5	1	2
1416	S	15	3	1	2	6	2	1	4	8	9	2	1	14	8	6	10	2	5	1	1	2	9	11	11	10	12	14	5
1417	O	12	1	3	7	10	17	17	9	6	8	2	0	8	16	6	2	0	1	2	1	17	3	8	16	9	8	7	9
1418	N	8	8	10	9	16	7	0	3	12	10	9	10	9	5	3	0	0	0	1	6	1	7	20	4	8	5	7	10
1419	D	5	7	7	7	3	1	1	1	8	8	7	1	2	1	2	1	4	8	1	2	1	1	1	12	18	7	3	2
1937																													
1420	J	1	1	7	6	4	2	1	16	6	12	14	10	9	6	2	1	1	2	2	2	7	11	3	1	2	0	2	13
1421	J	28	10	7	11	2	6	7	19	12	12	11	6	4	16	12	10	11	11	10	9	8	7	10	16	10	11	8	2
1422	F	24	4	6	3	5	6	15	13	1	4	18	6	1	0	4	4	1	1	10	13	16	7	8	5	3	2	5	15
1423	M	23	9	7	4	9	17	12	6	7	19	5	14	12	7	3	4	4	1	1	1	7	15	8	1	4	1	4	8
1424	A	19	9	5	8	4	5	18	20	20	18	20	12	12	10	4	6	13	19	1	1	3	11	9	9	3	4	9	6
1425	M	16	8	3	3	6	1	7	7	3	5	12	10	13	17	14	7	6	6	6	3	7	13	16	8	6	3	9	4
1426	J	12	1	13	7	6	8	10	6	2	13	8	12	4	9	7	2	16	9	4	3	7	6	3	3	9	12	11	2
1427	J	9	11	9	10	7	4	15	8	5	4	5	15	13	8	15	12	15	12	7	2	2	1	6	4	7	18	12	15
1428	A	5	5	7	5	2	2	2	3	2	1	4	7	1	2	2	4	2	4	19	3	0	1	4	11	9	5	1	1
1429	S	1	9	2	1	5	6	5	4	4	4	13	15	2	9	9	6	8	6	9	5	2	6	5	7	8	2	6	5
1430	S	28	3	1	17	14	5	14	19	6	8	12	16	19	16	17	15	9	7	12	5	1	2	3	0	5	10	16	16
1431	O	25	11	14	12	10	4	2	3	3	7	2	1	1	1	9	11	10	3	10	8	4	1	0	0	5	15	15	12
1432	N	21	8	14	14	12	4	3	6	11	16	15	11	7	3	1	3	6	10	8	5	8	7	2	1	1	2	1	1
1433	D	18	16	15	12	5	7	17.	13	7	9	1	1	2	2	10													

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INDEX OF SUBJECTS

This index is divided into eleven parts, each associated with a one-letter or two-letter reference symbol; these symbols, and the scope and order of the divisions, are as follows:

Reference symbol	*Scope*
G	General physics and miscellaneous topics.
AN	Analysis: statistical, harmonic, and spherical harmonic.
AS	Astronomical, other than solar physics, namely geometrical particulars concerning the sun, moon, and the earth—rotation, orbital motion, time, and the seasons.
AT	Atmospheric topics, including atmospheric electricity, the ionosphere, and radio data, but not auroral topics, nor the current-systems discussed in divisions D and DV.
AU	Auroral topics.
D	Disturbance and activity of the geomagnetic field.
DV	Daily variations, solar and lunar, of the geomagnetic field.
EE	Earth-electricity (earth currents, earth potentials, earth resistivity, but not atmospheric electricity, which is included in AT).
IM	Instruments, magnetic measurements, methods and programmes of magnetic observation, and magnetic prospecting.
M	Main geomagnetic field, including its anomalies, secular variation, and analysis.
SN	Solar physical data, including sunspots and other disturbances on the sun, and the sunspot cycle.

Within each division the items (but not necessarily the sub-items) are arranged alphabetically. The more important items are subdivided; the sub-items begin with a capital letter, and in some cases are themselves subdivided: the sub-sub-items do not begin with a capital. Cross references are given in italic type. For some very general items (e.g. Magnetic, in division G) all or almost all the sub-items are thus italicized, indicating that they appear elsewhere as separate items, mostly themselves subdivided.

The letters F or P indicate that a Figure or Plate illustrates the item indexed.

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